

## Lecture19

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# CSE 417 Algorithms and Complexity

Lecture 19, Autumn 2024  
Dynamic Programming

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# Announcements

- No class Monday, November 11
- Homework 7 – Due Wednesday, November 20
- **Dynamic Programming Reading:**
  - 6.1-6.2, Weighted Interval Scheduling
  - 6.4 Knapsack and Subset Sum
  - 6.6 String Alignment
    - 6.7\* String Alignment in linear space
  - 6.8 Shortest Paths (again)
  - 6.9 Negative cost cycles
    - How to make an infinite amount of money

# Dynamic Programming

- The most important algorithmic technique covered in CSE 417
- Key ideas
  - Express solution in terms of a polynomial number of sub problems
  - Order sub problems to avoid recomputation

# Recursion vs Iteration

```
Factorial(n){  
  if (n <= 1)  
    return 1;  
  else  
    return n*Factorial(n-1);  
}
```

7!

$7! = 7 \cdot 6!$

```
Factorial(n){  
  v = 1;  
  for (i = 2; i <= n; i++)  
    v = v*i;  
  return v;  
}
```

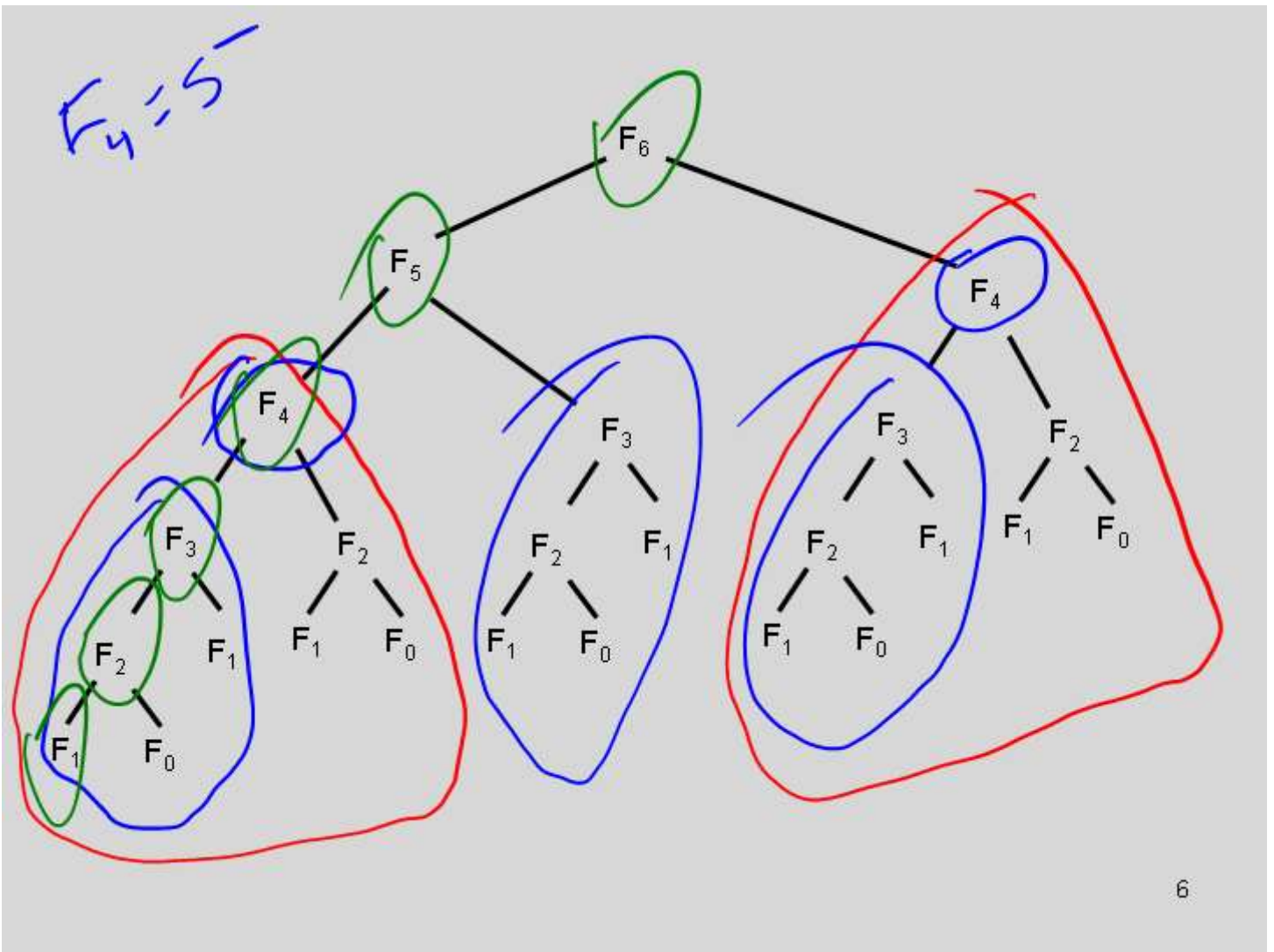
$7! = 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$

