

Argument Outlines

Most common

If the correct answer is YES, then our algorithm says YES.
 And If our algorithm says YES, then the correct answer is YES

Less common but sometimes:

If our algorithm says NO, then the correct answer is NO.
 And If our algorithm says YES, then the correct answer is YES

OR

If the correct answer is YES, then our algorithm says YES.
 And If the correct answer is NO, then our algorithm says NO

Works, but rarely the best:

If our algorithm says NO, then the correct answer is NO.
 And If the correct answer is NO, then our algorithm says NO

P (stands for "Polynomial")

The set of all decision problems that have an algorithm that runs in time $O(n^k)$ for some constant k .

NP (stands for "nondeterministic polynomial")

The set of all decision problems such that if the answer is YES, there is a proof of that which can be verified in polynomial time.

NP-hard

The problem B is NP-hard if
 for all problems A in NP, A reduces to B.

NP-Complete

The problem B is NP-complete if B is in NP
 and B is NP-hard

Verify vs. Solve

A **solver** takes as input an instance (and nothing else) and determines the correct answer (yes or no?)

Given a graph, is there an s, t path of length at most k ?

Given a graph, is it 2-colorable?

A **verifier** takes as input an instance AND a proposed solution and verifies that the proposed solution is valid.

Given a graph AND a proposed path, does it connect s to t in length k ?

Given a graph AND a proposed 2-coloring (i.e. a labeling) is the labeling valid?

Some New Problems

Here are some new problems. Are they in NP?

If they're in NP, what is the "certificate" when the answer is yes?

COMPOSITE – given an integer n is it composite (i.e. not prime)?

MAX-FLOW – find a maximum flow in a graph.

VERTEX-COVER – given a graph G and an integer k , does G have a vertex cover of size at most k ?

NON-3-Color – given a graph G , is it not 3-colorable?