Lecture20

CSE 417 Algorithms and Complexity

Lecture 20, Autumn 2024

Dynamic Programming, Part II

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Announcements

- Dynamic Programming Reading:
  - 6.1-6.2, Weighted Interval Scheduling
  - 6.4 Knapsack and Subset Sum
  - 6.6 String Alignment
    - 6.7* String Alignment in linear space
  - 6.8 Shortest Paths (again)
  - 6.9 Negative cost cycles
    - How to make an infinite amount of money

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Key Ideas for Dynamic Programming

- Give a recursive solution for the problem in terms of optimizing an objective function
- Order sub-problems to avoid duplicate computation
- Determine the elements that form the solution

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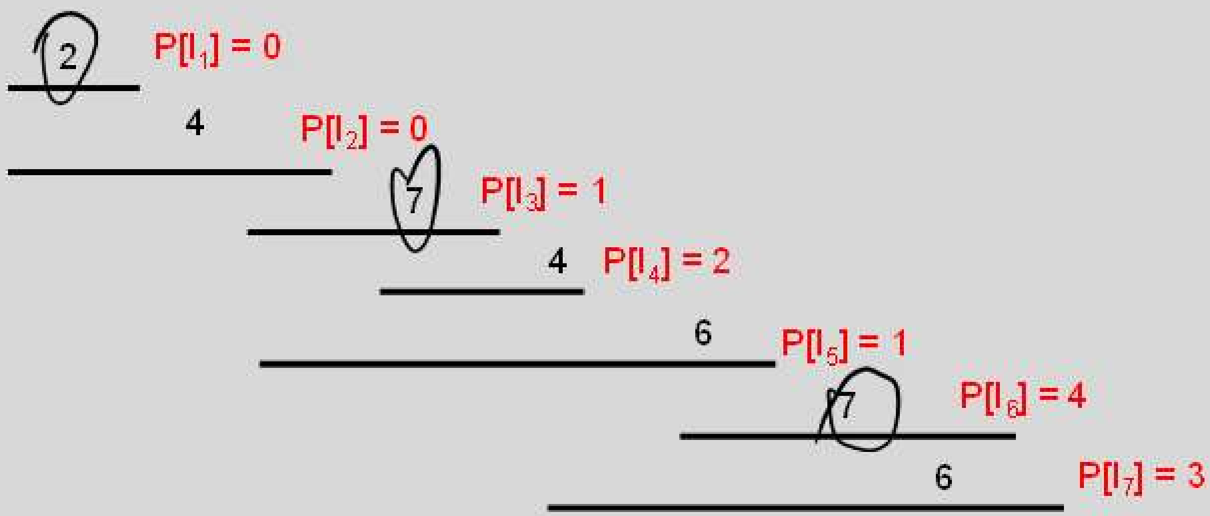
Intervals sorted by end time

Weighted Interval Scheduling

- Given a collection of intervals $I_1,\ldots,I_n$ with weights $w_1,\ldots,w_n$, choose a maximum weight set of non-overlapping intervals



