CSE 417
Dynamic Programming (pt 6)
Parsing Algorithms

UNIVERSITY of WASHINGTON

Reminders

> **HW9 due on Friday**
  - start early
  - program will be slow, so debugging will be slow...
  - should run in 2-4 minutes

> **Please fill out course evaluations**

Dynamic Programming Review

optimal substructure: (small) set of solutions, constructed from solutions to sub-problems that is guaranteed to include the optimal one

- Apply the steps...
  1. Describe solution in terms of solution to *any* sub-problems
  2. Determine all the sub-problems you'll need to apply this recursively
  3. Solve every sub-problem (once only) in an appropriate order

- Key question:
  1. Can you solve the problem by combining solutions from sub-problems?

- Count sub-problems to determine running time
  - total is number of sub-problems times time per sub-problem

Review From Previous Lectures

> Previously…

> Find opt substructure by considering how opt solution could use the last input
  - given multiple inputs, consider how opt uses last of either or both
  - given clever choice of sub-problems, find opt substructure by considering new options

> Alternatively, consider the shape of the opt solution in general: e.g., tree structured

Today

- Dynamic programming algorithms for parsing
  - CKY is an important algorithm and should be understandable
  - (everything after that is out of scope)

- If you want to see more examples, my next two favorites are...
  1. Optimal code generation (compilers)
  2. System R query optimization (databases)

Outline for Today

- Grammars
- CKY Algorithm
- Earley's Algorithm
- Leo Optimization

Grammars

> Grammars are used to understand languages

> Important examples:
  - natural languages
  - programming languages

Natural Language Grammar

> Example:

