CSE 417
Algorithms and Complexity
Greedy Algorithms
Winter 2023
Lecture 8
Announcements

• Reading
  – For today, sections 4.1, 4.2,
  – For next week sections 4.4, 4.5, 4.7, 4.8

• Homework 3 is available
  – Random Graphs
Highlight from last lecture:
Topological Sort Algorithm

While there exists a vertex v with in-degree 0

Output vertex v

Delete the vertex v and all out going edges
Greedy Algorithms

• Solve problems with the simplest possible algorithm
• The hard part: showing that something simple actually works
• Pseudo-definition
  – An algorithm is Greedy if it builds its solution by adding elements one at a time using a simple rule
Scheduling Theory

• **Tasks**
  – Processing requirements, release times, deadlines

• **Processors**

• **Precedence constraints**

• **Objective function**
  – Jobs scheduled, lateness, total execution time
Interval Scheduling

- Tasks occur at fixed times
- Single processor
- Maximize number of tasks completed

\[
\begin{align*}
\text{s}(1) & < \text{s}(2) & < \text{s}(3) \\
\text{f}(1) & > \text{f}(2) & > \text{f}(3)
\end{align*}
\]

- Tasks \{1, 2, \ldots, N\}
- Start and finish times: \text{s}(i), \text{f}(i)
What is the largest solution?
Greedy Algorithm for Scheduling

Let T be the set of tasks, construct a set of independent tasks I, A is the rule determining the greedy algorithm

\[
I = \{ \}
\]

While (T is not empty)

- Select a task t from T by a rule A
- Add t to I
- Remove t and all tasks incompatible with t from T
Simulate the greedy algorithm for each of these heuristics

Schedule earliest starting task

Schedule shortest available task

Schedule task with fewest conflicting tasks
Greedy solution based on earliest finishing time

Example 1

Example 2

Example 3
Theorem: Earliest Finish Algorithm is Optimal

• Key idea: Earliest Finish Algorithm stays ahead

• Let \( A = \{i_1, \ldots, i_k\} \) be the set of tasks found by EFA in increasing order of finish times

• Let \( B = \{j_1, \ldots, j_m\} \) be the set of tasks found by a different algorithm in increasing order of finish times

• Show that for \( r \leq \min(k, m) \), \( f(i_r) \leq f(j_r) \)
Stay ahead lemma

• A always stays ahead of B, \( f(i_r) \leq f(j_r) \)
• Induction argument
  – \( f(i_1) \leq f(j_1) \)
  – If \( f(i_{r-1}) \leq f(j_{r-1}) \) then \( f(i_r) \leq f(j_r) \)
Completing the proof

• Let $A = \{i_1, \ldots, i_k\}$ be the set of tasks found by EFA in increasing order of finish times
• Let $O = \{j_1, \ldots, j_m\}$ be the set of tasks found by an optimal algorithm in increasing order of finish times
• If $k < m$, then the Earliest Finish Algorithm stopped before it ran out of tasks
Scheduling all intervals

- Minimize number of processors to schedule all intervals
How many processors are needed for this example?
Prove that you cannot schedule this set of intervals with two processors.

_______    _________
_______    _________  ______
__________________  ______
____________________  ______
Depth: maximum number of intervals active
Algorithm

- Sort by start times
- Suppose maximum depth is $d$, create $d$ slots
- Schedule items in increasing order, assign each item to an open slot

- Correctness proof: When we reach an item, we always have an open slot
Greedy Graph Coloring

Theorem: An undirected graph with maximum degree $K$ can be colored with $K+1$ colors
Let $k$ be the largest vertex degree
Choose $k+1$ colors

for each vertex $v$
    Color[$v$] = uncolored

for each vertex $v$
    Let $c$ be a color not used in $N[v]$
    Color[$v$] = $c$
for each vertex v
    Color[v] = uncolored

for each vertex v
    Let c be the smallest color not used in N[v]
    Color[v] = c
Scheduling tasks

- Each task has a length $t_i$ and a deadline $d_i$
- All tasks are available at the start
- One task may be worked on at a time
- All tasks must be completed

- Goal minimize maximum lateness
  - Lateness = $f_i - d_i$ if $f_i \geq d_i$
## Example

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<thead>
<tr>
<th>Time</th>
<th>Deadline</th>
<th>Lateness 1</th>
<th>Lateness 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 3</td>
<td></td>
<td>Lateness 1</td>
<td>Lateness 3</td>
</tr>
<tr>
<td>3 2</td>
<td></td>
<td>Lateness 3</td>
<td></td>
</tr>
</tbody>
</table>
Determine the minimum lateness

<table>
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<th>Deadline</th>
</tr>
</thead>
<tbody>
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