CSE 417 Autumn 2025

Lecture 20: Non-optimal greedy

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Some interesting greedy ideas

One example from before: A* search

Pick the next point with smallest

distance from source + estimated distance to end

(computed like Dijkstra)

(via distance formula with coordinates, e.g.)

A* search *is* optimal for shortest paths, if you only ever underestimate distance.

Second example: University timetabling

Students submit to a university the courses they would like to take during the next quarter.

The university has k class blocks, and needs to determine if scheduling is possible with no students in two classes at once.

Graph representation:

- Vertices: Courses
- Edges: Courses that cannot be scheduled at the same time

Graph coloring

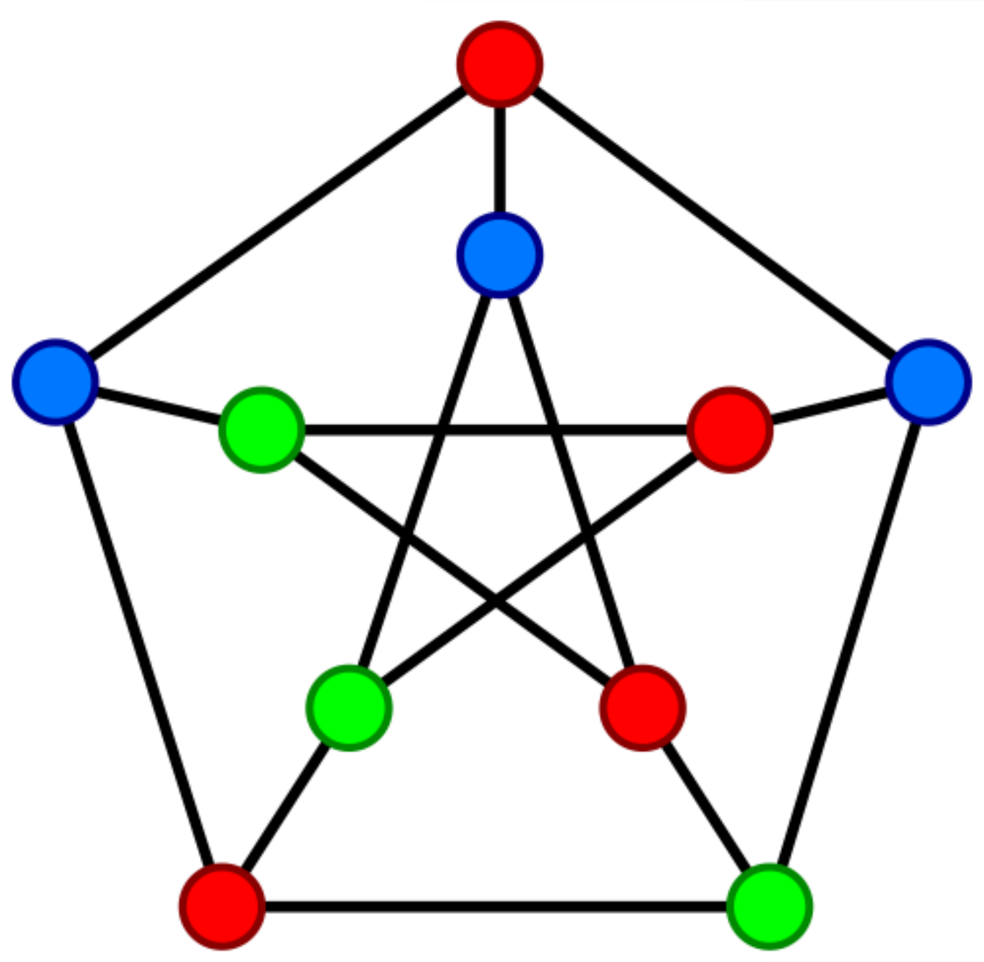
This is an instance of **graph coloring**. A valid graph coloring

- Assigns a color (timeslot) to each vertex (course)
- Such that no two adjacent vertices have the same color

Input: An undirected graph

Goal: What is the minimum number of colors needed?

Graph coloring



Why try a greedy algorithm?

Graph coloring is **hard**.

Mathematicians believe that it is impossible to solve graph coloring optimally in polynomial time: $O(n^c)$ for any c.

