Design and Analysis of Algorithms I

Asymptotic Analysis

Big-Oh: Basic Examples

Example #1

Claim : if $T(n) = a_k n^k + ... + a_1 n + a_0$ then

$$T(n) = O(n^k)$$

Proof : Choose $n_0 = 1$ and $c = |a_k| + |a_{k-1}| + .. + |a_1| + |a_0|$

Need to show that $\forall n \geq 1, T(n) \leq c \cdot n^k$

We have, for every $n \geq 1$,

$$\begin{aligned} T(n) &\leq |a_k|n^k + ... + |a_1|n + |a_0| \\ &\leq |a_k|n^k + ... + |a_1|n^k + |a_0|n^k \\ &= c \cdot n^k \end{aligned}$$

Example #2

<u>Claim</u> : for every $k \geq 1,\ n^k$ is not $O(n^{k-1})$

<u>Proof</u> : by contradiction. Suppose $n^k = O(n^{k-1})$

Then there exist constants c, n_0 such that

$$n^k \leq c \cdot n^{k-1} \quad \forall n \geq n_0$$

But then [cancelling n^{k-1} from both sides]:

$$n \leq c \quad \forall n \geq n_0$$

Which is clearly False [contradiction].

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