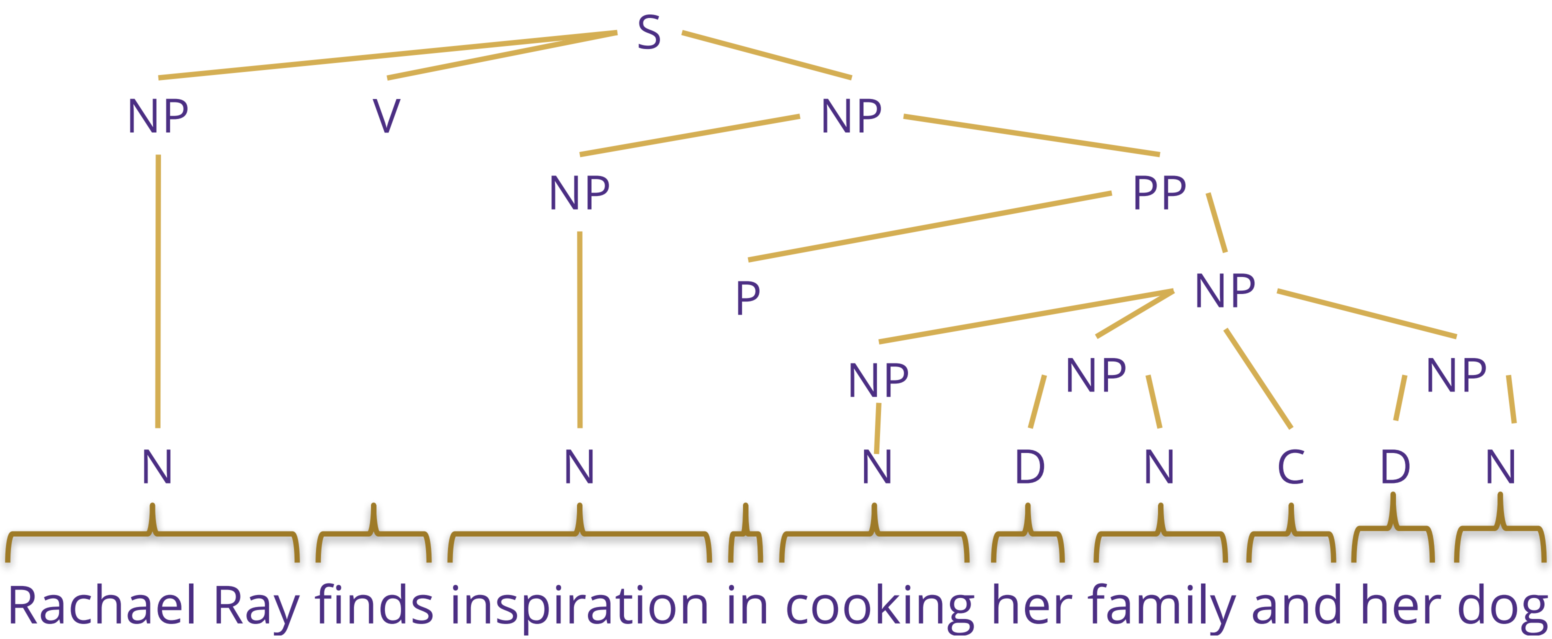
Natural Language Grammar

- Input is a list of parts of speech
  - noun (N), verb (V), preposition (P), determiner (D), conjunction (C), etc.

N	V	N	P	N	D	N	C	D	N
Rachael Ray	finds	inspiration	in	cooking	her	family	and	her	dog

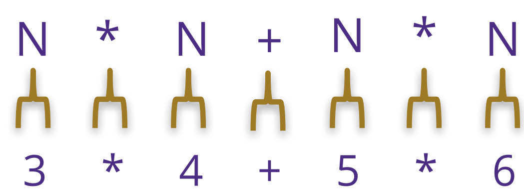
Natural Language Grammar

- Output is a tree showing structure



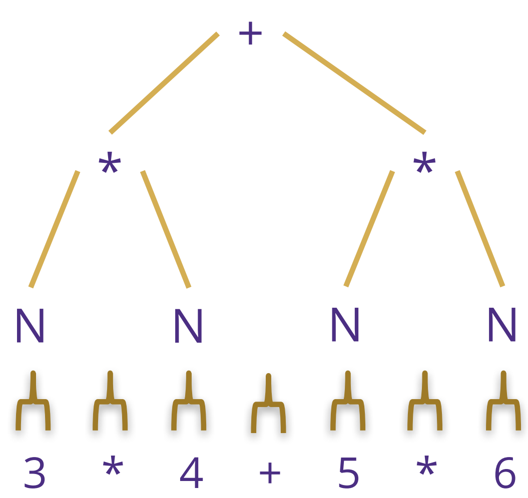
Programming Language Grammar

- Input is a list of ”tokens”
  - identifiers, numbers, +, -, *, /, etc.



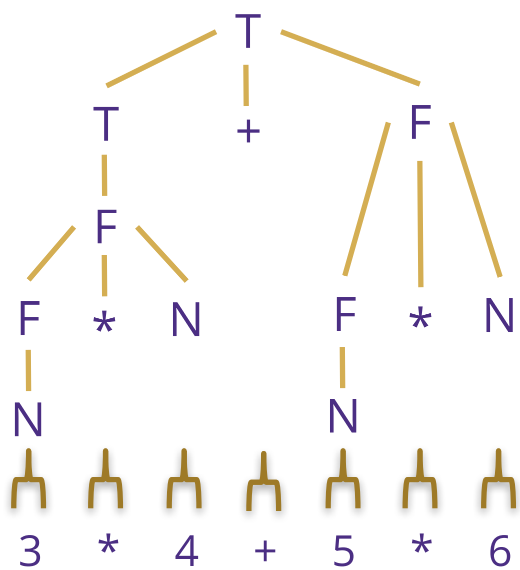
Programming Language Grammar

- Output is a tree showing structure



Programming Language Grammar

> Output is a tree showing structure



Context Free Grammars

- **Definition**: A context free grammar is a set of rules of the form

$$A \rightarrow B_1\ B_2\ \ldots\ B_k$$

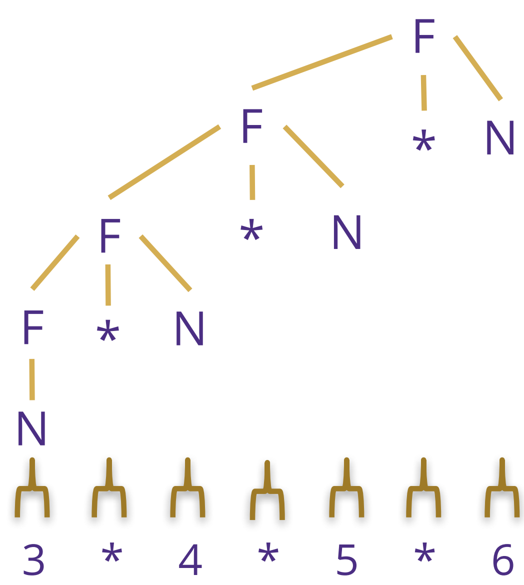
where each B_i can be either a token (a "terminal") or another symbol appearing on the left-hand side of one of the rules (a "non-terminal")

