CSE 417
Algorithms and Computational Complexity

Richard Anderson

Autumn 2024

Lecture 1

CSE 417 Course Introduction

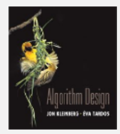
- CSE 417, Algorithms and Computational Complexity
  - MWF 10:30-11:20 AM
  - CSE2 G10
- Instructor
  - Richard Anderson, anderson@cs.washington.edu
  - Office hours:
    - Office hours: Monday 2:00-3:00 pm, Wednesday 3:00-4:00 pm, CSE2 344
- Teaching Assistants
  - Ananditha Raghunath, Kaiyuan Liu, Vinay Pritamani, Siddanth Varanasi

Announcements

- It's on the course website
  - https://courses.cs.washington.edu/courses/cse417/24au/
- Homework weekly
  - Usually due Fridays
  - HW 1, Due Friday, October 4.
  - It's on the website
- Homework is to be submitted electronically
  - Due at 11:59 pm, Fridays. Five late days.
- Edstem Discussion Board

Textbook

- Algorithm Design
- Jon Kleinberg, Eva Tardos
  - Only one edition
- Read Chapters 1 & 2
- Expected coverage:
  - Chapter 1 through 7
- Book available at:
  - Ebay ($13.62 to $229.94)
  - Amazon ($108.99/$30.60)
  - PDF



Course Mechanics

- Homework
  - Due Fridays
  - Mix of written problems and programming
  - Target: 1-week turnaround on grading
- Exams
  - Midterm, Friday, November 1
  - Final, Monday, December 9, 8:30-10:20 AM
  - Approximate grade weighting:
    - HW: 50, MT: 15, Final: 35
- Course web
  - Slides, Handouts, Discussion Board
- Canvas
  - Panopto videos

All of Computer Science is the Study of Algorithms

How to study algorithms

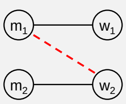
- Zoology
- Mine is faster than yours is
- Algorithmic ideas
  - Where algorithms apply
  - What makes an algorithm work
  - Algorithmic thinking
- Algorithm practice

Introductory Problem: Stable Matching

- Setting:
  - Assign TAs to Instructors
  - Avoid having TAs and Instructors wanting changes
    - E.g., Prof A. would rather have student X than her current TA, and student X would rather work for Prof A. than his current instructor.

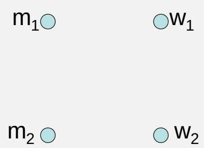
Formal notions

- Perfect matching
- Ranked preference lists
- Stability



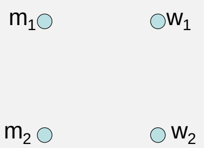
Example (1 of 3)

m_1: w_1 w_2
m_2: w_2 w_1
w_1: m_1 m_2
w_2: m_2 m_1



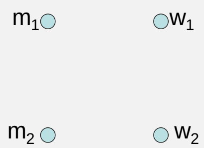
Example (2 of 3)

m_1: w_1 w_2
m_2: w_1 w_2
w_1: m_1 m_2
w_2: m_1 m_2



Example (3 of 3)

m_1: w_1 w_2
m_2: w_2 w_1
w_1: m_2 m_1
w_2: m_1 m_2



Formal Problem

- Input
  - Preference lists for $m_1, m_2, \ldots, m_n$
  - Preference lists for $w_1, w_2, \ldots, w_n$
- Output
  - Perfect matching M satisfying stability property:

If $(m', w') \in M$ and $(m'', w'') \in M$ then
(m' prefers w' to w'') or (w'' prefers m'' to m')

Idea for an Algorithm

m proposes to w
  If w is unmatched, w accepts
  If w is matched to m_2
    If w prefers m to m_2 w accepts m, dumping m_2
    If w prefers m_2 to m, w rejects m

Unmatched m proposes to the highest w on its preference list that it has not already proposed to

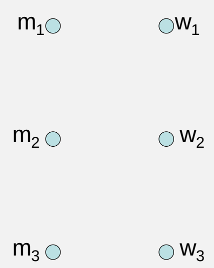
Algorithm

Initially all m in M and w in W are free
While there is a free m
  w highest on m's list that m has not proposed to
  if w is free, then match (m, w)
  else
    suppose (m_2, w) is matched
    if w prefers m to m_2
      unmatch (m_2, w)
      match (m, w)

Example

m_1: w_1 w_2 w_3
m_2: w_1 w_3 w_2
m_3: w_1 w_2 w_3

w_1: m_2 m_3 m_1
w_2: m_3 m_1 m_2
w_3: m_3 m_1 m_2



Does this work?

- Does it terminate?
- Is the result a stable matching?

- Begin by identifying invariants and measures of progress
  - m's proposals get worse (have higher m-rank)
  - Once w is matched, w stays matched
  - w's partners get better (have lower w-rank)

Claim: If an m reaches the end of its list, then all the w's are matched

Claim: The algorithm stops in at most n^2 steps

When the algorithms halts, every w is matched

Why?

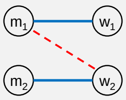
Hence, the algorithm finds a perfect matching

The resulting matching is stable

Suppose

$(m_1, w_1) \in M$, $(m_2, w_2) \in M$

m_1 prefers w_2 to w_1



How could this happen?

Result

- Simple, $O(n^2)$ algorithm to compute a stable matching
- Corollary
  - A stable matching always exists