CSE 417: Algorithms and Computational Complexity

7,8: Dyn. Programming, IV
String Edit Distance

Winter 2005

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Sequence Comparison: Edit Distance

- Given:
  - Two strings $A=a_1\ a_2\ \ldots\ a_n$ and $B=b_1\ b_2\ \ldots\ b_m$
- Find: The minimum number of edit steps to transform A into B where a step can be:
  - insert a single character
  - delete a single character
  - substitute one character by another
  - (you can copy a single character for free)

Example

- A = castle
- B = chattel

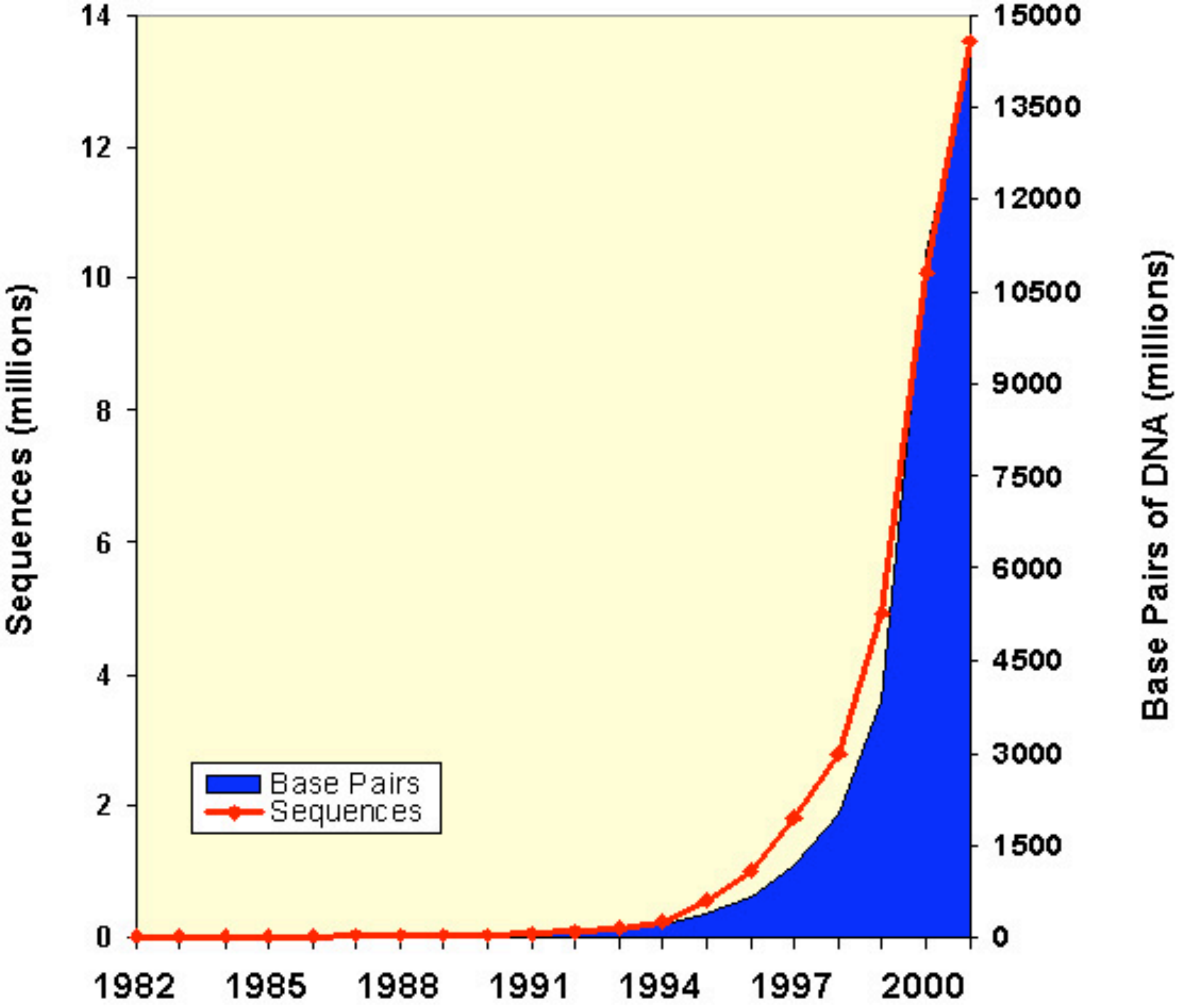
A		B
c	→	c
	insert	h
a	→	a
s	subst	t
t	→	t
l	delete	
e	→	e
	insert	l

Cost: 4

Applications

- "diff" utility – where do two files differ
- Version control & patch distribution – save/send only changes
- Molecular biology
  - Similar sequences often have similar origin and function
  - Similarity often recognizable despite millions or billions of years of evolutionary divergence