- The output of parsing is a tree with leaves labeled by terminals, internal nodes labeled by non-terminals, and the children of internal nodes matching some rule from the grammar
  - e.g., can have a node labeled A with children B_1, B_2, …, B_k
  - want a specific non-terminal ("start" symbol) as the **root**

Context Free Grammars

- Example grammar for only multiplication:

F → F * N
F → N

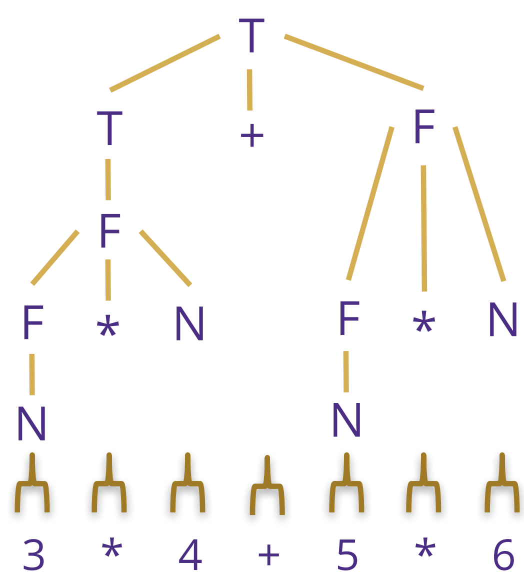


Context Free Grammars

> Example grammar for simple arithmetic expressions:

F ➞ F * N
F ➞ N

T ➞ T + F
T ➞ F



Context Free Grammars

> Called "context free" because the rule $A \rightarrow B_1\ B_2\ \ldots\ B_k$ says that A look like $B_1\ B_2\ \ldots\ B_k$ anywhere

> There are more general grammars called "context sensitive"
  - parsing those grammars is harder than NP-complete
  - (it is PSPACE-complete like generalized chess or go)

Context Free Grammars

> We will limit the sorts of grammars we consider…

> **Definition**: A grammar is in Chomsky normal form if *every* rule is in one of these forms:
1. A ➞ B, where B is a terminal
2. A ➞ B_1 B_2, where both B_1 and B_2 are non-terminals

> In particular, this rules out empty rules: A ➞
   – removal of those simplifies things *a lot*

Context Free Grammars

> **Definition**: A grammar is in Chomsky normal form if every rule is in one of these forms:
1. A ➞ C, where C is a terminal
2. A ➞ B_1 B_2, where both B1 and B2 are non-terminals

> **Fact**: Any context free grammar can be rewritten into an equivalent one in Chomsky normal form
- hence, we can assume this without loss of generality
- (there can be some blowup in the size of the grammar though…)

Context Free Grammars

> Example grammar for arithmetic in Chomsky normal form
  - step 1: remove terminals on right hand side

F → F * N F → N T → T + F T → F	T → T + F T → F	F → F * N F → N

Context Free Grammars

- Example grammar for arithmetic in Chomsky normal form
  - step 1: remove terminals on right hand side

F → F * N	T → T P F	F → F M N
F → N	T → F	F → N
T → T + F		
T → F	M → *	P → +

Context Free Grammars

- Example grammar for arithmetic in Chomsky normal form
  - step 2: introduce new non-terminals to replace 3+ on right hand side

F → F * N	T → T P F	F → F M N
F → N	T → F	F → N
T → T + F		
T → F	M → *	P → +

Context Free Grammars

- Example grammar for arithmetic in Chomsky normal form
  - step 2: introduce new non-terminals to replace 3+ on right hand side

$F \rightarrow F * N$	$T \rightarrow T_1\ F$	$F \rightarrow F_1\ N$
$F \rightarrow N$	$T_1 \rightarrow T\ P$	$F_1 \rightarrow F\ M$
$T \rightarrow T + F$	$T \rightarrow F$	$F \rightarrow N$
$T \rightarrow F$		
	$M \rightarrow *$	$P \rightarrow +$

