CSE 417 Algorithms and Complexity

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Lecture 22

Longest Common Subsequence

Longest Common Subsequence

- $C=c_1 \ldots c_g$ is a subsequence of $A=a_1 \ldots a_m$ if C can be obtained by removing elements from A (but retaining order)
- LCS(A, B): A maximum length sequence that is a subsequence of both A and B

```
ocurranec        attacggct
occurrence       tacgacca
```

Determine the LCS of the following strings

BARTHOLEMEWSIMPSON

KRUSTYTHECLOWN

String Alignment Problem

- Align sequences with gaps

```
CAT TGA  AT
CAGAT AGGA
```

- Charge δ_x if character x is unmatched
- Charge γ_{xy} if character x is matched to character y

Note: the problem is often expressed as a minimization problem, with $\gamma_{xx} = 0$ and $\delta_x > 0$

LCS Optimization

- $A = a_1 a_2 \ldots a_m$
- $B = b_1 b_2 \ldots b_n$

- Opt[j, k] is the length of $LCS(a_1 a_2 \ldots a_j, b_1 b_2 \ldots b_k)$

Optimization recurrence

If $a_j = b_k$, Opt[j,k] = 1 + Opt[j-1, k-1]

If $a_j \mathrel{!}= b_k$, Opt[j,k] = max(Opt[j-1,k], Opt[j,k-1])

Give the Optimization Recurrence for the String Alignment Problem

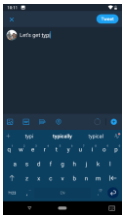
- Charge δ_x if character x is unmatched
- Charge γ_{xy} if character x is matched to character y

Opt[j, k] =

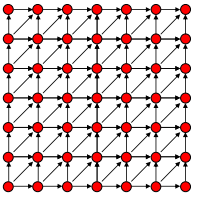
Let $a_j = x$ and $b_k = y$
Express as minimization

String edit with Typo Distance

- Find closest dictionary word to typed word
- Dist('a', 's') = 1
- Dist('a', 'u') = 6
- Capture the likelihood of mistyping characters
- Different distance model for T9 on basic mobile phone



Dynamic Programming Computation



Code to compute Opt[n, m]

```
for (int i = 0; i < n; i++)
  for (int j = 0; j < m; j++)
    if (A[ i ] == B[ j ] )
      Opt[ i,j ] = Opt[ i-1, j-1 ] + 1;
    else if (Opt[ i-1, j ] >= Opt[ i, j-1 ])
       Opt[ i, j ] := Opt[ i-1, j ];
    else
       Opt[ i, j ] := Opt[ i, j-1];
```

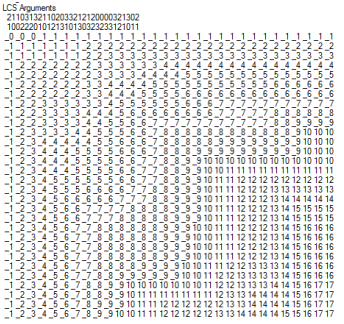
Storing the path information

```
A[1..m],  B[1..n]
for i := 1 to m     Opt[i, 0] := 0;
for j := 1 to n     Opt[0,j] := 0;
Opt[0,0] := 0;
for i := 1 to m
        for j := 1 to n
                if A[i] = B[j]  {  Opt[i,j] := 1 + Opt[i-1,j-1];  Best[i,j] := Diag; }
                else if Opt[i-1, j] >= Opt[i, j-1]
                        {  Opt[i, j] := Opt[i-1, j], Best[i,j] := Left; }
                else    {  Opt[i, j] := Opt[i, j-1], Best[i,j] := Down; }
```

$b_1 \ldots b_n$

$a_1 \ldots a_m$

Reconstructing Path from Distances



How good is this algorithm?