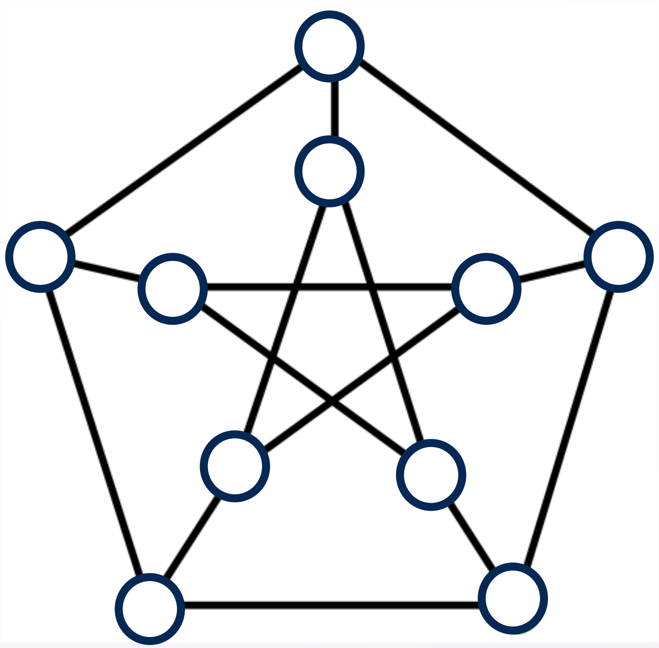
More about this later in course: graph coloring is **NP-complete**.

Easy optimal exponential time algorithm:

- For every k, check every way to color n vertices with k colors.
- But there are k^n possible colorings each iteration!

Greedy approach to graph coloring

1. Think of colors as numbers 1, 2, 3, …
2. **while** some vertex is not yet colored,
3. Color it with the smallest number that is not used by its neighbors.



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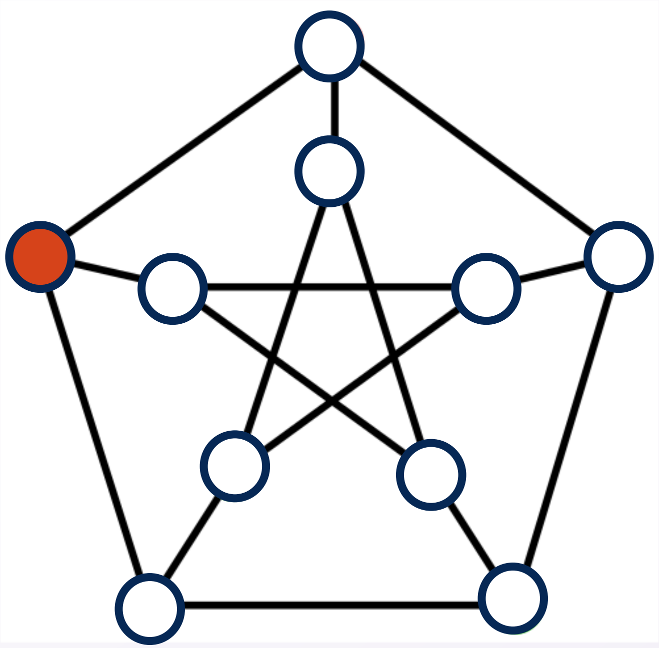

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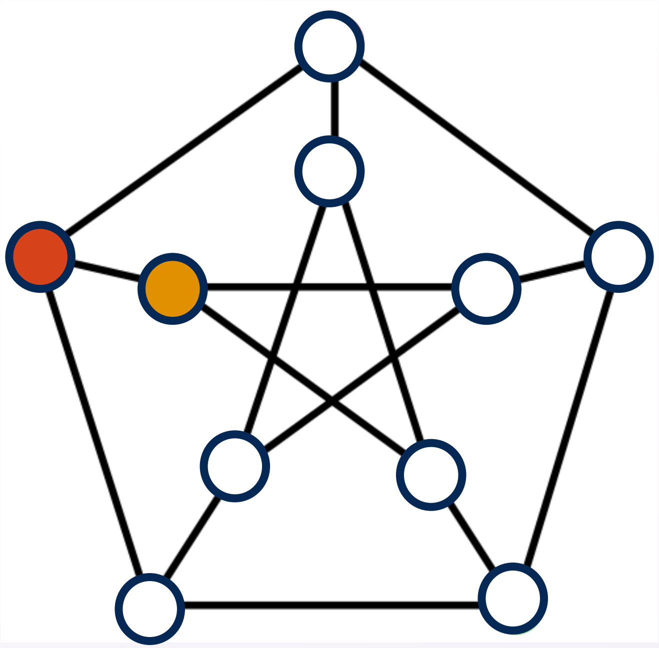

