University of Washington
Department of Computer Science and Engineering
CSE 417, Autumn 2024

October 29, 2024

Homework 5, Due Friday, November 1, 11:59 PM, 2024

Turn in instructions: Electronic submission on GradeScope. Submit as a PDF, with each problem on a separate page. While we have encouraged typeset solutions (e.g., Word or LaTex) in the homework, you may hand write solutions to these problems, as they are designed as exam problems.

It is strongly recommended that you do this assignment before the midterm!

Problem 1 Graph Theory (5 points):

a) *True or false*: Let $G = (V, E)$ be an undirected graph. If G is a tree, then G is bipartite. Justify your answer[1].

b) *True or false*: Let $G = (V, E)$ be a directed graph with n vertices and m edges. It is possible to determine if G has a cycle in $O(n + m)$ time. Justify your answer.

Problem 2 Stable Matching (5 points):

Step through the Gale-Shapely stable matching algorithm on the instance below. (You may choose the proposals in any order.) The preference lists are:

$$M = \begin{bmatrix} m_1: & w_1 & w_2 & w_3 & w_4 \\ m_2: & w_1 & w_3 & w_4 & w_2 \\ m_3: & w_2 & w_1 & w_3 & w_4 \\ m_4: & w_2 & w_1 & w_3 & w_4 \end{bmatrix}$$

$$W = \begin{bmatrix} w_1: & m_3 & m_4 & m_1 & m_2 \\ w_2: & m_1 & m_2 & m_4 & m_3 \\ w_3: & m_3 & m_4 & m_1 & m_2 \\ w_4: & m_1 & m_2 & m_3 & m_4 \end{bmatrix}$$

Fill in the following table to trace the algorithm. The first two rows are given.

[1] "Justify" means give a short and convincing explanation. Depending on the situation, justifications can involve counter examples, or cite results established in the text or in lecture.

Round	Proposal	Result	Current Matching
0			$(m_1, *), (m_2, *), (m_3, *), (m_4, *)$
1	m_1 proposes to w_1	w_1 accepts m_1	$(m_1, w_1), (m_2, *), (m_3, *), (m_4, *)$
2	m_2 proposes to w_1	w_1 rejects m_2	$(m_1, w_1), (m_2, *), (m_3, *), (m_4, *)$
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Problem 3 Shortest Cycle (5 points):

Let $G = (V, E)$ be an undirected graph. Let $e = \{u, v\}$ be an edge in G. Give an $O(n + m)$ time algorithm that finds the shortest cycle in G which contains the edge e. Explain why your algorithm is correct.

Problem 4 Connected Components (5 points):

Suppose $G = (V, E)$ is an undirected graph with n vertices and n edges. (Note: G is not allowed to have self loops or parallel edges.)

a) What is the minimum number of connected components that G can have? Justify your answer.

b) What is the maximum number of connected components that G can have? Justify your answer.

Problem 5 Interval Scheduling (5 points):

The input for an interval scheduling problem is a set of intervals $I = \{i_1, \ldots, i_n\}$ where i_k has start time s_k, and finish time f_k. The problem is to find a set of non-overlapping intervals that satisfies a given criteria.

a) Suppose that you want to maximize the total length of the selected intervals. *True or false*: The greedy algorithm based on selecting intervals in order of decreasing length finds an optimal solution. Justify your answer.

b) Suppose that all intervals have the same length, and you want to maximize the total length of the selected intervals. *True or false*: The greedy algorithm based on selecting intervals in order of increasing start time finds an optimal solution. Justify your answer.

Problem 6 Recurrences (5 points):

Solve the following recurrences by unrolling the recursion tree. Express your answers as $O(f(n))$.

a)

$$T(n) = \begin{cases} 5T(\frac{n}{3}) + n & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

b)