Intervals sorted by end time

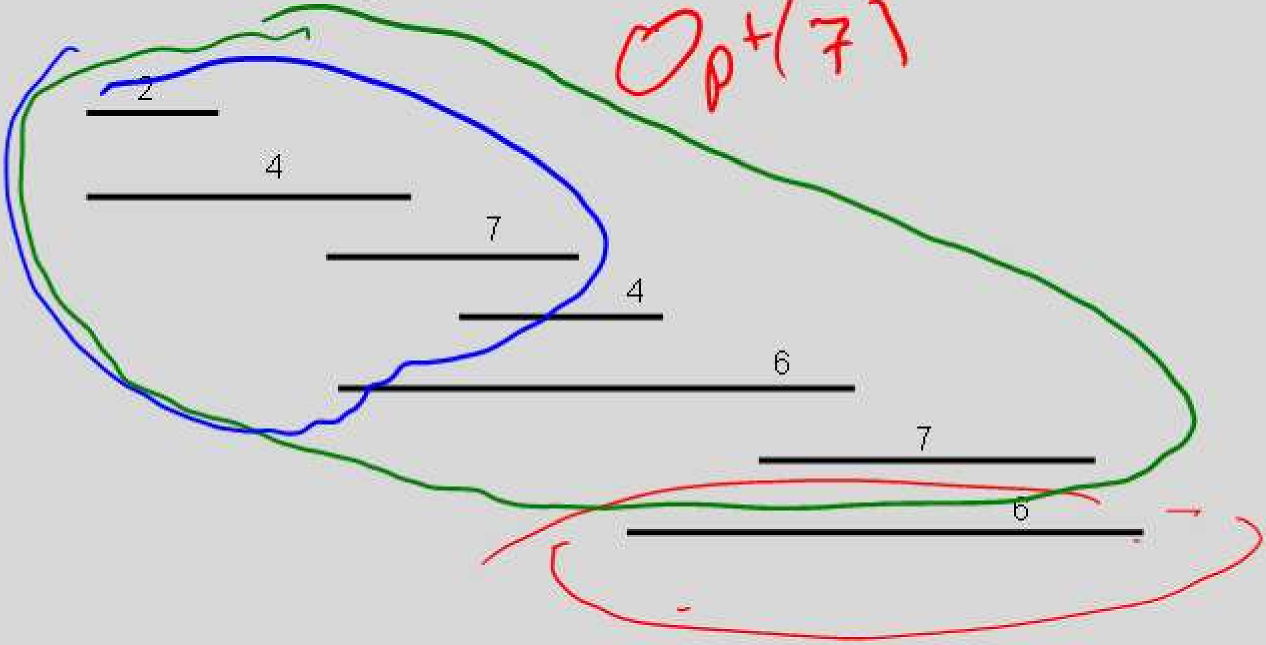
$I_1, I_2, \ldots I_n$

Recursive Solution

Opt(n)

Express the solution to a problem of size n in terms of solutions to problems of size k, where k < n

Opt(7)



Intervals sorted by end time

Optimality Condition

- Opt[j] is the maximum weight independent set of intervals I_1, I_2, . . ., I_j
- Opt[j] = max(Opt[j – 1], w_j + Opt[p[j]])
  - Where p[j] is the index of the last interval which finishes before I_j starts

- Convert to iterative algorithm to compute: Opt[1], Opt[2], Opt[3],…, Opt[n-1], Opt[n]

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Iterative Algorithm

```
MaxValue(n){
    int[ ] M = new int[n+1];
    M[0] = 0;
    for (int i = 1; i <= n; i++){
        M[ j ] = max(M[j-1], w_j + M[p[ j ]]);
    }
    return M[n];
}
```

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How many different ways can I walk to work?

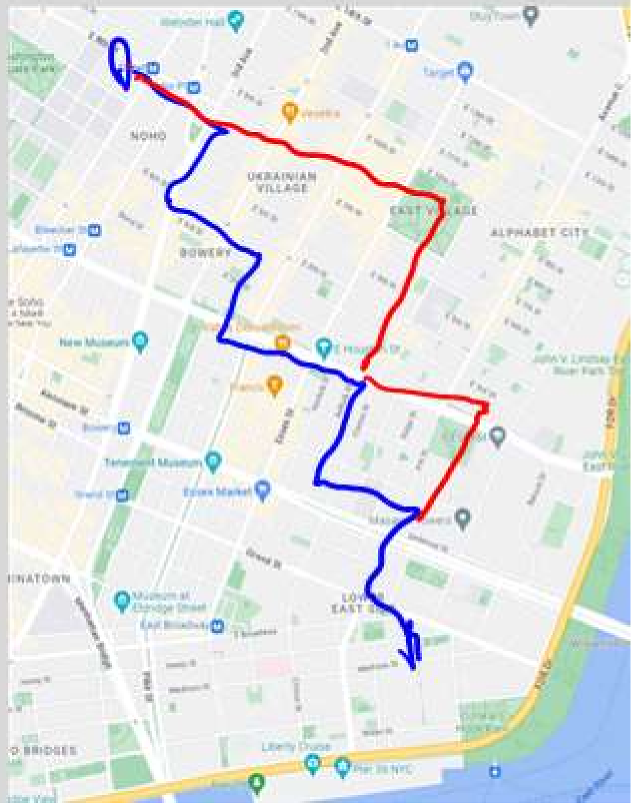
Only taking "efficient" routes

Make the problem discrete

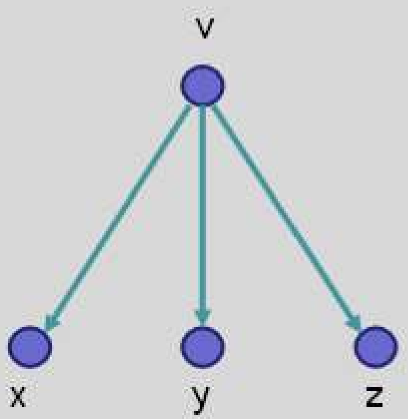
Directed Graph model:
Intersections and streets

Assume the graph is a
directed acyclic graph (DAG)

Problem: compute the number
of paths from vertex h to
vertex w



P[v]: Number of paths from v to v_0



How do you compute P[v] if you know P[x], P[y], and P[z]?