# Counting Rabbits

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, . . .

$$F_0 = 0; \quad F_1 = 1; \quad F_n = F_{n-1} + F_{n-2}$$

```
Fib(n){  
  if (n == 0)  
    return 0;  
  else if (n == 1)  
    return 1;  
  else  
    return Fib(n-1) + Fib(n-2);  
}
```



# Fibonacci with Memoization

```
Fib(n){  
  if (n == 0)  
    return 0;  
  else if (n == 1)  
    return 1;  
  else  
    return Fib(n-1) + Fib(n-2);  
}
```

$F_0$	0
$F_1$	1
2	1
3	2
4	3
5	
6	
7	

# Reordering computation

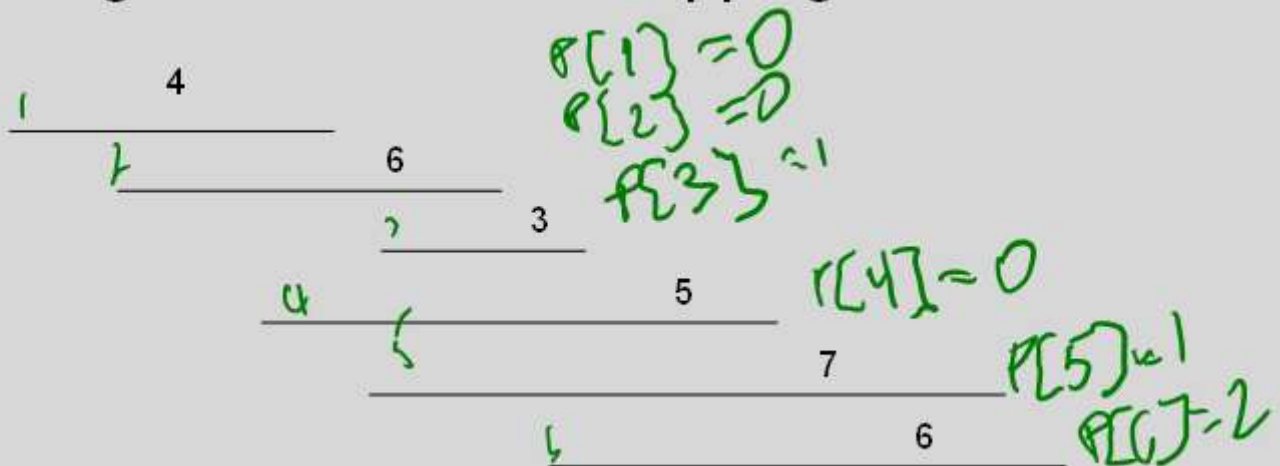
```
Fib(n){  
    int[] F = new int[n+1]  
  
    F[0] = 0;  
    F[1] = 1;  
    for (i = 2; i <= n; i++)  
        F[i] = F[i-1] + F[i-2];  
    return F[n];  
}
```



