CSE 417 Algorithms and Complexity
Lecture 19, Autumn 2023
Dynamic Programming, Part II

Announcements
• Dynamic Programming Reading:
  – 6.1-6.2, Weighted Interval Scheduling
  – 6.4 Knapsack and Subset Sum
  – 6.6 String Alignment
    • 6.7* String Alignment in linear space
  – 6.8 Shortest Paths (again)
  – 6.9 Negative cost cycles
    • How to make an infinite amount of money

Key Ideas for Dynamic Programming
• Give a recursive solution for the problem in terms of optimizing an objective function
• Order sub-problems to avoid duplicate computation
• Determine the elements that form the solution

Weighted Interval Scheduling
• Given a collection of intervals I₁,…,Iₙ with weights w₁,…,wₙ, choose a maximum weight set of non-overlapping intervals

Recursive Solution
Express the solution to a problem of size n in terms of solutions to problems of size k, where k < n

Optimality Condition
• Opt[ j ] is the maximum weight independent set of intervals I₁, I₂, . . . , I₉
  – Where p[ j ] is the index of the last interval which finishes before Iₖ starts
• Convert to iterative algorithm to compute:
  Opt[1], Opt[2], Opt[3],…, Opt[n-1], Opt[n]
Iterative Algorithm

```java
MaxValue(n) {
    int[] M = new int[n+1];
    M[0] = 0;
    for (int i = 1; i <= n; i++) {
        M[i] = max(M[i-1], w[i] + M[p[i]]);
    }
    return M[n];
}
```

How many different ways can I walk to work?

Only taking “efficient” routes

Make the problem discrete

Directed Graph model: Intersections and streets

Assume the graph is a directed acyclic graph (DAG)

Problem: compute the number of paths from vertex h to vertex w

P[v]: Number of paths from v to v₀

How do you compute P[v] if you know P[x], P[y], and P[z]?

Recursive Algorithm

```java
PC(v) {
    if (v == v₀) return 1;
    count = 0;
    foreach (w in N⁺(v)) {
        count = count + PC(w);
    }
    return count;
}
```

Ordering the vertices

How do you order the vertices of a DAG such that if there is an edge from v to w, w comes before v in the ordering?

Path Counting

G=(V,E) is an n node directed acyclic graph, with Vₙ₋₁, Vₙ₋₂, ..., V₁, V₀ a topological order of the vertices. An array is computed giving the number of paths from each vertex to V₀.

CountPaths(G, P) {
    P[0] = 1;
    for (i = 1 to n-1) {
        P[i] = 0;
        foreach (w in N⁺(vᵢ)) {
            P[i] = P[i] + P[w];
        }
    }
}
Typesetting

- Layout text on a page to optimize readability and aesthetic measures
- Skilled profession replaced by computing
- Goal – give text a uniform appearance which is primarily done by choosing line breaks to balance white space
  - Interword spacing can stretch or shrink
  - Hyphenation is sometimes available

Optimal line breaking

- Element distinctness has been a particular focus of lower bound analysis. The first time-space tradeoff lower bound for the problem apply to structured algorithms. Boros et al. [13] gave a time-space tradeoff lower bound for computing $ED$ on comparison branching programs of $T \in \Omega(n^{1/2}/S^{1/2})$ and, since $S^{1/2} \leq \log_{\alpha}T$, Yao [32] improved this to a near-optimal $T \in \Omega(n^{1/2}/S^{1/2})$, where $S(n) = 5/((1/3)^{1/2}$). Since these lower bounds apply to the average case for randomly ordered inputs, by Yao's lemma, they also apply to randomized comparison branching programs. These bounds also trivially apply to all frequency moments since, for $k \neq 1$, $ED(x) = n$ if $F_k(x) = n$. This near-quadratic lower bound seemed to suggest that the complexity of $ED$ and $F_k$ should closely track that of sorting.

Optimal Line Breaking

- Words have length $w_i$, line length $L$
- Penalty related to white space or overflow of line
  - Quadratic measure often used
- $\text{Pen}(i, j)$: Penalty for putting $w_i, w_{i+1}, \ldots, w_j$ on the same line
- $\text{Opt}[m]$: minimum penalty for ending a line with $w_m$

The quick brown fox jumped over the lazy dog.
The quick brown fox jumped over the lazy dog.

Pen("The quick brown") = 1
Pen("fox jumped over") = 2
Pen("fox jumped") = 8
Pen("the lazy dog") = 6
Pen("over the lazy dog.") = 4

Pen(i, j): Penalty for putting $w_i, w_{i+1}, \ldots, w_j$ on the same line

Optimal Line Breaking

Optimal score for ending a line with $w_m$

$\text{Opt}[m] = \min_{i} \{ \text{Opt}[i] + \text{Pen}(i+1,m) \}$ for $0 < i < m$

For words $w_1, w_2, \ldots, w_n$, we compute $\text{Opt}[n]$ to find the optimal layout

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