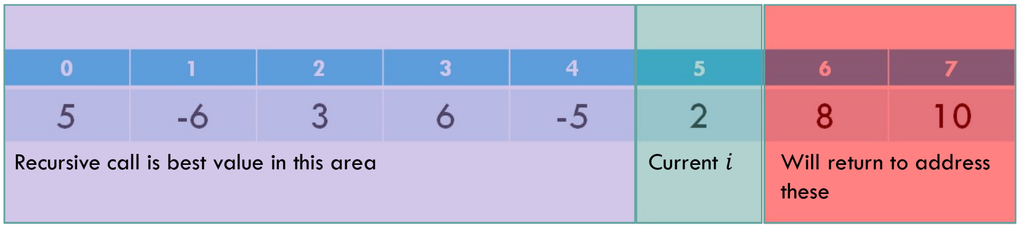
Recurrence



0	1	2	3	4	5	6	7
5	-6	3	6	-5	2	8	10

Recursive call is best value in this area

Current i

Will return to address these

Need recursive answer to the left

Currently processing i

Recursive calls to the left are needed to know optimum from $1 \ldots i$

Will move i to the right in our iterative algorithm

Longest Increasing Subsequence

$LIS(i,j)$ is "Number of elements of the maximum increasing subsequence from $0, \ldots, i$ where every element of the sequence is at most $A[j]$"

Need a recurrence

$$LIS(i,j) = \begin{cases} 0 & \text{if } i < 0 \\ \mathbb{I}[A[i] \leq A[j]] & \text{if } i = 0 \\ LIS(i-1,j) & \text{if } A[i] > A[j] \\ \max\{1 + LIS(i-1,i), LIS(i-1,j)\} & \text{otherwise} \end{cases}$$

If $A[i] > A[j]$ element i cannot be included in an increasing subsequence where every element is at most $A[j]$. So taking the largest among the first $i-1$ suffices.

If $A[i] \leq A[j]$, then if we include i, we may include elements to the left only if they are less than $A[i]$ (since $A[i]$ will now be the last, and therefore largest, of elements $0 \ldots i$. If we don't include i we want the maximum increasing subsequence among $0 \ldots i-1$.

LIS

$LIS(1,2)$ $A[1] \le A[2]$ allowed to add:
$1 + LIS(0,1)$ or $LIS(0,2)$

$$LIS(i,j) = \begin{cases} 0 & \text{if } i < 0 \\ \mathbb{I}[A[i] \le A[j]] & \text{if } i = 0 \\ LIS(i-1,j) & \text{if } A[i] > A[j] \\ \max\{1 + LIS(i-1,i), LIS(i-1,j)\} & \text{otherwise} \end{cases}$$

j

i

	0, 5	1, -6	2, 3	3, 6	4, -5	5, 2	6, 8	7, 10
0, 5	1	0	0	1	0	0	1	1
1, -6	1	1	1					
2, 3								
3, 6								
4, -5								
5, 2								
6, 8								
7, 10								

Vertex Cover

Vertex Cover

A set S of vertices is a vertex cover if for every edge (u, v): u is in S, or v is in S, (or both)

Find the minimum vertex cover in a tree.

Give every **vertex** a weight, find the minimum weight vertex cover