Context Free Grammars

> Example grammar for arithmetic in Chomsky normal form
  – step 3: eliminate 1 non-terminal on RHS by substitution

$F \rightarrow F * N$	$T \rightarrow T_1\ F$	$F \rightarrow F_1\ N$
$F \rightarrow N$	$T_1 \rightarrow T\ P$	$F_1 \rightarrow F\ M$
$T \rightarrow T + F$	$T \rightarrow F$	$F \rightarrow N$
$T \rightarrow F$		
	$M \rightarrow *$	$P \rightarrow +$

Context Free Grammars

> Example grammar for arithmetic in Chomsky normal form
  – step 3: eliminate 1 non-terminal on RHS by substitution

$F \rightarrow F * N$
$F \rightarrow N$
$T \rightarrow T + F$
$T \rightarrow F$

$T \rightarrow T_1\ F$
$T_1 \rightarrow T\ P$
$T_1 \rightarrow F\ P$
$T \rightarrow F_1\ N$
$T \rightarrow N$

$F \rightarrow F_1\ N$
$F_1 \rightarrow F\ M$
$F \rightarrow N$

$M \rightarrow *$

$P \rightarrow +$

Outline for Today

> Grammars
> CKY Algorithm
> Earley's Algorithm
> Leo Optimization

Parsing Context Free Grammars

- Trying to find a tree...

- **Q**: What technique do we know that might be helpful?
- **A**: Dynamic programming!

Parsing Context Free Grammars

- Apply dynamic programming...
  - to find any tree that matches the data
  - (can be generalized to find the “most likely” parse also...)

- Think about what the parse tree for tokens 1 .. n might look like
  - root corresponds to some rule $A \rightarrow B_1\ B_2$ (Chomsky Normal Form)
  - child B_1 is root of parse tree for some 1 .. k
  - child B_2 is root of parse tree for k+1 .. n
  - (or it could be a leaf $A \rightarrow C$, where C is a terminal, if n=1)

Parsing Context Free Grammars

- In general, parse tree for tokens i .. j might look like
  - $A \rightarrow C$ if i = j OR
  - $A \rightarrow B_1\ B_2$ where
    - child B_1 is root of parse tree for some i .. k
    - child B_2 is root of parse tree for k+1 .. j

- Try each of those possibilities (at most |G|) for each (i,j) pair
  - each requires checking j – i + 1 possibilities for k
  - need answers to sub-problem with j – i smaller
    - can fill in the table along the diagonals, for example

Cocke-Kasami-Younger (CKY)

> Example table from arithmetic example:

	3	*	4	+	5	*	6
3	F/T						
*		M					
4			F/T				
+				P			
5					F/T		
*						M	
6							F/T

$T \rightarrow T_1\ F$

$T \rightarrow F_1\ N$

$T_1 \rightarrow T\ P$

$T_1 \rightarrow F\ P$

$F \rightarrow F_1\ N$

$F_1 \rightarrow F\ M$

$T \rightarrow N$

$F \rightarrow N$

$M \rightarrow *$

$P \rightarrow +$

Cocke-Kasami-Younger (CKY)

> Example table from arithmetic example:

	3	*	4	+	5	*	6
3	F/T	F_1					
*		M					
4			F/T	T_1			
+				P			
5					F/T	F_1	
*						M	
6							F/T

$T \rightarrow T_1\ F$

$T \rightarrow F_1\ N$

$T_1 \rightarrow T\ P$

$T_1 \rightarrow F\ P$

$F \rightarrow F_1\ N$

$F_1 \rightarrow F\ M$

$T \rightarrow N$

$F \rightarrow N$

$M \rightarrow *$

$P \rightarrow +$

Cocke–Kasami–Younger (CKY)

> Example table from arithmetic example:

	3	*	4	+	5	*	6
3	F/T	F_1	F/T				
*		M					
4			F/T	T_1	T		
+				P			
5					F/T	F_1	F/T
*						M	
6							F/T

$T \rightarrow T_1\ F$
$T \rightarrow F_1\ N$
$T_1 \rightarrow T\ P$
$T_1 \rightarrow F\ P$
$F \rightarrow F_1\ N$
$F_1 \rightarrow F\ M$
$T \rightarrow N$
$F \rightarrow N$
$M \rightarrow *$
$P \rightarrow +$