Intervals sorted by end time

# Dynamic Programming

- Weighted Interval Scheduling
- Given a collection of intervals  $I_1, \dots, I_n$  with weights  $w_1, \dots, w_n$ , choose a maximum weight set of non-overlapping intervals



Intervals sorted by end time

# Optimality Condition

- $\text{Opt}[j]$  is the maximum weight independent set of intervals  $I_1, I_2, \dots, I_j$
- $\text{Opt}[j] = \max(\text{Opt}[j-1], w_j + \text{Opt}[p[j]])$ 
  - Where  $p[j]$  is the index of the last interval which finishes before  $I_j$  starts

$$\text{Opt}[j] = \begin{cases} \text{if } I_j \text{ is used} \\ w_j + \text{Opt}[p[j]] \\ \text{if } I_j \text{ is not used} \\ \text{Opt}[j-1] \end{cases}$$

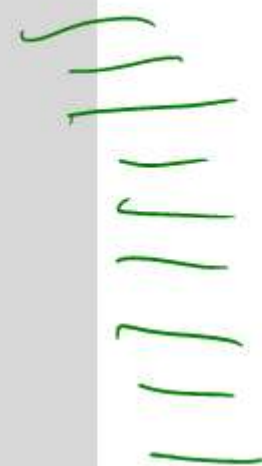
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# Algorithm

```
MaxValue(j) =  
  if j = 0 return 0  
  else  
    return max( MaxValue(j-1),  
               wj + MaxValue(p[ j ]))
```

Worst case run time:  $2^n$

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# A better algorithm

$M[j]$  initialized to -1 before the first recursive call for all  $j$

MaxValue( $j$ ) =

if  $j = 0$  return 0;

else if  $M[j] \neq -1$  return  $M[j]$ ;

else {

$M[j] = \max(\text{MaxValue}(j-1), w_j + \text{MaxValue}(p[j]));$

return  $M[j]$ ;

}

# Iterative Algorithm

```
MaxValue(n){
    int[ ] M = new int[n+1];

    M[0] = 0;

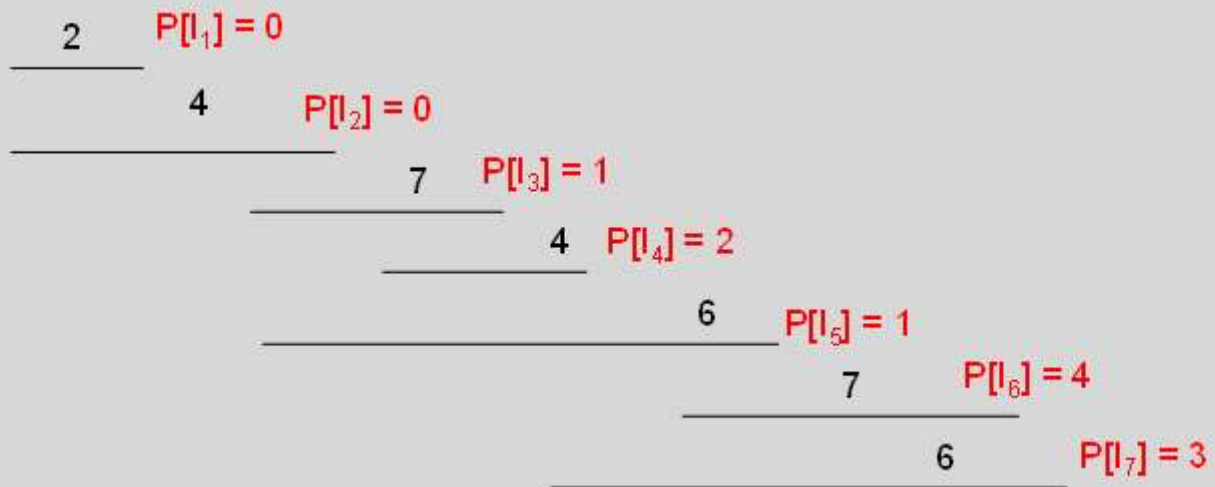
    for (int i = 1; i <= n; i++){
        M[ i ] = max(M[i-1], wi + M[p[ i ]]);
    }

    return M[n];
}
```