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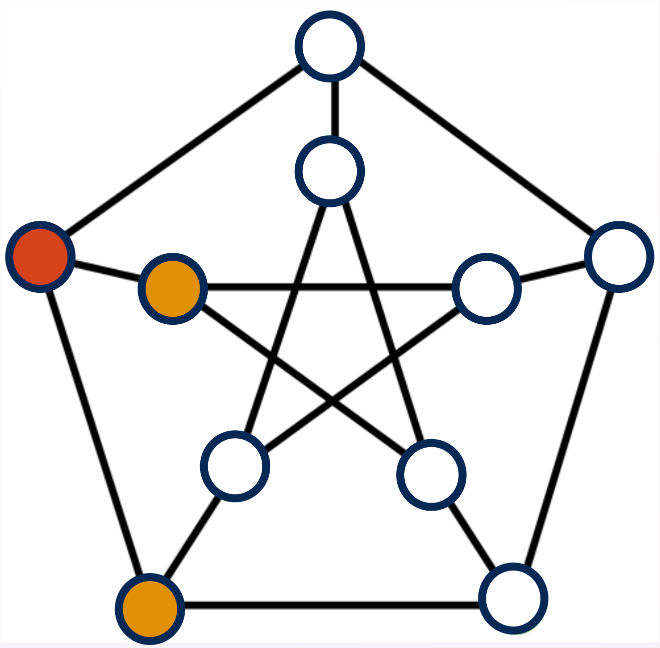

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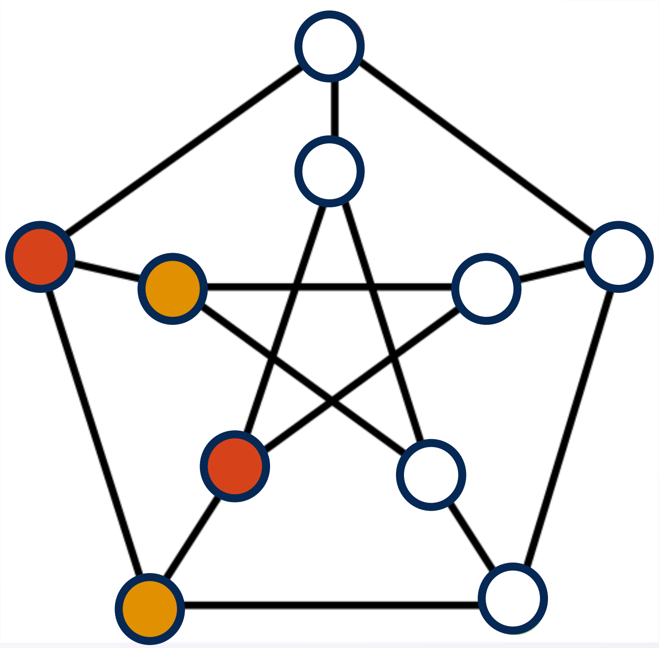

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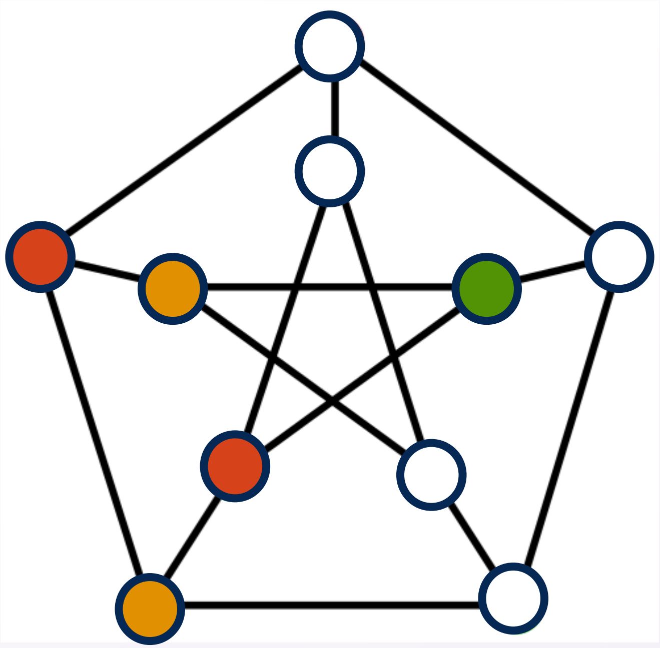

