University of Washington
Department of Computer Science and Engineering
CSE 421, Fall 2005

December 12, 2005

Final Exam, December 12, 2005

NAME: ____________________________

Instructions:

- Closed book, closed notes, no calculators
- Time limit: 1 hour 50 minutes
- Answer the problems on the exam paper.
- If you need extra space use the back of a page
- Problems are not of equal difficulty, if you get stuck on a problem, move on.

1	/15
2	/10
3	/15
4	/20
5	/25
6	/10
7	/15
8	/15
9	/15
10	/15
Total	/155

Problem 1 (15 points):

Give solutions to the following recurrences. Justify your answers.

a)

$$T(n) = \begin{cases} T(n-1) + n & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases}$$

b)

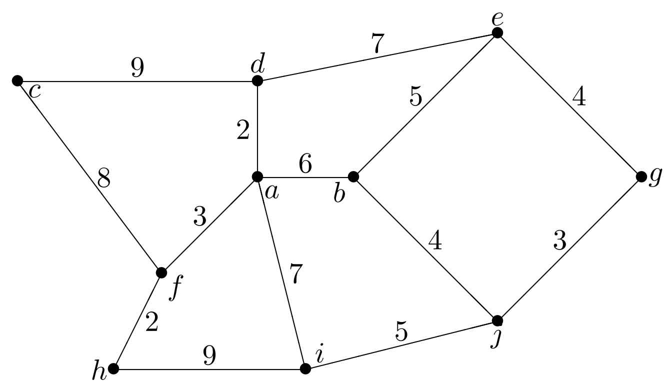
$$T(n) = \begin{cases} 17T(n/17) + n & \text{if } n > 1 \\ 0 & \text{if } n = 0 \end{cases}$$

c)

$$T(n) = \begin{cases} 3T(n/4) + n & \text{if } n > 1 \\ 0 & \text{if } n = 0 \end{cases}$$

Problem 2 (10 points):

Argue that the edge (a, b) is in a Minimum Spanning Tree for G without computing the Minimum Spanning Tree.



Problem 3 (15 points):

How can you:

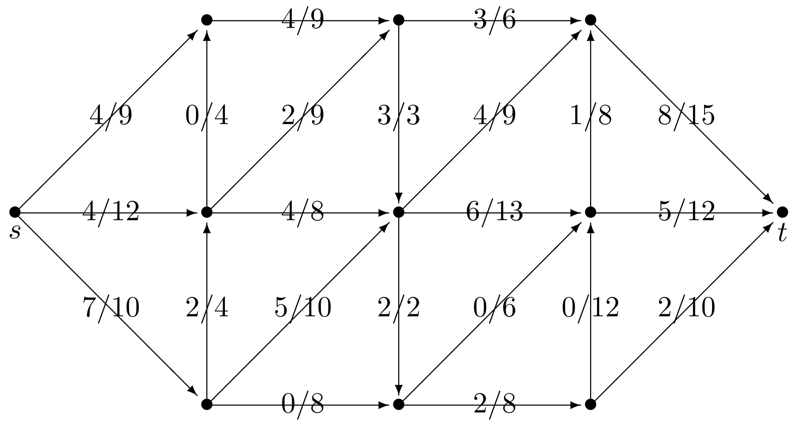
a) Find a maximum flow in an undirected graph, using an algorithm for computing maximum flow in a directed graph.

b) Determine if a directed graph has a cycle using an algorithm for Topological Sort.