# Fill in the array with the Opt values

$$\text{Opt}[j] = \max(\text{Opt}[j-1], w_j + \text{Opt}[p[j]])$$

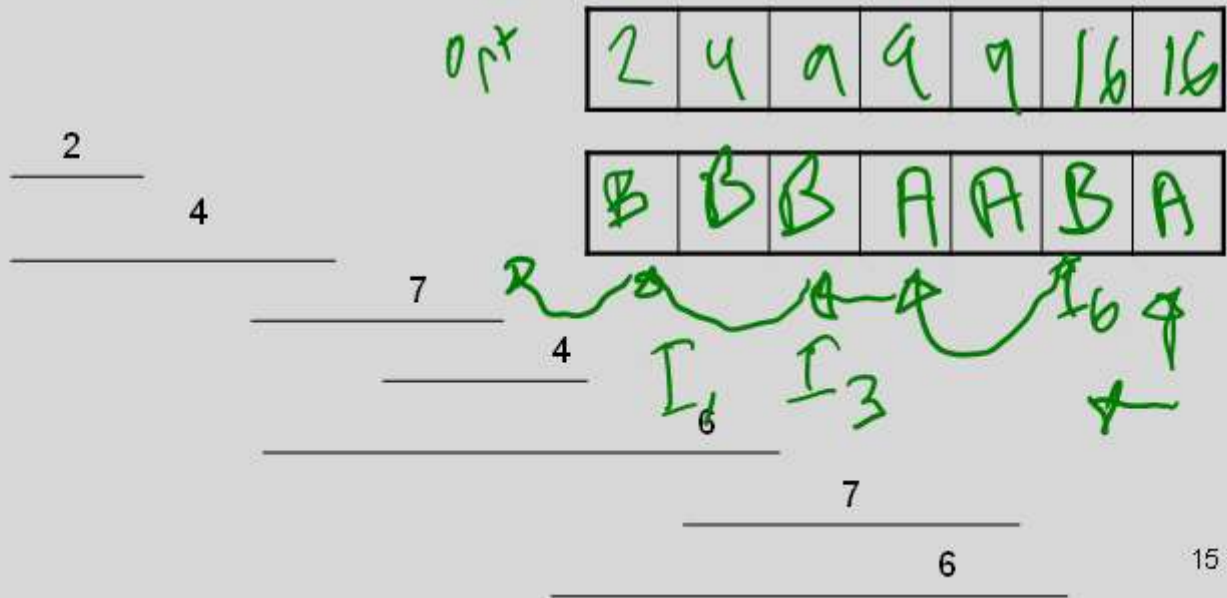
1	2	3	4	5	6	7
2	4	9	9	9	16	16



# Computing the solution

$$\text{Opt}[j] = \max(\overset{A}{\text{Opt}[j-1]}, w_j + \overset{B}{\text{Opt}[p[j]]})$$

Record which case is used in Opt computation



# Iterative Algorithm

```
int[] M = new int[n+1];
char[] R = new char[n+1];

M[0] = 0;
for (int j = 1; j < n+1; j++){
    v1 = M[j-1];
    v2 = W[j] + M[P[j]];
    if (v1 > v2) {
        M[j] = v1;
        R[j] = 'A';
    }
    else {
        M[j] = v2;
        R[j] = 'B';
    }
}
```



# Algorithm Summary

- $O(n)$  time algorithm for finding maximum weight independent set of intervals
- Key idea: Creating an Opt function to express optimal set of  $I_1, I_2, \dots, I_k$  in terms of optimal set of  $I_1, I_2, \dots, I_{k-1}$
- Organize computation to avoid recomputation