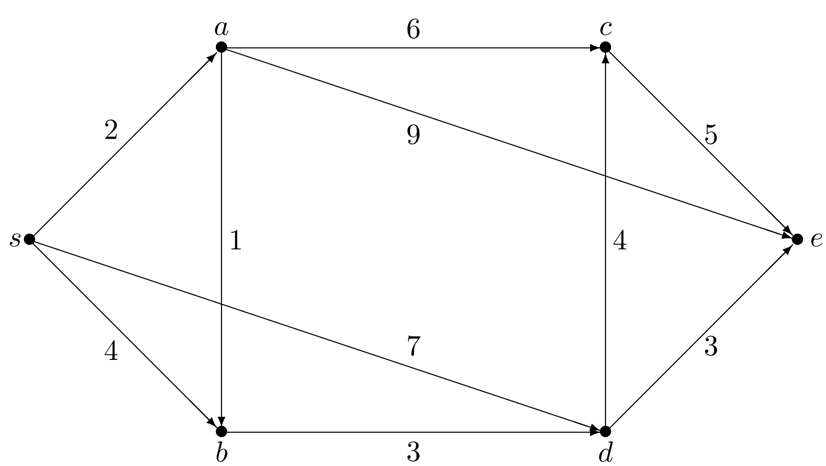
$$T(n) = \begin{cases} T(\frac{4n}{5}) + n & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

c)

$$T(n) = \begin{cases} 16T(\frac{n}{4}) + n^2 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

Problem 7 Dijkstra's Algorithm (5 points):

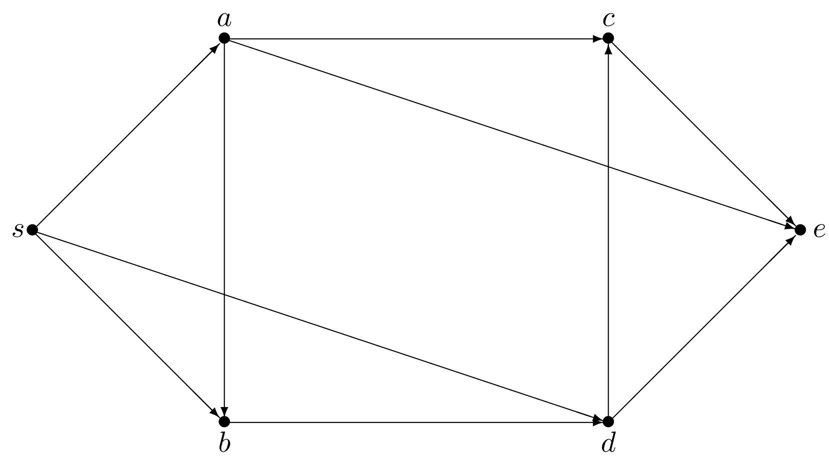
Use the following graph to simulate Dijkstra's algorithm starting from the vertex s.



a) Simulate Dijkstra's shortest path algorithm on the graph above by filling in the table. The entries should contain the preliminary distance values.

Round	Vertex	s	a	b	c	d	e
1							
2							
3							
4							
5							
6							

b) Draw the back edges found by your simulation of Dijkstra's algorithm.

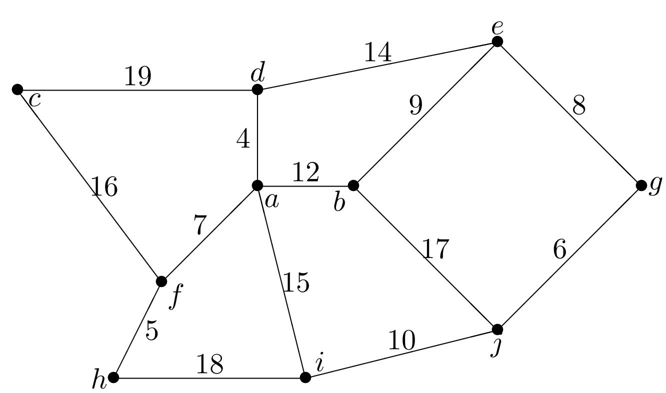


Problem 8 Minimum Spanning Trees (5 points):

Let $G = (V, E)$ be an undirected graph with edge weights, where w_i is the weight of edge e_i. Let $G' = (V', E')$ be a graph with the same sets of edges and vertices, with edge weight $w'_i = w_i + 2$. What can you say about the total weight of the minimum spanning tree for G versus the total weight of the minimum spanning tree for G'. Justify your answer.

Problem 9 Prim and Kruskal (5 points):

Consider the following undirected graph G.



a) List the edges in the order they are selected for a minimum spanning tree by Prim's algorithm. (For convenience, you can just give the edge weights.)

b) List the edges in the order they are selected for a minimum spanning tree by Kruskal's algorithm. (For convenience, you can just give the edge weights.)

Problem 10 Recurrence Solution (5 points):

Give an exact solution to the following recurrence. Show your work:

$$T(n) = \begin{cases} 2T(n-1) & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$