Contrapositives

CSE 417 22AU
Lecture 4

Announcements

HW1 out tonight!

Due in just more than one-week, 11:59 PM on Fri. Oct. 14.

There are two types of problems:

“mechanical” usually: execute an algorithm, come up with an example of something or a very short proof.

“long-form” usually: design an algorithm, write code for an algorithm, think about an algorithm in the real world, or write a longer proof.

The directions include how many of each count. You can submit extras, we count the best ones.

Announcements

We try to keep the problems in each category approximately the same difficulty, but that isn't always possible. It's a good idea to read all of them.

Coding questions (since they're autograded) can be submitted at any time (through the last day of classes) without using a resubmission (i.e., you can think about it as having infinitely many resubmissions).

Otherwise, on each later homework, you can submit up to two old problems to be (re-)graded.

Announcements

Collaboration policy (in brief)

PLEASE collaborate!

- The types of questions in this course *really* benefit from bouncing ideas off of others.

But you must submit your own independent writeup

- That means waiting 30 minutes between discussing with others and producing your writeup.
- And not relying on notes/pictures, etc. from the discussion (more details on the webpage).
- If you can't solve it after 30 minutes, then you couldn't solve it if you get a similar problem later.

When is an implication false?

Computer scientists think of every implication as true or false.

Implications are promises – promises can be broken (or wrong), so they can be false!

The implication $p \rightarrow q$ is false when we can show the promise has been broken. That is when p is true, but q is false.

True or False?

For the purposes of this slide, Alice always carries an umbrella, Bob never carries an umbrella, and it is sunny out right now.

If it is sunny right now, then Alice has her umbrella.

If it is raining right now, then Alice has her umbrella.

If Bob has his umbrella, then it is raining right now.

If it is sunny right now, then Bob has his umbrella.

True or False?

For the purposes of this slide, Alice always carries an umbrella, Bob never carries an umbrella, and it is sunny out right now.

If it is sunny right now, then Alice has her umbrella.	True
If it is raining right now, then Alice has her umbrella.	True
If Bob has his umbrella, then it is raining right now.	True
If it is sunny right now, then Bob has his umbrella.	False

Vacuous Truth

Some of those probably felt weird.

The implication $p \to q$ is vacuously true when p is false.

It's true, but only as a "default" value – it's true precisely because we could not actually use it for anything.

Why is this the rule? See the extra slides.

p	q	$p \to q$
True	True	True
True	False	False
False	True	True
False	False	True

Contrapositives

There are two equivalent ways to write an implication

$p \to q$ and $\neg q \to \neg p$

How do I know they're equivalent?

p	q	$p \to q$
True	True	True
True	False	False
False	True	True
False	False	True

$\neg p$	$\neg q$	$\neg q \to \neg p$
False	False	
False	True	
True	False	
True	True	

Contrapositives

There are two equivalent ways to write an implication

$p \to q$ and $\neg q \to \neg p$

How do I know they're equivalent?

p	q	$p \to q$
True	True	True
True	False	False
False	True	True
False	False	True

$\neg p$	$\neg q$	$\neg q \to \neg p$
False	False	True
False	True	False
True	False	True
True	True	True

Contrapositives

To take a contrapositive, switch the “if-part” and “then-part” and negate them both.

If it is raining, then I have my umbrella.

If I do not have my umbrella, then it is not raining

Try it yourself:

If I’m on campus, then I have my Husky card.

Why take contrapositives?

Some implications are easier to prove in their contrapositive form.

Let’s practice some more direct proofs.

Even/Odd

Let a be an integer. If a^2 if even, then a is even.

$a^2 = 2\mathrm{k}$ for some integer k.

Even/Odd

Let a be an integer. If a^2 if even, then a is even.

Try taking the contrapositive and proving that instead!

Even/Odd

Let a be an integer. If a^2 if even, then a is even.

What's the contrapositive?

If a is odd, then a^2 is odd.

$a = 2k + 1$ for some integer k,

$a^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$

One more vocab word

Converse

The converse of $p \rightarrow q$ is the implication $q \rightarrow p$

The converse is not necessarily the same as the original implication!

Consider:

If it’s raining, then I have my umbrella vs. If I have my umbrella, then it’s raining.

Gale-Shapley

If r is not matched to h by Gale-Shapley, then at least one of h or r does not have the other as their first choice.

Sums

Let a, b be integers.

If $a + b \geq 15$ then $a \geq 8$ or $b \geq 8$

Optional: Extra slides

Why vacuous truth?

Why do we call vacuous implications true? Why not call them “false”? Or “neither true nor false”?

Answer 1: It’s the convention. Everyone else does it; if you try to call it something else, everyone else will be very confused.

Answer 2: Implications can be general enough to be sometimes vacuous and sometimes not. Consider
“If a number is even and prime, then it is equal to 2”
Depending on what you choose for “number” the implication might be vacuous or not!

We’d really rather not think of the implication as “sometimes true, and sometimes neither true nor false” or worse yet “sometimes true and sometimes false” – “true” is the only realistic choice.