University of Washington
Department of Computer Science and Engineering
CSE 417, Autumn 2020

October 7, 2020

Homework 2, Due Wednesday, October 14, 1:30 pm, 2020

Turnin instructions: Electronics submission on GradeScope. Submit as a PDF, with each problem on a separate page.

Problem 1 (10 points):

Order the following functions in increasing order by their growth rate:

1. n^3
2. $(\log n)^{\log n}$
3. $n^{\sqrt{\log n}}$
4. $2^{n/10}$

Explain how you determined the ordering.

Problem 2 (10 points):

Prove that $2n^2 + 4n \log n + 6n + 20 \log^2 n + 11$ is $O(n^2)$.

Problem 3 (10 points):

The *diameter* of an undirected graph is the maximum distance between any pair of vertices. If a graph is not connected, its diameter is infinite. Let G be an n node undirected graph, where n is even. Suppose that every vertex has degree at least $n/2$. Show that G has diameter at most 2.

Problem 4 (10 points):

Let $G = (V, E)$ be an undirected graph with n vertices such that the degree of every vertex of G is at most k. Give an algorithm to color the edges of G with at most $2k - 1$ colors such that any pair of edges e and f which are incident to the same vertex have distinct colors. Explain why your algorithm successfully colors the edges of the graph.

Programming Problem 5 (10 points):

The *Coupon Collector* problem is: There are n types of coupons. Each time you get a coupon, you are given a coupon of a random type (with equal probability of receiving each coupon). The question is how many coupons do you expect to receive, on the average, before you have collected the full set of coupons.

Your programming assignment is to write a simulator of the Coupon Collector Problem, and run simulations to see how long how many coupons are needed to complete the set. You should run your program for values of n up to $4,000$. Determine the average number of coupons required to complete the set. How does this relate to the results of programming problem 5 of homework 1? (Consider both the total number of coupons C, as well as C/n.)

Programming Problem 6 (10 points):

We now consider a variation of the Coupon Collector Problem where there are n types of coupons, and each coupon has a value associated with it. The value of a coupon is a random integer between 1 and n. You want to put together a complete set of coupons of *minimum value*, so that of all of the coupons you receive of a certain type c_i, you keep the one of minimum value. You collect coupons until you have a full set of coupons and then you determine the value of the set of coupons.

Your programming assignment is to write a simulator of the Coupon Collector problem with values, and run simulations to see what the average value is for a complete set of coupons. You should run this up to $n = 4,000$. How does this relate to the results of programming problem 5 of homework 1? (Consider both the total value of the coupons V, as well as V/n.)