Comparison

Sample values of k	Brute Force	Recurse by edge
3	$O(n^3(n+m))$	$O(n+m)$
$\log n$	$O\left(n^{\log n}(n+m)\right)$	$O(n(n+m))$
$n/2$	$O\left(2^{\frac{n}{2}}(n+m)\right)$	$O\left(2^{\frac{n}{2}}(n+m)\right)$
$n-3$	$O(n^3(n+m))$	$O(2^{n-3}(n+m))$

Approximation Ratio

For a minimization problem (find the shortest/smallest/least/etc.)

If $OPT(\mathrm{G})$ is the value of the best solution for G, and $ALG(G)$ is the value that your algorithm finds, then ALG is an α approximation algorithm if for every G,

$$\alpha \cdot OPT(G) \geq ALG(G)$$

i.e. you're within an α factor of the real best.

Finding an approximation for Vertex Cover

Take the idea from the clever exponential time algorithm.

But instead of checking which of u, v a good idea to add, just add them both!

```
While(G still has edges)
    Choose any edge (u,v)
    Add u to VC, and v to VC
    Delete u v and any edges touching them
EndWhile
```

So, what if the graph isn't bipartite?

Big idea:

Just round!

If $x_u \geq \frac{1}{2}$, round up to 1.

If $x_u < \frac{1}{2}$, round down to 0

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Minimize $\sum w(u) \cdot x_u$
Subject to:
$x_u + x_v \geq 1$ for all $(u, v) \in E$
$0 \leq x_u \leq 1$ for all u.

Two questions – is it a vertex cover? How far are we from the true minimum?