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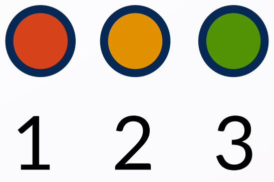

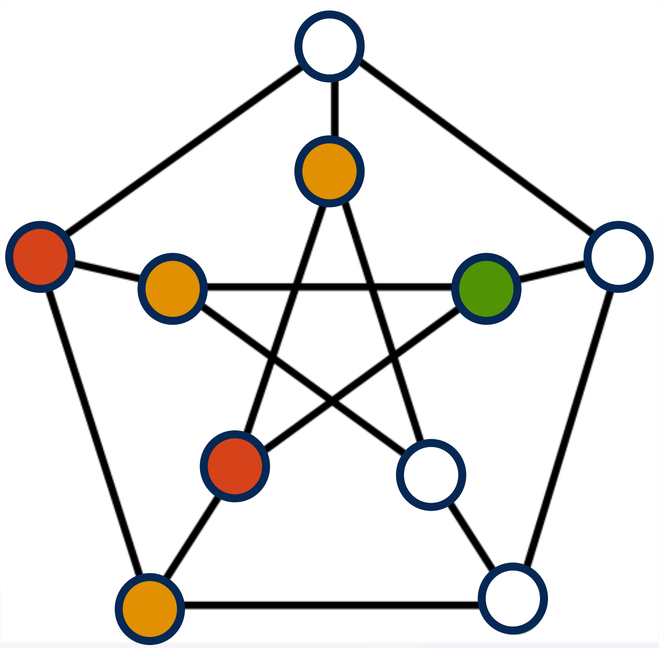

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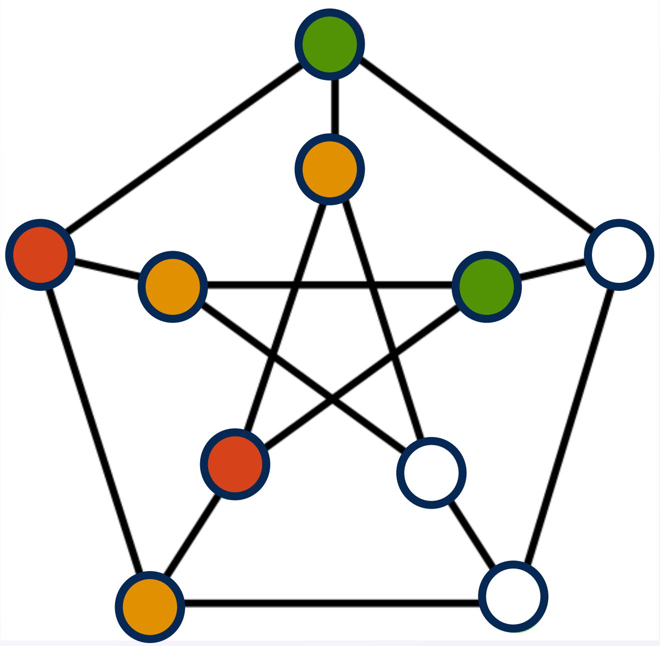

