CSE 417: Algorithms and Computational Complexity

2: Algorithms and Efficiency

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Algorithms: definition

- Procedure to accomplish a task or solve a well-specified problem
  - Well-specified: know what all possible inputs look like and what output looks like given them
  - "accomplish" via simple, well-defined steps
  - Ex: sorting names (via comparison)
  - Ex: checking for primality (via +, -, *, /)

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Algorithms: a sample problem

- Printed circuit-board company has a robot arm that solders components to the board
- Time to do it depends on
  - total distance the arm must move from initial rest position around the board and back to the initial positions
- For each board design, must figure out good order to do the soldering

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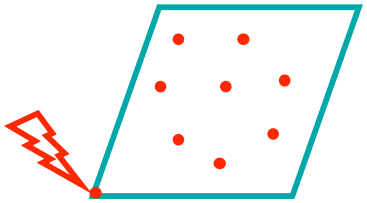
Printed Circuit Board



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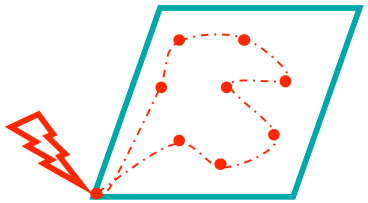
Printed Circuit Board



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A well-defined Problem

- Input: Given a set **S** of **n** points in the plane
- Output: The shortest cycle tour that visits each point in the set **S**.
- Better known as “TSP”
- How might you solve it?

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Nearest Neighbor Heuristic

- Start at some point p_0
- Walk first to its nearest neighbor p_1
- Repeatedly walk to the nearest unvisited neighbor until all points have been visited
- Then walk back to p_0

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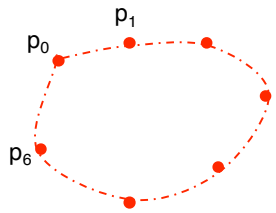
Nearest Neighbor Heuristic



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An input where it works badly

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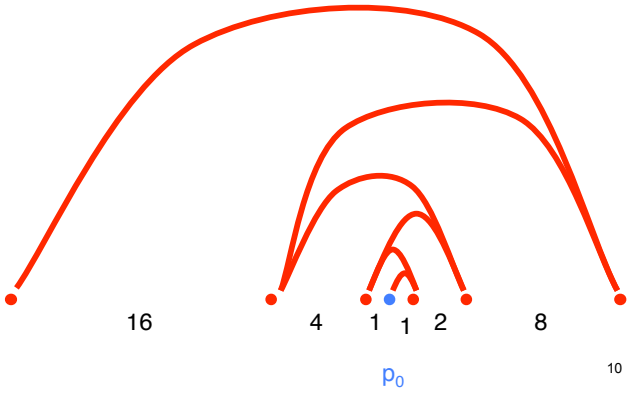
An input where it works badly



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Revised idea - Closest pairs first

- Repeatedly pick the closest pair of points to join so that the result can still be part of a single loop in the end
  - can pick endpoints of line segments already created
- How does this work on our bad example?

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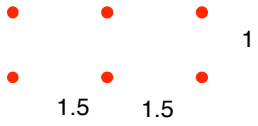
Another bad example



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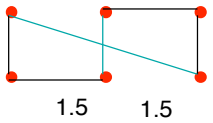
Another bad example



$1+\sqrt{10}$ vs 3

Something that works

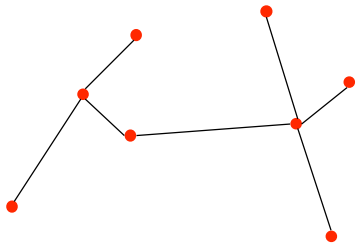
- For each of the n! orderings of the points check the length of the cycle you get
- Keep the best one

Two Notes

- The two incorrect algorithms were **greedy**
  - Often very natural & tempting ideas
  - they make choices that look great "locally" (and never reconsidered them)
  - often does not work - you get boxed in
    - when it does the algorithms are typically efficient
- Our correct algorithm avoids this, but is incredibly slow
  - 20! is so large that counting to one billion in a second it would still take 2.4 billion seconds
    - (around 70 years!)

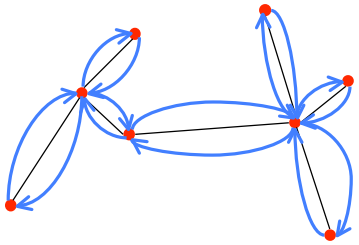
Something that "works" (differently)

1. Find Min Spanning Tree



Something that "works" (differently)

2. Walk around it



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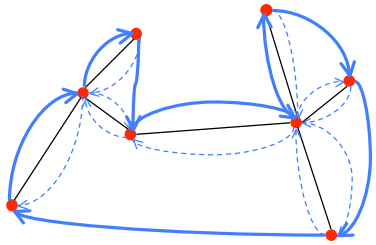
Something that "works" (differently)

3. Take shortcuts (instead of revisiting)



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Something that "works" (differently): Guaranteed Approximation

- Does it seem wacky?
- Maybe, but it's *always* within a factor of 2 of the best tour!
  - deleting one edge from best tour gives *a* spanning tree, so *Min* spanning tree < best tour
  - best tour ≤ wacky tour ≤ 2 * MST ≤ 2 * best

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The Morals of the Story

- Simple problems can be hard
  - Factoring, TSP
- Simple ideas don't always work
  - Nearest neighbor, closest pair heuristics
- Simple algorithms can be very slow
  - Brute-force factoring, TSP
- Changing your objective can be good
  - Guaranteed approximation for TSP

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