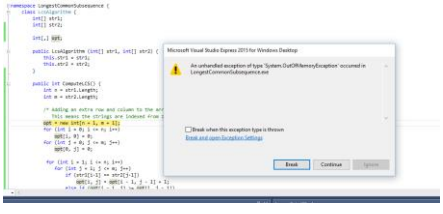
- Is it feasible to compute the LCS of two strings of length 300,000 on a standard desktop PC? Why or why not.

Implementation 1

```
public int ComputeLCS() {
            int n = str1.Length;
            int m = str2.Length;

            int[,] opt = new int[n + 1, m + 1];
            for (int i = 0; i <= n; i++)
                opt[i, 0] = 0;
            for (int j = 0; j <= m; j++)
                opt[0, j] = 0;

             for (int i = 1; i <= n; i++)
                for (int j = 1; j <= m; j++)
                    if (str1[i-1] == str2[j-1])
                        opt[i, j] = opt[i - 1, j - 1] + 1;
                    else if (opt[i - 1, j] >= opt[i, j - 1])
                        opt[i, j] = opt[i - 1, j];
                    else
                        opt[i, j] = opt[i, j - 1];

            return opt[n,m];
        }
```

N = 17000

Runtime should be about 5 seconds*



* Personal PC, 6 years old

Manufacturer:	Dell
Model:	Optiplex 990
Processor:	Intel(R) Core(TM) i5-2400 CPU @ 3.10GHz 3.10 GHz
Installed memory (RAM):	8.00 GB (7.88 GB usable)
System type:	64-bit Operating System, x64-based processor

Implementation 2

```
public int SpaceEfficientLCS() {
            int n = str1.Length;
            int m = str2.Length;
            int[] prevRow = new int[m + 1];
            int[] currRow = new int[m + 1];

            for (int j = 0; j <= m; j++)
                prevRow[j] = 0;

            for (int i = 1; i <= n; i++) {
                currRow[0] = 0;
                for (int j = 1; j <= m; j++) {
                    if (str1[i - 1] == str2[j - 1])
                        currRow[j] = prevRow[j - 1] + 1;
                    else if (prevRow[j] >= currRow[j - 1])
                        currRow[j] = prevRow[j];
                    else
                        currRow[j] = currRow[j - 1];
                }
                for (int j = 1; j <= m; j++)
                    prevRow[j] = currRow[j];
            }

            return currRow[m];
        }
```

N = 300000

N: 10000 Base 2 Length: 8096 Gamma: 0.8096 Runtime:00:00:01.86
N: 20000 Base 2 Length: 16231 Gamma: 0.81155 Runtime:00:00:07.45
N: 30000 Base 2 Length: 24317 Gamma: 0.8105667 Runtime:00:00:16.82
N: 40000 Base 2 Length: 32510 Gamma: 0.81275 Runtime:00:00:29.84
N: 50000 Base 2 Length: 40563 Gamma: 0.81126 Runtime:00:00:46.78
N: 60000 Base 2 Length: 48700 Gamma: 0.8116667 Runtime:00:01:08.06
N: 70000 Base 2 Length: 56824 Gamma: 0.8117715 Runtime:00:01:33.36

N: 300000 Base 2 Length: 243605 Gamma: 0.8120167 Runtime:00:28:07.32

Observations about the Algorithm

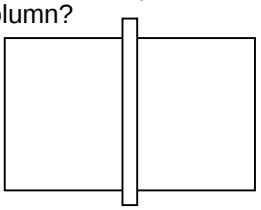
- The computation can be done in O(m+n) space if we only need one column of the Opt values or Best Values
- The computation requires O(nm) space if we store all of the string information

Computing LCS in O(nm) time and O(n+m) space

- Divide and conquer algorithm
- Recomputing values used to save space
- Section 6.7 of the text, but we will not have time to cover in detail (so you are not responsible for section 6.7)

Divide and Conquer Algorithm

- Where does the best path cross the middle column?



- For a fixed i, and for each j, compute the LCS that has a_i matched with b_j

Algorithm Analysis

- T(m,n) = T(m/2, j) + T(m/2, n-j) + cnm
- Solution: T(m,n) <= 2cnm