Growth of GenBank



Recursive Solution

- Sub-problems: Edit distance problems for all prefixes of A and B that don't include all of both A and B

- Let D(i,j) be the number of edits required to transform $a_1\ a_2 \ldots a_i$ into $b_1\ b_2 \ldots b_j$

- Clearly D(0,0)=0

Computing D(n,m)

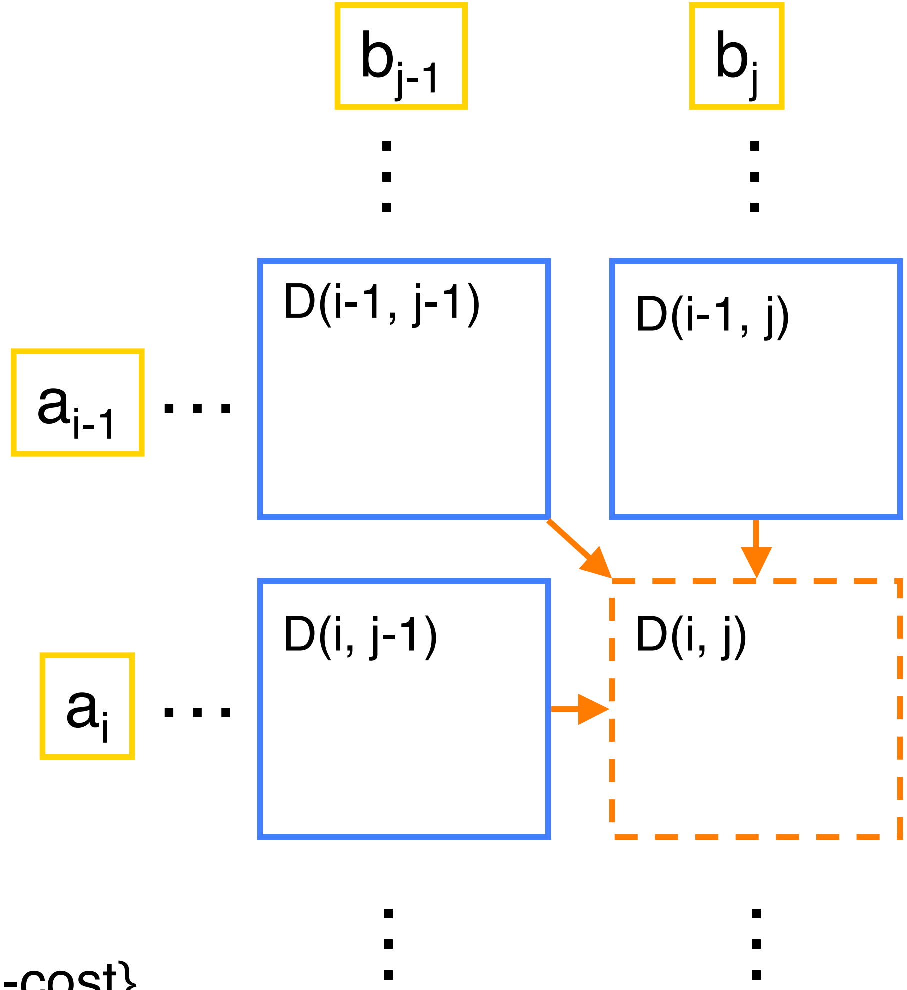
- Imagine how best sequence handles the last characters a_n and b_m
- If best sequence of operations
  - deletes a_n then $D(n,m)=D(n-1,m)+1$
  - inserts b_m then $D(n,m)=D(n,m-1)+1$
  - replaces a_n by b_m then $D(n,m)=D(n-1,m-1)+1$
  - matches a_n and b_m then $D(n,m)=D(n-1,m-1)$

Recursive algorithm D(n,m)

```
if  n=0  then
    return (m)
elseif  m=0  then
    return(n)
else
    if  $a_n=b_m$  then
        replace-cost=0
    else
        replace-cost=1
    endif
    return(min{ D(n-1, m) + 1,
                D(n, m-1) +1,
                D(n-1, m-1) + replace-cost})
```

Dynamic Programming

```
for j = 0 to m;  $D(0,j) \leftarrow j$; endfor
for i = 1 to n;   $D(i,0) \leftarrow i$; endfor
for i = 1 to n
    for j = 1 to m
        if  $a_i=b_j$ then
            replace-cost $\leftarrow$ 0
        else
            replace-cost $\leftarrow$ 1
        endif
        $D(i,j) \leftarrow$  min { D(i-1, j) + 1,
                                D(i, j-1) +1,
                                D(i-1, j-1) + replace-cost}
    endfor
endfor
```



b_{j-1}

b_j

a_{i-1}

a_i

D(i-1, j-1)