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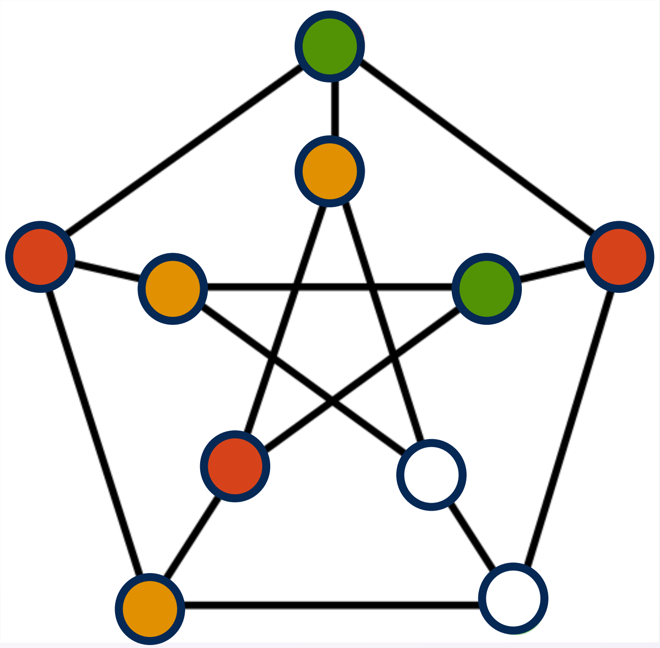

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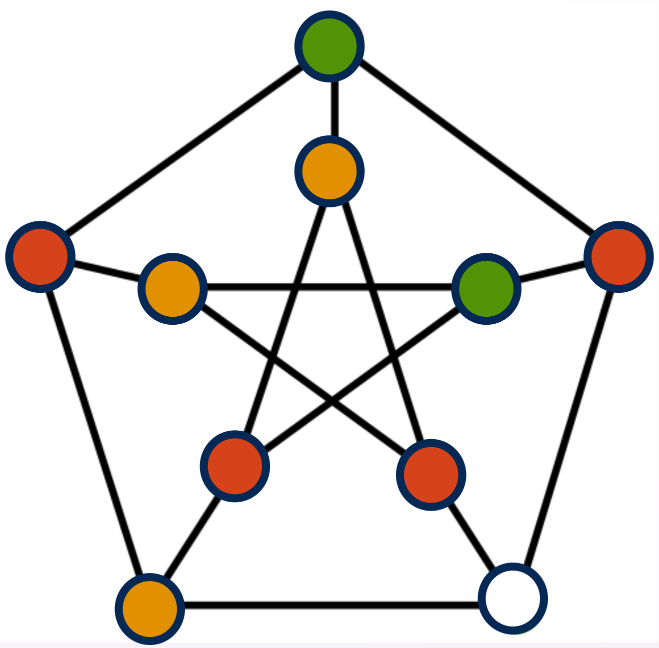