Cocke-Kasami-Younger (CKY)

> Example table from arithmetic example:

	3	*	4	+	5	*	6
3	F/T	F_1	F/T	T_1			
*****		M					
4			F/T	T_1	T		
+				P			
5					F/T	F_1	F/T
*****						M	
6							F/T

- $T \rightarrow T_1\ F$
- $T \rightarrow F_1\ N$
- $T_1 \rightarrow T\ P$
- $T_1 \rightarrow F\ P$
- $F \rightarrow F_1\ N$
- $F_1 \rightarrow F\ M$
- $T \rightarrow N$
- $F \rightarrow N$
- $M \rightarrow *$
- $P \rightarrow +$

Cocke-Kasami-Younger (CKY)

> Example table from arithmetic example:

	3	*	4	+	5	*	6
3	F/T	F_1	F/T	T_1	T		
*****		M					
4			F/T	T_1	T		
+				P			
5					F/T	F_1	F/T
*****						M	
6							F/T

$T \rightarrow T_1\ F$

$T \rightarrow F_1\ N$

$T_1 \rightarrow T\ P$

$T_1 \rightarrow F\ P$

$F \rightarrow F_1\ N$

$F_1 \rightarrow F\ M$

$T \rightarrow N$

$F \rightarrow N$

$M \rightarrow *$

$P \rightarrow +$

Cocke-Kasami-Younger (CKY)

> Example table from arithmetic example:

	3	*	4	+	5	*	6
3	F/T	F_1	F/T	T_1	T		T
*		M					
4			F/T	T_1	T		
+				P			
5					F/T	F_1	F/T
*						M	
6							F/T

$T \rightarrow T_1\ F$

$T \rightarrow F_1\ N$

$T_1 \rightarrow T\ P$

$T_1 \rightarrow F\ P$

$F \rightarrow F_1\ N$

$F_1 \rightarrow F\ M$

$T \rightarrow N$

$F \rightarrow N$

$M \rightarrow *$

$P \rightarrow +$

Cocke-Kasami-Younger (CKY)

> Can reconstruct the tree from the table as usual.

	3	*	4	+	5	*	6
3	F/T	F_1	F/T	T_1	T		T
*****		M					
4			F/T	T_1	T		
+				P			
5					F/T	F_1	F/T
*****						M	
6							F/T

$T \rightarrow T_1\ F$

$T \rightarrow F_1\ N$

$T_1 \rightarrow T\ P$

$T_1 \rightarrow F\ P$

$F \rightarrow F_1\ N$

$F_1 \rightarrow F\ M$

$T \rightarrow N$

$F \rightarrow N$

$M \rightarrow *$

$P \rightarrow +$

Cocke-Kasami-Younger (CKY)

- Running time is $O(|G|\ n^3)$
  - in NLP, $|G| >> n$, so this is great
  - in PL, $|G| < n$, so this is not great
  - in algorithms, this is usually considered $O(n^3)$ since $|G|$ is a "constant"
    - I will follow this convention for the rest of the lecture...

- Algorithm easily generalizes to find "most likely" parse tree
  - frequently used in NLP case

Outline for Today

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Improving CKY (out of scope)

- CKY is not optimal even for general grammars...
  - (can be improved using fast matrix multiplication)

- PLUS we know that certain grammars can be parsed much faster
- In particular, there exist O(n) algorithms for typical PL grammars
  - $O(n^3)$ was out of the question in 1965...

- Arithmetic example is one of those
  - notice how the table is mostly blank
  - that's a lot of wasted effort

Improving CKY

- To get to O(n), we cannot fill in an n x n table
  - doing so always requires $\Omega(n^2)$ time

Improving CKY

> **Idea**: coalesce columns...