D(i-1, j)

D(i, j-1)

D(i, j)

Example run with AGACATTG and GAGTTA

		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
0									
G 1									
A 2									
G 3									
T 4									
T 5									
A 6									

Example run with AGACATTG and GAGTTA

			A	G	A	C	A	T	T	G
		0	1	2	3	4	5	6	7	8
	0	0	1	2	3	4	5	6	7	8
G	1	1								
A	2	2								
G	3	3								
T	4	4								
T	5	5								
A	6	6								

Example run with AGACATTG and GAGTTA

			A	G	A	C	A	T	T	G
		0	1	2	3	4	5	6	7	8
	0	0	1	2	3	4	5	6	7	8
G	1	1	1	1	2	3	4	5	6	7
A	2	2								
G	3	3								
T	4	4								
T	5	5								
A	6	6								

Example run with AGACATTG and GAGTTA

			A	G	A	C	A	T	T	G
		0	1	2	3	4	5	6	7	8
	0	0	1	2	3	4	5	6	7	8
G	1	1	1	1	2	3	4	5	6	7
A	2	2	1	2	1					
G	3	3								
T	4	4								
T	5	5								
A	6	6								

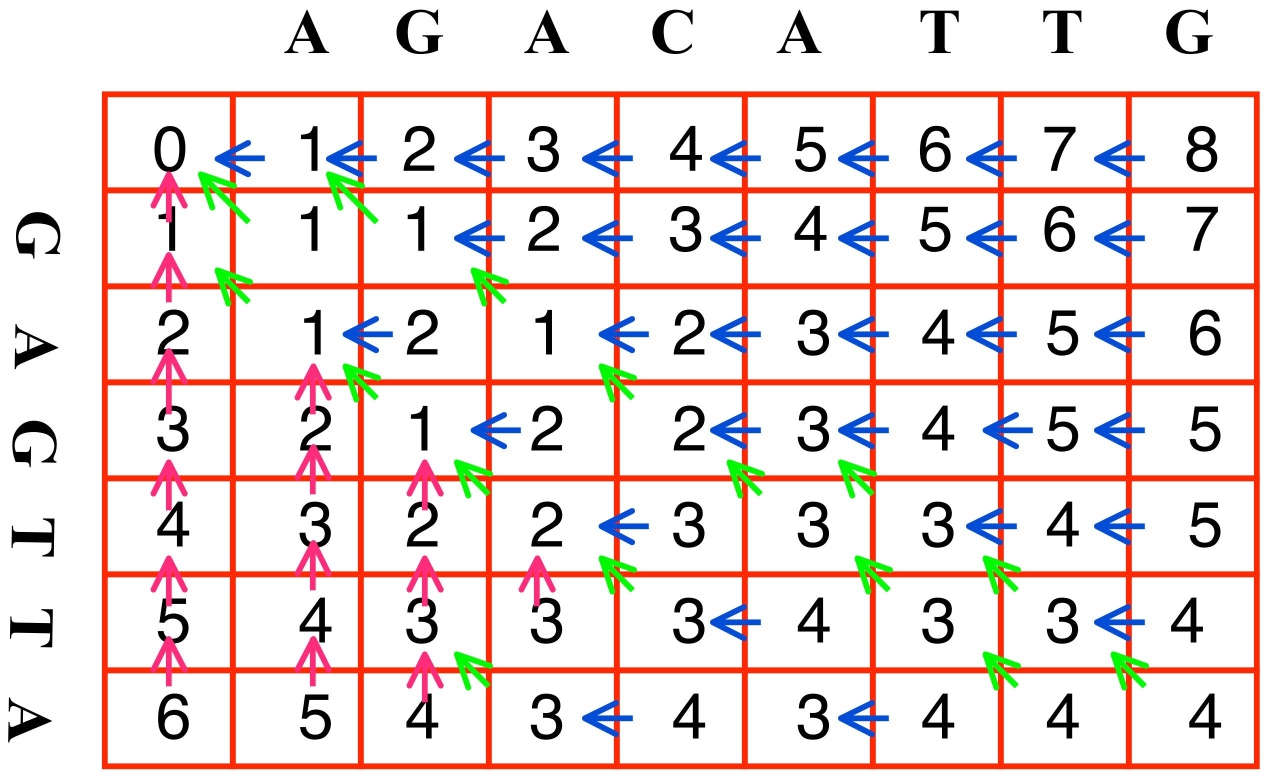
Example run with AGACATTG and GAGTTA

			A	G	A	C	A	T	T	G
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	0	0	1	2	3	4	5	6	7	8
G	**1**	1	1	1	2	3	4	5	6	7
A	**2**	2	1	2	1	2	3	4	5	6
G	**3**	3	2	1	2	2	3	4	5	5
T	**4**	4								
T	**5**	5								
A	**6**	6								