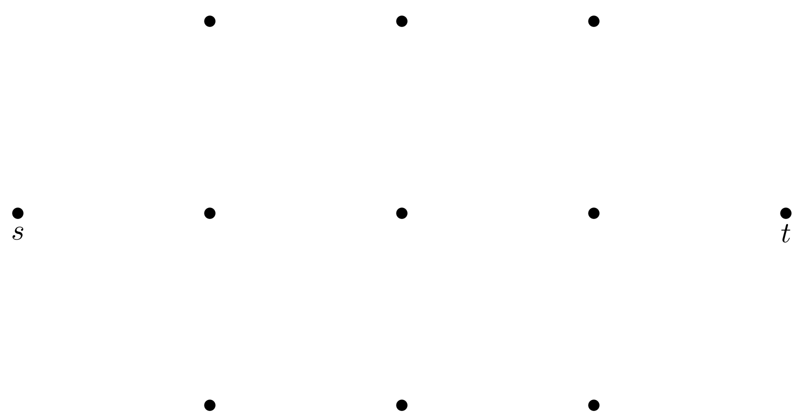
c) Determine if a undirected graph is connected, using an algorithm for finding a minimum spanning tree.

Problem 4 (20 points):

Consider the following flow graph G, with an assigned flow f. The pair x/y indicates that the edge is carrying a flow x and has capacity y.



a) Draw the residual graph G_f for the flow f.



b) Give as tight an upperbound on the number of iterations remaining as you can, assuming that the Ford-Fulkerson algorithm is being used to compute a maximum flow. Justify your answer.

Problem 5 (25 points):

What is the fastest known algorithm for each of the following problems? Give a short description or citation (no more than two sentences each). What is the run time of the fastest algorithm? (You may interpret "fastest algorithm" as "fastest algorithm discussed in class". There are theoretical results on some of these problems that were beyond the scope of the course."

1. Given a directed graph G and vertices s, and t, determine the smallest number of edges to remove to separate the graph into two pieces, one containing s, the other t.

2. Solving the single source shortest paths problem on a directed graph with n vertices and m edges.

3. Multiplying two n-bit numbers.

4. Determining if a directed graph has a negative cost cycle.

5. Finding a simple cycle containing all of the vertices of a directed graph.

Problem 6 (10 points):

Give short answers to the following questions about reductions. Make sure your conclusions are as complete as possible.

a) Suppose that you show that 3-SAT $<_P$ NETWORK-FLOW. What do you conclude?

b) Suppose that you show that REVERSE-WIFFLE-WIDGETS $<_P$ 3-SAT. What do you conclude about REVERSE-WIFFLE-WIDGETS?

Problem 7 (15 points):

Suppose you have a collection of items $i_1, \ldots, i_n$, with weights $w_1, \ldots, w_n$ and a bag of capacity W.

a) Describe a simple, efficient algorithm to select as many items as possible to fit inside the bag, e.g., the maximum cardinality set of items that have weights sum to at most W.

b) Give a concise and rigorous argument that your algorithm is correct.

Problem 8 (15 points):

Let $B = b_1, \ldots, b_n$ be a binary sequence. Give a polynomial time algorithm that finds the longest alternating subsequence $0, 1, 0, 1, 0, 1, \ldots$ of B. A subsequence does not require the digits to be consecutive. The subsequence should start with 0.

Problem 9 (15 points):

You have been placed in charge of hiring at Moogle. You have job classifications $J_1, \ldots, J_m$, and it has been determined that for $k = 1, \ldots, m$, d_k new employees need to be hired for job J_k. The HR department has idenitified candidates $C_1, \ldots, C_n$ the company is willing to hire, and for each candidate C_i has a list $l_{i1}, \ldots, l_{ij_i}$ of jobs that the candidate could fill.
Since you were hired by Moogle primarily because of your prowess in algorithms, you realize this is the perfect opportunity to demonstrate your worth to the company. Show how you can use network flow to determine if the pool of candidates can fill the available jobs, and if so, assign each candidate to a job they are qualified for. (Don't forget to show how you would actually compute the assignment of people to jobs.)

Problem 10 (15 points):

Give an algorithm to compute the longest common subsequence of three sequences. Given sequences $A = a_1, \ldots, a_n$, $B = b_1, \ldots, b_n$, and $C = c_1, \ldots, c_n$ the algorithm should find a sequence $D = d_1, \ldots, d_k$ of maximum length that is a subsequence of A, B, and C.
For this problem, it is only necessary to have your algorithm compute the length of the Longest Common Subseqence. Give a short justification of why your algorithm works.