CSE 417 Algorithms

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Autumn 2023
Lecture 4
Announcements

• Reading
  – Chapter 2.1, 2.2
  – Chapter 3 (Mostly review)
  – Start on Chapter 4
• Homework Guidelines
  – Submit homework with Gradescope
  – Describing an algorithm
    • Clarity is most important
    • Pseudocode generally preferable to just English
      – But sometimes both methods combined work best
  – Prove that your algorithm works
    • A proof is a “convincing argument”
  – Give the run time for your algorithm
    • Justify that the algorithm satisfies the runtime bound
  – You may lose points for style
  – Homework assignments will (probably) be worth the same amount
Five Problems

Scheduling
Weighted Scheduling
Bipartite Matching
Maximum Independent Set
Competitive Facility Location
Summary – Five Problems

• Scheduling
• Weighted Scheduling
• Combinatorial Optimization
• Maximum Independent Set
• Competitive Scheduling
What does it mean for an algorithm to be efficient?
Definitions of efficiency

• Fast in practice

• Qualitatively better worst case performance than a brute force algorithm
Polynomial time efficiency

• An algorithm is efficient if it has a polynomial run time
• Run time as a function of problem size
  – Run time: count number of instructions executed on an underlying model of computation
  – $T(n)$: maximum run time for all problems of size at most $n$
Polynomial Time

- Algorithms with polynomial run time have the property that increasing the problem size by a constant factor increases the run time by at most a constant factor (depending on the algorithm)
Why Polynomial Time?

• Generally, polynomial time seems to capture the algorithms which are efficient in practice

• The class of polynomial time algorithms has many good, mathematical properties
Polynomial vs. Exponential Complexity

• Suppose you have an algorithm which takes $n!$ steps on a problem of size $n$

• If the algorithm takes one second for a problem of size 10, estimate the run time for the following problems sizes:

  12  14  16  18  20
Ignoring constant factors

- Express run time as $O(f(n))$
- Emphasize algorithms with slower growth rates
- Fundamental idea in the study of algorithms
- Basis of Tarjan/Hopcroft Turing Award
Why ignore constant factors?

• Constant factors are arbitrary
  – Depend on the implementation
  – Depend on the details of the model

• Determining the constant factors is tedious and provides little insight
Why emphasize growth rates?

• The algorithm with the lower growth rate will be faster for all but a finite number of cases
• Performance is most important for larger problem size
• As memory prices continue to fall, bigger problem sizes become feasible
• Improving growth rate often requires new techniques
Formalizing growth rates

• $T(n)$ is $O(f(n))$ \hspace{1cm} [T : Z^+ \rightarrow R^+]$
  - If $n$ is sufficiently large, $T(n)$ is bounded by a constant multiple of $f(n)$
  - Exist $c, n_0$, such that for $n > n_0$, $T(n) < c f(n)$

• $T(n)$ is $O(f(n))$ will be written as:
  $T(n) = O(f(n))$
  - Be careful with this notation
Prove $3n^2 + 5n + 20$ is $O(n^2)$

Let $c =$

Let $n_0 =$

$T(n)$ is $O(f(n))$ if there exist $c$, $n_0$, such that for $n > n_0$, $T(n) < c \cdot f(n)$
Order the following functions in increasing order by their growth rate

a) \( n \log^4 n \)
b) \( 2n^2 + 10n \)
c) \( 2^{n/100} \)
d) \( 1000n + \log^8 n \)
e) \( n^{100} \)
f) \( 3^n \)
g) \( 1000 \log^{10} n \)
h) \( n^{1/2} \)
Lower bounds

- $T(n)$ is $\Omega(f(n))$
  - $T(n)$ is at least a constant multiple of $f(n)$
  - There exists an $n_0$, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$

- Warning: definitions of $\Omega$ vary

- $T(n)$ is $\Theta(f(n))$ if $T(n)$ is $O(f(n))$ and $T(n)$ is $\Omega(f(n))$
Useful Theorems

• If \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = c \) for \( c > 0 \) then \( f(n) = \Theta(g(n)) \)

• If \( f(n) \) is \( O(g(n)) \) and \( g(n) \) is \( O(h(n)) \) then \( f(n) \) is \( O(h(n)) \)

• If \( f(n) \) is \( O(h(n)) \) and \( g(n) \) is \( O(h(n)) \) then \( f(n) + g(n) \) is \( O(h(n)) \)
Ordering growth rates

• For $b > 1$ and $x > 0$
  – $\log^b n$ is $O(n^x)$

• For $r > 1$ and $d > 0$
  – $n^d$ is $O(r^n)$