Example run with AGACATTG and GAGTTA

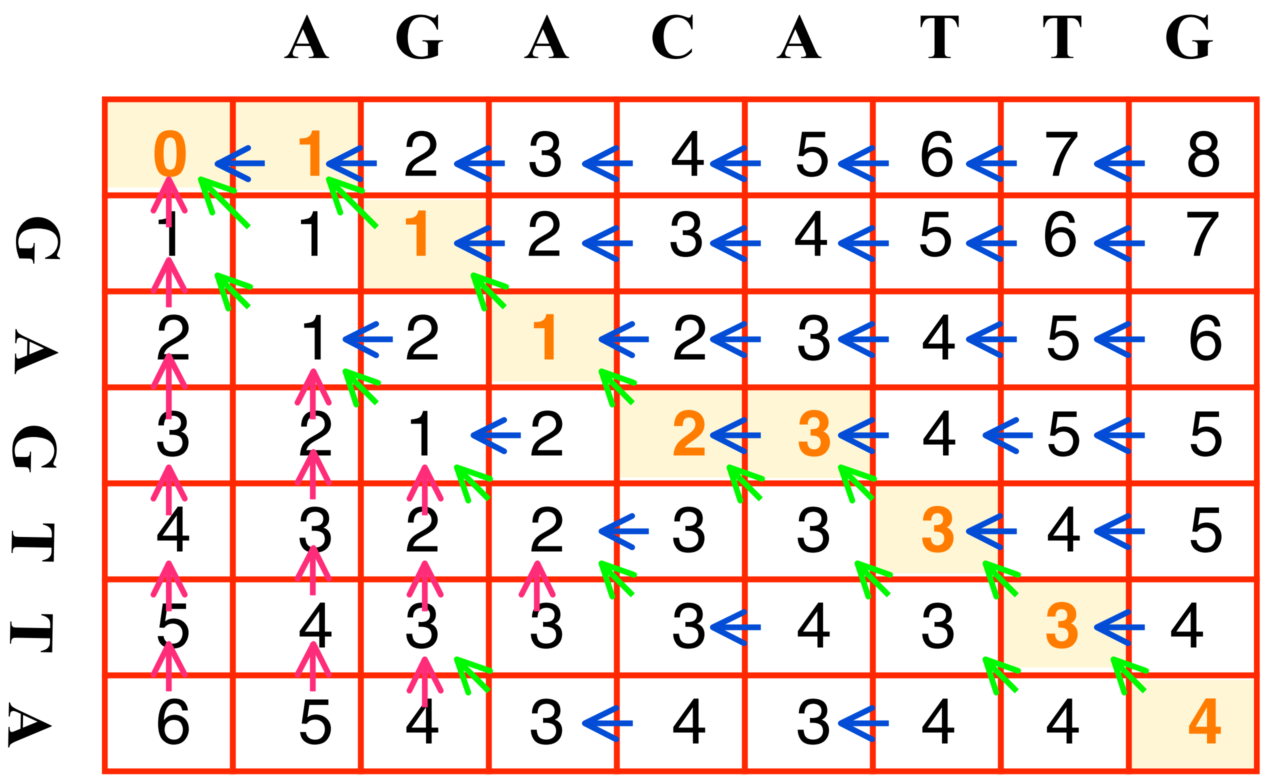
			A	G	A	C	A	T	T	G
		0	1	2	3	4	5	6	7	8
	0	0	1	2	3	4	5	6	7	8
G	1	1	1	1	2	3	4	5	6	7
A	2	2	1	2	1	2	3	4	5	6
G	3	3	2	1	2	2	3	4	5	5
T	4	4	3	2	2	3	3	3	4	5
T	5	5	4	3	3	3	4	3	3	4
A	6	6	5	4	3	4	3	4	4	4

Example run with AGACATTG and GAGTTA



		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
G	1	1	1	2	3	4	5	6	7
A	2	1	2	1	2	3	4	5	6
G	3	2	1	2	2	3	4	5	5
T	4	3	2	2	3	3	3	4	5
T	5	4	3	3	3	4	3	3	4
A	6	5	4	3	4	3	4	4	4

Example run with AGACATTG and GAGTTA



		A	G	A	C	A	T	T	G
	0	1	2	3	4	5	6	7	8
G	1	1	1	2	3	4	5	6	7
A	2	1	2	1	2	3	4	5	6
G	3	2	1	2	2	3	4	5	5
T	4	3	2	2	3	3	3	4	5
T	5	4	3	3	3	4	3	3	4
A	6	5	4	3	4	3	4	4	4

Reading off the operations

- Follow the sequence and use color/ direction of arrows to tell what operation was performed.
  - ← Insert
  - ↑ Delete
  - ↖ Copy or substitute