Problem 1 (10 points):
Answer the following questions with “yes”, “no”, or “unknown, as this would resolve the P vs. NP question.” Give a brief explanation of your answer.

Define the decision version of the Interval Scheduling Problem as follows: Given a collection of intervals on a time-line, and a bound $k$, does the collection contain a subset of nonoverlapping intervals of size at least $k$?

a) Question: Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

b) Question: Is it the case that Independent Set $\leq_P$ Interval Scheduling?

Problem 2 (10 points):
Suppose that you have an $O(n^3)$ time algorithm for the Hamiltonian Circuit Problem. Prove that $P = NP$.

Problem 3 (10 points):
(Kleinberg-Tardos, Page 507, Problem 7). Since the 3-Dimensional Matching Problem is NP-complete, it is natural to expect that the corresponding 4-Dimensional Matching Problem is at least as hard. Let us define 4-Dimensional Matching as follows. Given sets $W, X, Y,$ and $Z$, each of size $n$, and a collection $C$ of ordered 4-tuples of the form $(w_i, x_j, y_k, z_l)$, do there exist $n$ 4-tuples from $C$ so that no two have an element in common? Prove that 4-Dimensional Matching is NP-Complete.

Problem 4 (10 Points):
(Kleinberg-Tardos, Page 513, Problem 17). You are given a directed graph $G = (V, E)$ with weights $w_e$ on its edges $e \in E$. The weights can be negative or positive. The Zero-Weight-Cycle Problem is to decide if there is a simple cycle in $G$ so that the sum of the edge weights on this cycle is exactly 0. Prove that the Zero-Weight-Cycle problem is NP-Complete. (Hint: Hamiltonian PATH)