

Homework 7, Due Friday, November 18, 2023

On problems one and two provide justification of your answers. Provide a clear explanation of why your algorithm solves the problem, as well as a justification of the run time. Since this assignment is from the dynamic programming section - your algorithms should use dynamic programming!

**Problem 1 (10 points) Weighted Independent Set on a Path:**

The weighted independent set problem is: Given an undirected graph  $G = (V, E)$  with weights on the vertices, find an independent set of maximum weight. A set of vertices  $I$  is independent if there are no edges between vertices in  $I$ . This problem is known to be NP-Complete.

For this problem, we restrict attention to a graph that is a path. Suppose  $P$  is a path, where the vertices are  $v_1, v_2, \dots, v_n$ , with edges between  $v_i$  and  $v_{i+1}$ . Suppose that each node  $v_i$  has an associated weight  $w_i$ . Give an algorithm that takes an  $n$  vertex path  $P$  with weights and returns an independent set of maximum total weight. The run time of the algorithm should be polynomial in  $n$  (i.e.,  $O(n^k)$  for some  $k$ ).

**Problem 2 (10 points) Task Choice:**

Suppose that each week you have the choice of a high stress task, a low stress task, or no task. If you take a high stress task in week  $i$ , you are not allowed to take any task in week  $i + 1$ . For  $n$  weeks, the high stress tasks have payoff  $h_1, \dots, h_n$ , and the low stress tasks have payoff  $l_1, \dots, l_n$ , and not doing a task has payoff 0. (You may assume that the task payoffs are all greater than zero.) Give an algorithm which given the two lists of payoffs, maximizes the value of tasks that are performed over  $n$  weeks. The run time of the algorithm should be polynomial in  $n$  (i.e.,  $O(n^k)$  for some  $k$ ).

**Programming Problem 3 (10 points):**

Leetcode 509. Fibonacci Number

For problems from LeetCode, write a program that solves the given problem in one of the languages supported by LeetCode. Run the program in LeetCode and pass the tests. (Note that you can add your own test cases, which can be very helpful in debugging.) You should submit your source code, as well as submitting a screen shot that shows the solution has been accepted.

**Programming Problem 4 (10 points):**

Leetcode 121. Best Time to Buy and Sell Stock II. (Note: this problem initially listed, Leetcode 123. Best Time to Buy and Sell Stock III, which is apparently a harder problem. Solving problem 123 will also get credit.)

**Programming Problem 5 (10 points):**

Leetcode 45. Jump Game II.