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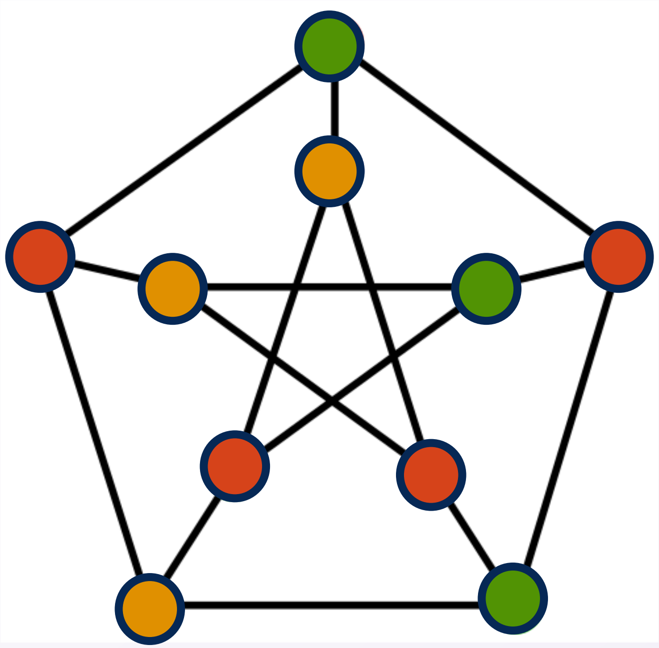
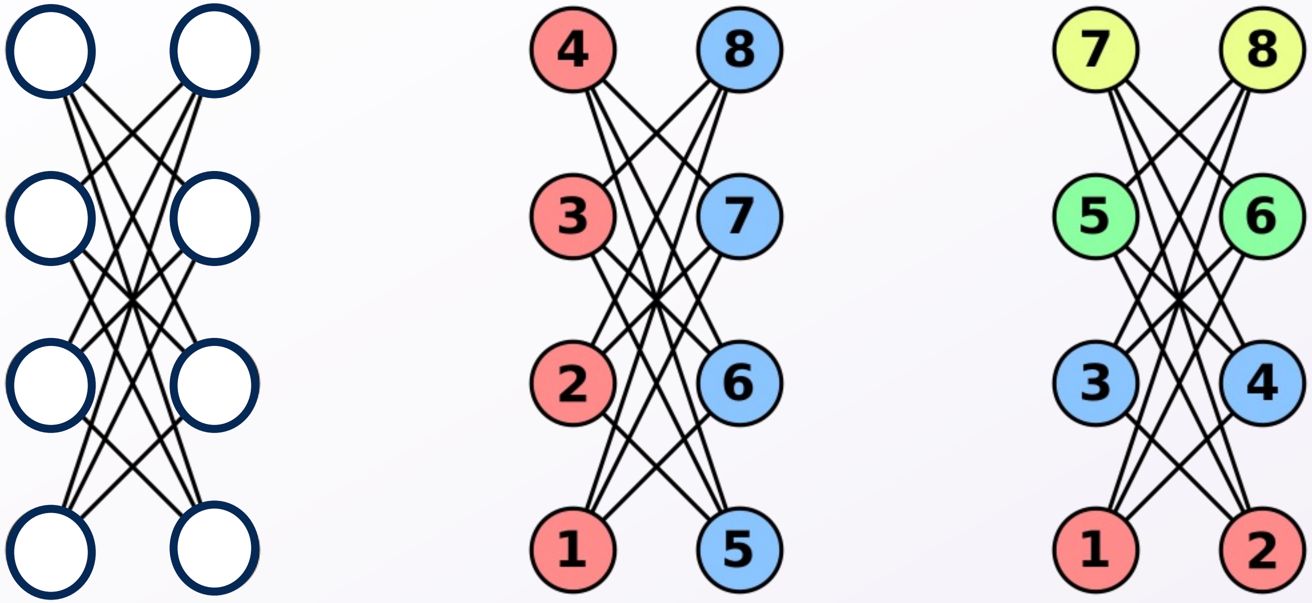


How bad can greedy be?

A **family** of counterexamples: crown graphs

- Take two set of vertices $x_1, \dots, x_n$ and $y_1, \dots, y_n$
- Connect every pair (x_i, y_j) except when $i = j$



Greedy: $n/2$ colors

Optimal: 2 colors

Gap: $n/4 = \Omega(n)$

How good is greedy?

The **degree** of a vertex is the number of neighbors it has.

Observation: The greedy algorithm uses at most 1 more color than the maximum degree of the graph!

- Because we pick the smallest number different from all neighbors.

Approximation algorithms

What is an approximation algorithm?

Suppose you are trying to solve a problem that asks you to **maximize** some quantity. On any particular input,

- Let $\boldsymbol{a}$ be the value obtained by your algorithm.
- Let $\boldsymbol{o}$ be the optimal value.

The **approximation ratio** for this particular input is $\boldsymbol{a/o}$.

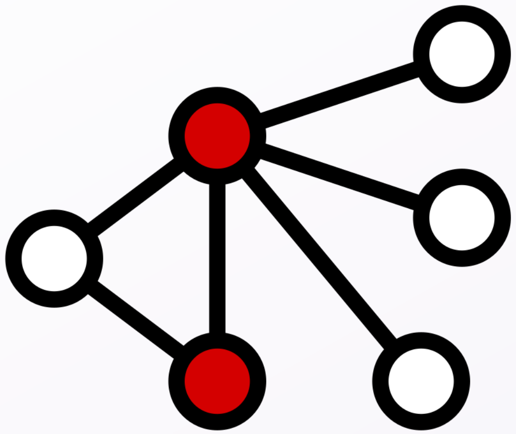
The approximation ratio for your overall algorithm is the largest $\boldsymbol{a/o}$ for any possible input.