	3	*	4	+	5	*	6
3	N/F/T	F_1	F/T	T_1	T		T
*****		M					
4			N/F/T				
+				P			
5					N/F/T	F_1	F/T
*****						M	
6							N/F/T

Improving CKY

- **Idea**: coalesce columns...
  - let I_j include everything in the column j
  - these are rules that parse i .. j for some i
- Need to remember i as well
  - write entries of I_j as “A (i)”, recording both symbol and where parsing started

Improving CKY

- Now we fill in the sets I_1, I_2, …, I_n
  - parsing left to right

- If we are lucky enough to get $|I_j| = O(1)$ for all j, this *could* be a linear time algorithm
  - assuming we can build I_j in O(1) time

- Latter means we cannot look at *all previous* I_j's
  - probably need to only look at I_{j-1}

Improving CKY: False Start

- Suppose I_j is the set of "A (i)" where A matches i .. j

- How do we build I_j?
- If N is the j-th symbol of input, add "A (j)" for every A → N rule

- What next?
- If "C (k)" is in I_j, we might need to add "A (i)" for any A → B C...
  - "A (i)" should be added if "B (i)" is in I_k
  - it takes O(n) time to try every k in 1 .. j-1
  - so we are back to $\Omega(n^2)$

Improving CKY

> To get O(n), we need to keep track of anything we might need to use later on in order to complete the parsing of a rule

> Specifically, if we have parsed "B (i)", we need to keep track of the fact that it could be used to get an A ➞ B C (i) if we later see C

> We write this fact as "A ➞ B · C (i)", which, in I_j, means that we have parsed the B part at i .. j
  – (the "C" part can be missing here
    i.e., if the rule is A ➞ B, where B is a non-terminal)

Improved Parser

- Let I_j be the set of elements like "A → B · C (i)", where:
  1. B matches input tokens i .. j
  2. It is possible for A to follow something that matches input tokens 1 .. i-1
- Note that "·" can be at beginning, middle, or end
  - (we may as well drop the limit of only 2 symbols on the RHS)
- Second part is another optimization
  - don't waste time trying to parse rules that aren't useful based on what came earlier

Improved Parser

> Let I_j be the set of elements like "A ➞ B · C (i)", where:

1. B matches input tokens i .. j
2. It is possible for A to follow something that matches input tokens 1 .. i-1

> Fill in I_j as follows:

- add anything that could follow I_{j-1} and matches input token j
  - > (if "A ➞ B · C (i)" is in I_{j-1}, then C could follow)
- for each added *complete* item "A ➞ B C · (i)" added: ← only part that is potentially slow...
  - > if I_i contains "A' ➞ B · A (i')", then add "A' ➞ B A · (i')" to I_j
  - > (likewise for "A' ➞ · A (i')")
- add all those items that could follow the ones already added

Improved Parser

- Fill in I_j as follows:
  - add anything that could follow I_{j-1} and matches input token j
    - (if "A → B · C (i)" is in I_{j-1}, then C could follow)
  - for each added *complete* item "A → B C · (i)" added:
    - if I_i contains "A' → B · A (i')", then add "A' → B A · (i')" to I_j
    - (likewise for "A' → · A (i')")
  - add all those items that could follow the ones already added
- If all $|I_j|$'s are size O(1), then this is O(1) time per item
  - hence, O(n) over all

Earley's algorithm

- This version is called Earley's algorithm
- It was developed independently of CKY by Earley
  - (relation to CKY was noted by Ruzzo et al.)
  - also considered a dynamic programming algorithm
    - the sub-problems being solved are not quite so obvious as in CKY

Earley's algorithm

- Can be shown that Earley's algorithm runs in $O(n^2)$ time for any unambiguous grammar
  - meaning there is only one possible parse tree
    - typical of PL grammars (though not NLP grammars)
- Can also be shown it runs in O(n) time for **nice** LR(k) grammars
- BUT not for all LR(k) grammars
  - latter can be parsed in O(n) time by other algorithms
- The running time is at least the sum of sizes of the I_j's...

Outline for Today

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> Earley's Algorithm
> Leo Optimization

Bad Cases for Earley

- **Q**: Can the Ij's be O(n) for some *unambiguous* grammar's?

