

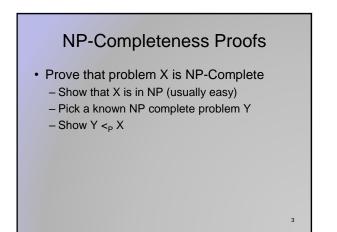
NP-Completeness and Beyond

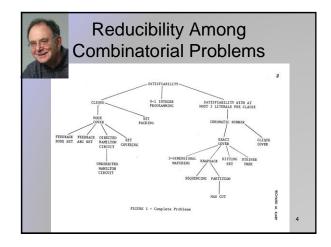
Announcements

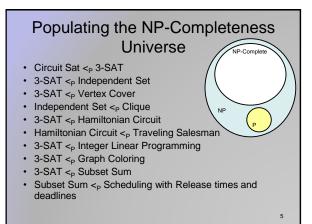
- Exam practice problems on course homepage
- Final Exam: Monday, December 9, 8:30 AM

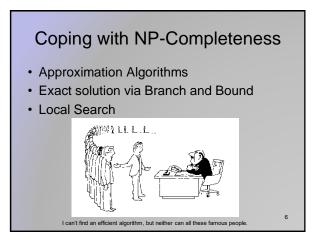
Mon, Dec 2	NP-Completeness
Wed, Dec 4	NP-Completeness
Fri, Dec 6	NP-Completeness and Beyond
Mon, Dec 9	Final Exam

This is my last lecture









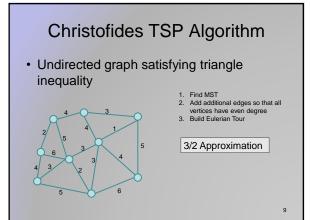
# Multiprocessor Scheduling

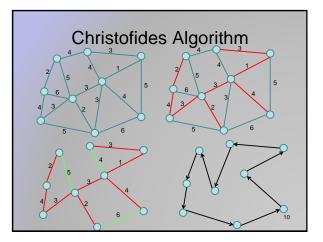
- Unit execution tasks
- Precedence graph
- K-Processors
- Polynomial time for k=2
- Open for k = constant
- NP-complete is k is part of the problem



## Highest level first is 2-Optimal

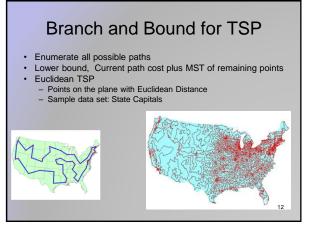
Choose k items on the highest level Claim: number of rounds is at least twice the optimal.





# Branch and Bound

- Brute force search tree of all possible solutions
- Branch and bound compute a lower bound on all possible extensions
  - Prune sub-trees that cannot be better than optimal

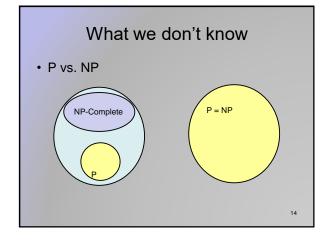


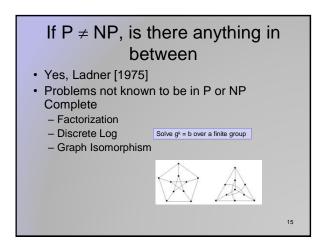
# Local Optimization

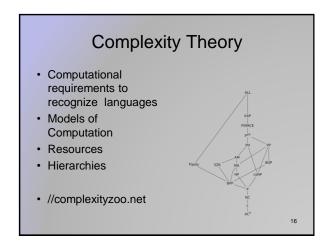
 Improve an optimization problem by local improvement

13

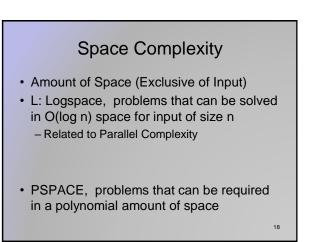
- Neighborhood structure on solutions
- Travelling Salesman 2-Opt (or K-Opt)
- Independent Set Local Replacement







# Difference of the provided and the provided and

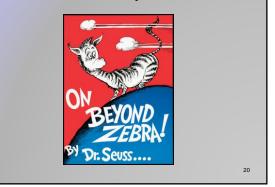


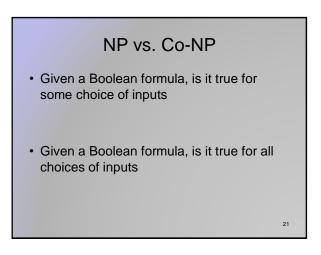
## Other types of computation

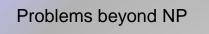
- Randomization
  - Can you solve problems faster with a random number generator?
- Quantum Computers
  - Can you solve problems faster if you have quantum bits which allow superposition?
    - Probably yes: Shor's Integer Factoring algorithm

19

## So what is beyond NP?





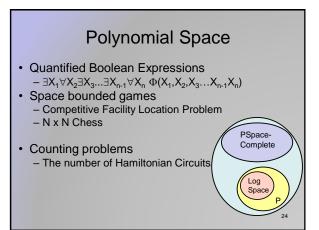


- Exact TSP, Given a graph with edge lengths and an integer K, does the minimum tour have length K
- Minimum circuit, Given a circuit C, is it true that there is no smaller circuit that computes the same function a C

## **Polynomial Hierarchy**

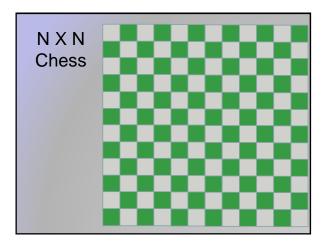
Level 1

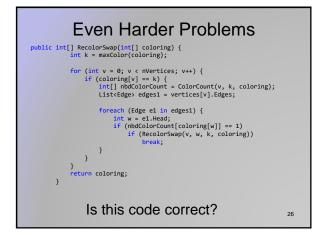
- $-\exists X_1 \Phi(X_1), \forall X_1 \Phi(X_1)$
- Level 2
  - $\forall \mathsf{X}_1 \exists \mathsf{X}_2 \ \Phi(\mathsf{X}_1, \mathsf{X}_2), \ \exists \mathsf{X}_1 \forall \mathsf{X}_2 \ \Phi(\mathsf{X}_1, \mathsf{X}_2)$
- Level 3  $- \forall X_1 \exists X_2 \forall X_3 \Phi(X_1, X_2, X_3), \exists X_1 \forall X_2 \exists X_3 \Phi(X_1, X_2, X_3)$

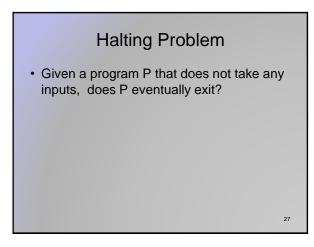


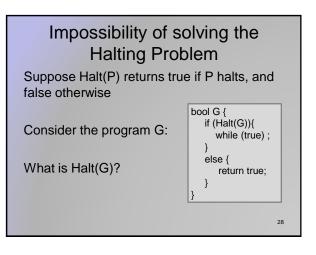
23

22









# **Undecidable Problems**

- The Halting Problem is undecidable
- Impossible problems are hard . . .