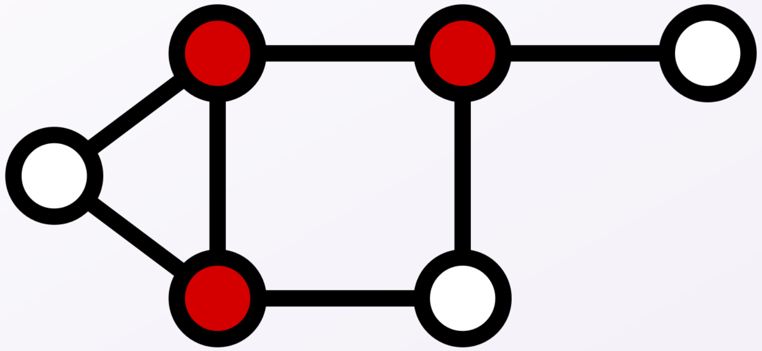
Use $\boldsymbol{o/a}$ for minimization problems.

Vertex cover

Input: An undirected graph

Goal: Select a smallest set of vertices so that every edge is covered

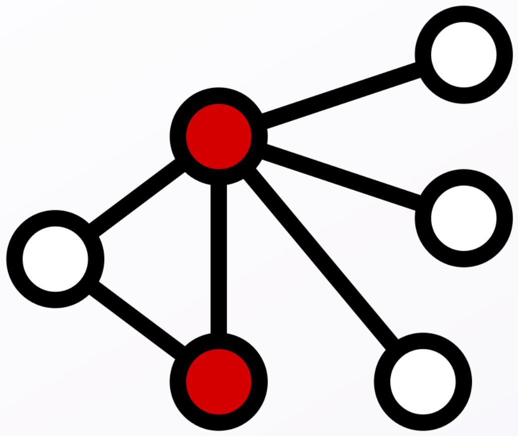




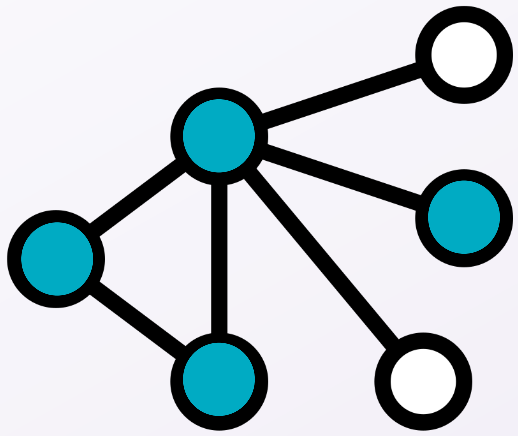
A simple greedy approach

1. **while** we don't have a vertex cover (some edge is uncovered),
2. Pick any uncovered edge and select ***both*** of its endpoints.

Optimal



Greedy



Calculating the approximation ratio

Claim. The greedy algorithm achieves an approximation ratio of 2.

- Because we picked uncovered edges, the chosen edges don’t touch each other.
- Any vertex cover must cover all the chosen edges.
- Because they don’t touch, every vertex can only cover one.
- Thus, any vertex cover must use at least half of what we used!

Load balancing

Input: There are m computers and n jobs, taking time $t_1, \dots, t_n$.

The **makespan** is the time it takes to finish on all computers.

Goal: Distribute the jobs to minimize makespan.

1:	$t_1 = 2$	$t_4 = 6$
2:	$t_2 = 3$	$t_5 = 2$
3:	$t_3 = 4$	$t_6 = 3$

A greedy approach

1. **for each** job in the order we received it,
2. Use the first available machine.

Greedy:

1:	$t_1 = 2$	$t_4 = 6$
2:	$t_2 = 3$	$t_5 = 2$
3:	$t_3 = 4$	$t_6 = 3$

Optimal:

