

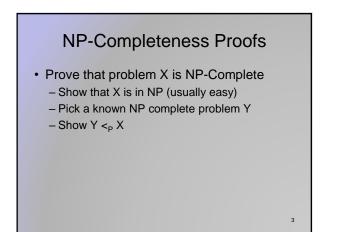
NP-Completeness and Beyond

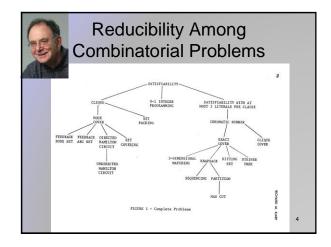
Announcements

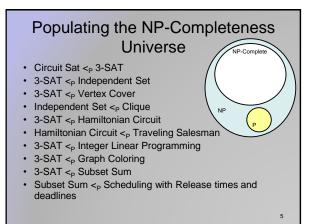
- Exam practice problems on course homepage
- Final Exam: Monday, December 9, 8:30 AM

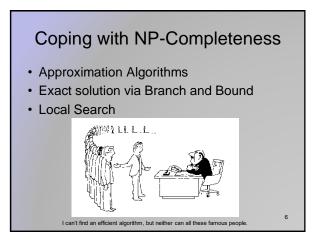
Mon, Dec 2	NP-Completeness
Wed, Dec 4	NP-Completeness
Fri, Dec 6	NP-Completeness and Beyond
Mon, Dec 9	Final Exam

This is my last lecture









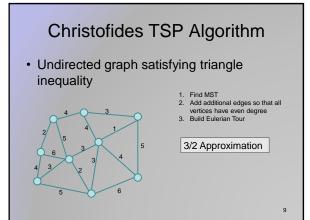
Multiprocessor Scheduling

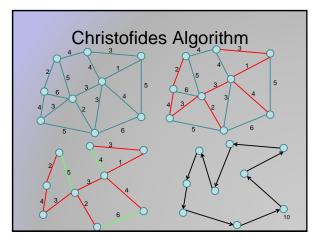
- Unit execution tasks
- Precedence graph
- K-Processors
- Polynomial time for k=2
- Open for k = constant
- NP-complete is k is part of the problem



Highest level first is 2-Optimal

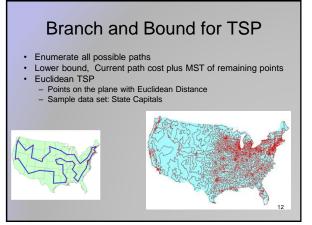
Choose k items on the highest level Claim: number of rounds is at least twice the optimal.





Branch and Bound

- Brute force search tree of all possible solutions
- Branch and bound compute a lower bound on all possible extensions
 - Prune sub-trees that cannot be better than optimal

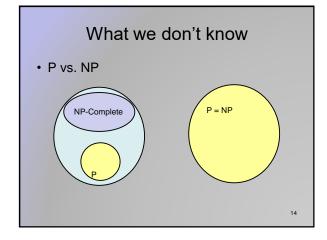


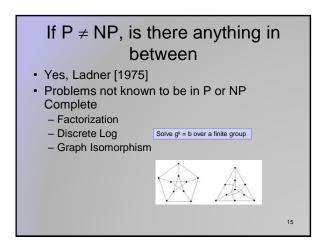
Local Optimization

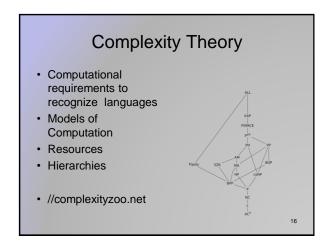
 Improve an optimization problem by local improvement

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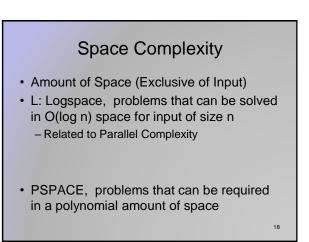
- Neighborhood structure on solutions
- Travelling Salesman 2-Opt (or K-Opt)
- Independent Set Local Replacement







Difference of the provided and the provided and

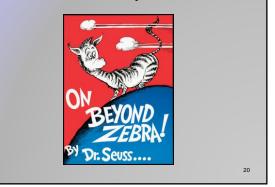


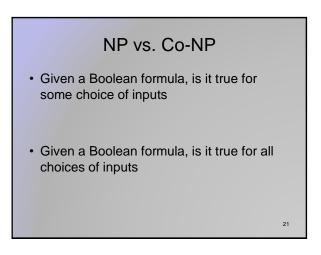
Other types of computation

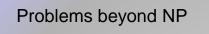
- Randomization
 - Can you solve problems faster with a random number generator?
- Quantum Computers
 - Can you solve problems faster if you have quantum bits which allow superposition?
 - Probably yes: Shor's Integer Factoring algorithm

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So what is beyond NP?





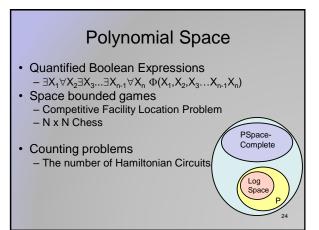


- Exact TSP, Given a graph with edge lengths and an integer K, does the minimum tour have length K
- Minimum circuit, Given a circuit C, is it true that there is no smaller circuit that computes the same function a C

Polynomial Hierarchy

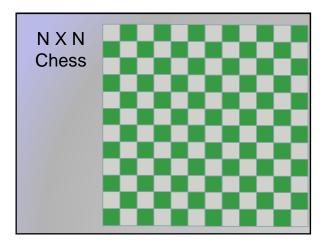
Level 1

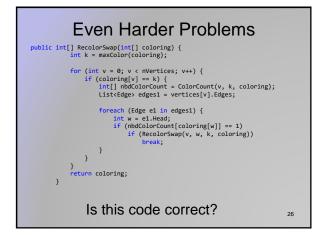
- $-\exists X_1 \Phi(X_1), \forall X_1 \Phi(X_1)$
- Level 2
 - $\forall \mathsf{X}_1 \exists \mathsf{X}_2 \ \Phi(\mathsf{X}_1, \mathsf{X}_2), \ \exists \mathsf{X}_1 \forall \mathsf{X}_2 \ \Phi(\mathsf{X}_1, \mathsf{X}_2)$
- Level 3 $- \forall X_1 \exists X_2 \forall X_3 \Phi(X_1, X_2, X_3), \exists X_1 \forall X_2 \exists X_3 \Phi(X_1, X_2, X_3)$

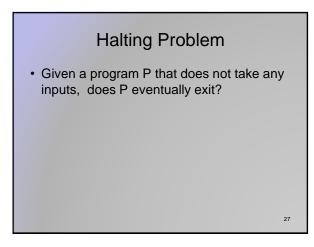


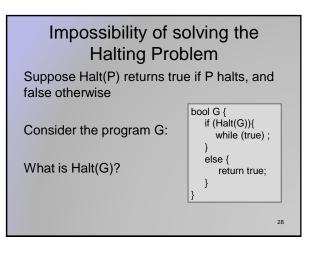
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Undecidable Problems

- The Halting Problem is undecidable
- Impossible problems are hard . . .

