P (stands for "Polynomial")

The set of all decision problems that have an algorithm that runs in time $O(n^k)$ for some constant k.

NP (stands for "nondeterministic polynomial")

The set of all decision problems such that if the answer is YES, there is a proof of that which can be verified in polynomial time.

NP-hard

The problem B is NP-hard if
for all problems A in NP, A reduces to B.

NP-Complete

The problem B is NP-complete if B is in NP
and B is NP-hard

I have a problem

My problem C is hard.

So hard, it's probably NP-hard. How do I show it?

What does it mean to be NP-hard?

We need to be able to reduce any problem A to C.

Let's choose B to be a known NP-hard problem. Since B is known to be NP-hard, $A \leq B$ for every possible A. So if we show $B \leq C$ too then $A \leq B \leq C \rightarrow A \leq C$ so every NP problem reduces to C!

3-Coloring ≤ 3-SAT

Variables: is this vertex red? Blue? Green? (can't have just one variable, let's just have three).

Constraints?

If (u, v) is an edge, then u and v are different colors.

u gets exactly one color.

Correctness

If the reduction returns YES, then G was 3-colorable.