1:	$t_4 = 6$		
2:	$t_2 = 3$	$t_5 = 2$	$t_1 = 2$
3:	$t_3 = 4$	$t_6 = 3$	

Calculating the approximation ratio

Take some time to

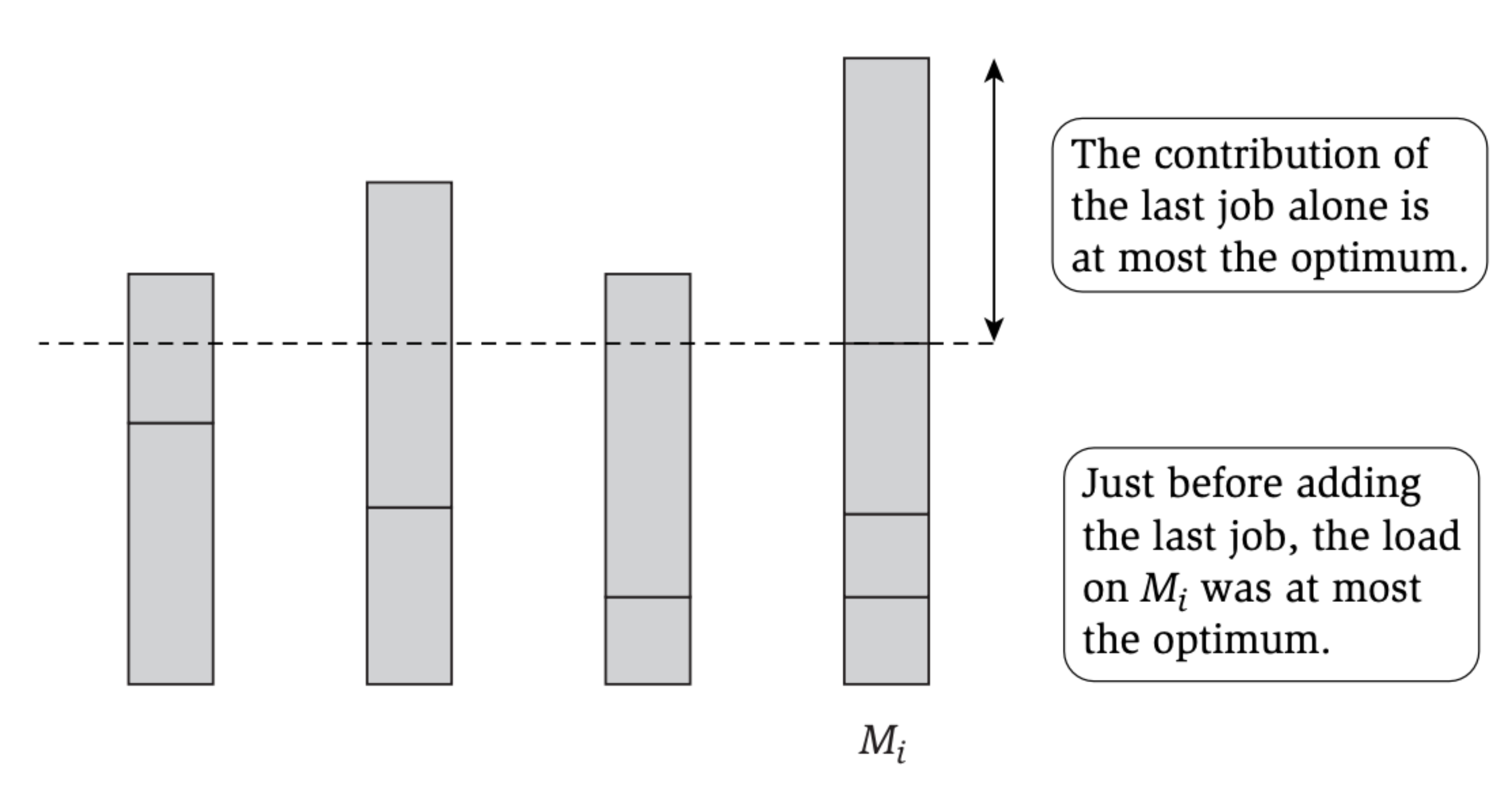
- Come up with some more examples where greedy is suboptimal, and try to make them as bad as possible.
- Think about how you might come up with an approximation ratio for this algorithm

A few observations

- If the input is a bunch of tasks of the same size, we are pretty close to optimal.
  - The optimal makespan is $\geq \frac{1}{m}(t_1 + \cdots + t_n)$.
- If the input has just one big task, we are actually optimal!
  - The optimal makespan is at least the maximum t_i.

Calculating the approximation ratio

Claim. The greedy algorithm achieves an approximation ratio of 2.



Final reminders

HW4 (Graphs) resubmissions close tonight @ 11:59pm!

I have OH now-12:30pm:

- Meet at front of classroom, we'll walk over together
- CSE (Allen) 214 if you're coming later

Nathan has online OH 12–1pm:

- https://washington.zoom.us/my/nathanbrunelle