## Approximation Ratio

For a minimization problem (find the shortest/smallest/least/etc.)

If $OPT(\mathrm{G})$ is the value of the best solution for $G$, and $ALG(G)$ is the value that your algorithm finds, then $ALG$ is an $\alpha$ approximation algorithm if for every $G$,

$$\alpha \cdot OPT(G) \geq ALG(G)$$

i.e. you're within an $\alpha$ factor of the real best.

## Finding an approximation for Vertex Cover

Take the idea from the clever exponential time algorithm.

But instead of checking which of $u, v$ a good idea to add, just add them both!

```
While(G still has edges)
    Choose any edge (u,v)
    Add u to VC, and v to VC
    Delete u v and any edges touching them
EndWhile
```

# Vertex Cover LP

Minimize $\sum w(u) \cdot x_u$

Subject to:

$x_u + x_v \geq 1$ for all $(u, v) \in E$

$0 \leq x_u \leq 1$ for all $u$.

Don't worry about the weights for today.

We got an exact solution for bipartite graphs….

# So, what if the graph isn't bipartite?

Big idea:

Just round!

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If $x_u \geq \frac{1}{2}$, round up to $1$.

If $x_u < \frac{1}{2}$, round down to $0$

Minimize $\sum w(u) \cdot x_u$
Subject to:
$x_u + x_v \geq 1$ for all $(u, v) \in E$
$0 \leq x_u \leq 1$ for all $u$.

Two questions – is it a vertex cover? How far are we from the true minimum?