

Homework 2, Due Friday, October 13, 2023

Turnin instructions: Electronics submission on GradeScope. Submit as a PDF, with each problem on a separate page.

Problem 1 (10 points):

Order the following functions in increasing order by their growth rate:

1. n^3
2. $(\log n)^{\log n}$
3. $n^{\sqrt{\log n}}$
4. $2^{n/10}$

Explain how you determined the ordering.

Problem 2 (10 points):

The *diameter* of an undirected graph is the maximum distance between any pair of vertices. If a graph is not connected, its diameter is infinite. Let G be an n node undirected graph, where n is even. Suppose that every vertex has degree at least $n/2$. Show that G has diameter at most 2.

Problem 3 (10 points):

Let $G = (V, E)$ be an undirected graph with n vertices such that the degree of every vertex of G is at most k . Describe an algorithm to color the edges of G with at most $2k - 1$ colors such that any pair of edges e and f which are incident to the same vertex have distinct colors. Explain why your algorithm successfully colors the edges of the graph.

You should describe your algorithm using pseudo-code, which allows you to use a mix of English language statements and control structures. For example, if you were asked to color a graph with maximum degree at most k with $k + 1$ colors you could give the following pseudo-code:

```
Set all vertices to uncolored
Foreach vertex v
    Select a color for v from [1,k+1] that is not used by any of v's neighbors
```

To show that the algorithm works, you would need to argue that there is always a color available for the select statement to choose.

Programming Problem 4 (10 points):

LeetCode problem 1971. Find if Path Exists in Graph

Note: Bi-directional graph means Undirected Graph.

For problems from LeetCode, write a program that solves the given problem in one of the languages supported by LeetCode. Run the program in LeetCode and pass the tests. (Note that you can add your own test cases, which can be very helpful in debugging.) You should submit your source code, as well as submitting a screen shot that shows the solution has been accepted.

Programming Problem 5 (10 points):

LeetCode problem 785, Is Graph Bipartite.