Bad Cases for Earley

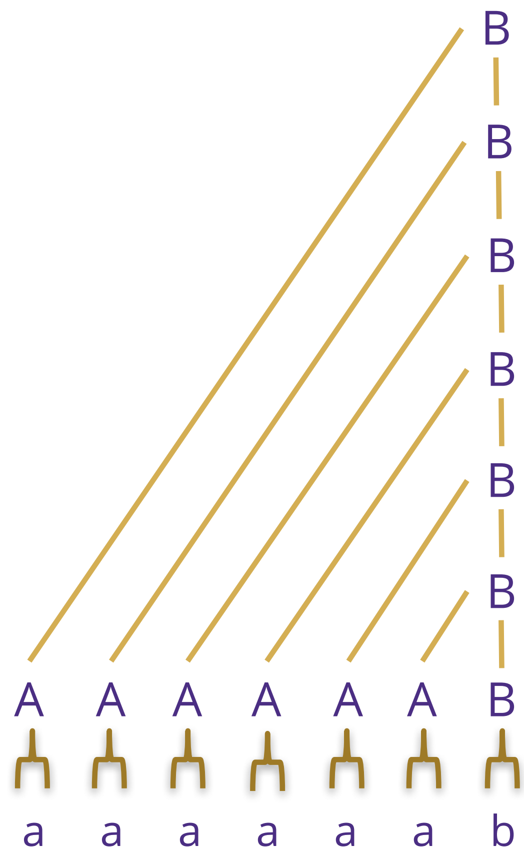
- **Q**: Can the Ij's be O(n) for some *unambiguous* grammar's?
- **A**: Unfortunately, yes

$A \rightarrow a$

$B \rightarrow b$

$B \rightarrow A\ B$

- All B's completed in I_n

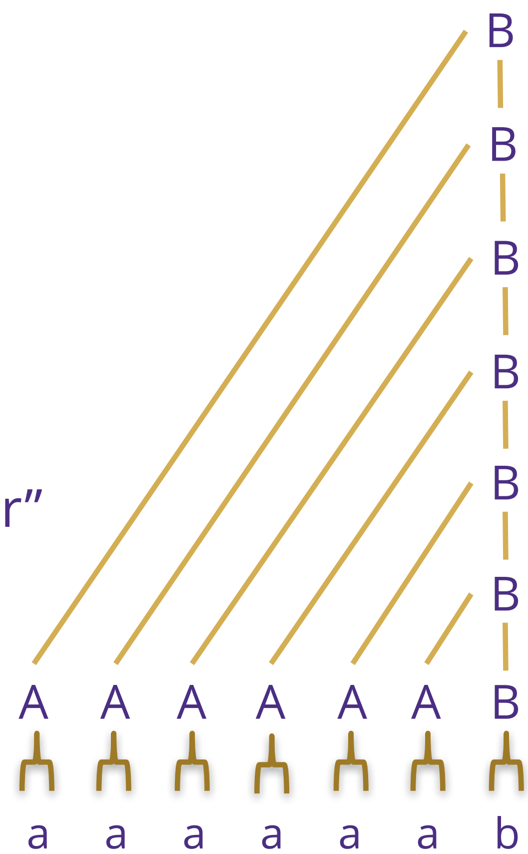


Bad Cases for Earley

- > **Q**: Can the Ij's be O(n) for some *unambiguous* grammar's?
- > **A**: Unfortunately, yes

- > This is a "right recursive grammar"
- > Fortunately, these are the only bad cases (O(n) otherwise)

- > Grammars can be *usually* be rewritten to avoid it



Joop Leo's Optimization

- Alternatively, we can improve the algorithm to handle those...
- Leo makes the following optimization:
  - only record the top-most item in a tall stack like this
  - (actually O(1) copies of it depending on how we might look for it later)
- Can then show that the I_j's are O(1) size
  - number with dot <u>not</u> at end is O(1) due to LR(k) property
  - clever argument shows number with dot at end is also O(1)
    - removing stacks leaves tree with all 2+ children and leaves those above
    - (furthermore, each is discovered only once for unambiguous grammars)

Joop Leo's Optimization

> Alternatively, we can improve the algorithm to handle those...

> Leo makes the following optimization:
  - only record the top-most item in a tall stack like this
  - (actually O(1) copies of it depending on how we might look for it later)

> Result is O(n) in the worst case for LR(k)
  - (i.e., for anything parsable by deterministic push-down automaton
  - covers almost every PL grammar

Parsers in Practice

> CKY and Earley are used in NLP
  – recall that |G| is usually larger there

> In PL, we typically use special grammars (e.g., LR(k)) that can be parsed in linear time
  – LR(k) was invented by Don Knuth
  – parses anything that can be parsed by a deterministic push-down automaton

> Earley + Leo gives the same asymptotic performance
  – expect it to see more use given speed of computers
  – (LR parsing was developed for machines 10k x slower)