$P[v] = P[x] + P[y] + P[z]$

v_0

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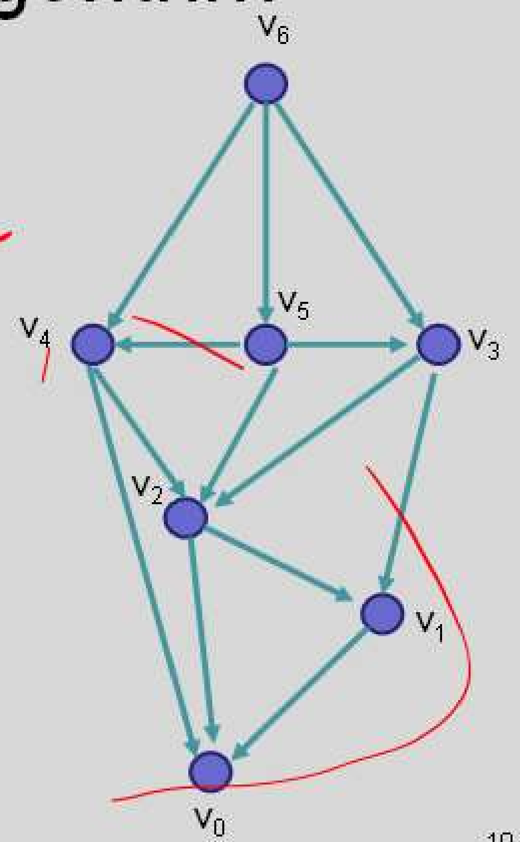
Recursive Algorithm

```
PC(v){
    if (v == v0)
        return 1;
    count = 0;
    foreach (w in N+(v)){
        count = count + PC(w);
    }
    return count;
}
```



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Ordering the vertices

How do you order the vertices of a DAG such that if there is an edge from v to w, w comes before v in the ordering?

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Path Counting

G=(V,E) is an n node directed acyclic graph, with $v_{n-1}, v_{n-2}, \ldots, v_1, v_0$ a topological order of the vertices. An array is computed giving the number of paths from each vertex to v_0.

```
CountPaths(G, P){
    P[0] = 1;
    for (i = 1 to n-1){
        P[i] = 0;
        foreach (w in N⁺(vᵢ)){
            P[i] = P[i] + P[w];
    }
}
```

Runtime

$O(n+m)$

Typesetting

- Layout text on a page to optimize readability and aesthetic measures
- Skilled profession replaced by computing
- Goal – give text a uniform appearance which is primarily done by choosing line breaks to balance white space
  - Interword spacing can stretch or shrink
  - Hyphenation is sometimes available

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Optimal line breaking

Element distinctness has been a particular focus of lower bound analysis. The first time-space tradeoff lower bounds for the problem apply to structured algorithms. Borodin et al. [13] gave a time-space tradeoff lower bound for computing ED on *comparison* branching programs of $T \in \Omega(n^{3/2}/S^{1/2})$ and, since $S \geq \log_2 n$, $T \in \Omega(n^{3/2}\sqrt{\log n}/S)$. Yao [32] improved this to a near-optimal $T \in \Omega(n^{2-\epsilon(n)}/S)$, where $\epsilon(n) = 5/(\ln n)^{1/2}$. Since these lower bounds apply to the average case for randomly ordered inputs, by Yao's lemma, they also apply to randomized comparison branching programs. These bounds also triv-ially apply to all frequency moments since, for $k \neq 1$, $ED(x) = n$ iff $F_k(x) = n$. This near-quadratic lower bound seemed to suggest that the complexity of ED and F_k should closely track that of sorting.

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$w_j, w_{j+1}, \ldots w_k$

Optimal Line Breaking

- Words have length w_i, line length L
- Penalty related to white space or overflow of the line
  - Quadratic measure often used
- Pen(i, j): Penalty for putting $w_i, w_{i+1}, \ldots, w_j$ on the same line
- Opt[m]: minimum penalty for ending a line with w_m $\quad w_1, \ldots w_m$

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The quick brown
fox jumped over
the lazy dog.

The quick brown
fox jumped
over the lazy dog.

Pen("The quick brown") = 1
Pen("fox jumped over") = 2
Pen("fox jumped") = 8
Pen("the lazy dog") = 6
Pen("over the lazy dog.") = 4

Pen(i, j): Penalty for putting $w_i, w_{i+1}, \ldots, w_j$ on the same line

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Optimal Line Breaking

Optimal score for ending a line with w_m

previous line ends at w_i

w_{m+1}

w_m

w_{m-2}

w_{m-1}, w_m

$$\text{Opt}[m] = \min_i \{ \text{Opt}[\,i\,] + \text{Pen}(i+1,m)\} \text{ for } 0 < i < m$$

Opt[m]

For words w_1, w_2, . . . , w_n, we compute Opt[n] to find the optimal layout

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Optimal Line Breaking

```
Opt[0] = 0;
for m = 1 to n {
    Find i that minimizes Opt [ i ] + Pen(i+1,m);
    Opt[m] = Opt [ i ] + Pen(i+1,m);
    Pred[m] = i;
}
```

n

x O(n)

O(n²)

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