CSE 417: Algorithms and Computational Complexity
Assignment #2
January 16, 2004
due: Friday, January 23

1. Use Strassen's algorithm to compute the matrix product

$$\begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 8 & 4 \\ 6 & 2 \end{pmatrix}.$$

   Show your work.

2. (a) Let m_c be the minimum number of multiplications to multiply two $c \times c$ matrices. (For instance, the "standard" method shows $m_2 \leq 8$, and Strassen showed $m_2 \leq 7$.) Determine the asymptotic complexity of $n \times n$ matrix multiplication, in terms of n, c, and m_c, which occurs if the matrices are decomposed into c^2 submatrices each of size $n/c \times n/c$, and the m_c-multiplication algorithm is used recursively. You may assume that $m_c > c^2$.

   (b) What is the greatest k such that, if you can multiply 3×3 matrices using k multiplications, then you can multiply $n \times n$ matrices in time asymptotically faster than Strassen's algorithm? What would the running time of your algorithm be? (For your interest, Laderman has shown that $m_3 \leq 23$.)

   (c) Victor Pan has discovered how to multiply 68×68 matrices using 132,464 multiplications, how to multiply 70×70 matrices using 143,640 multiplications, and how to multiply 72×72 matrices using 155,424 multiplications. Which method yields the best asymptotic running time when used in a divide-and-conquer matrix multiplication algorithm? What is its running time and how does it compare to Strassen's algorithm?

3. Let $a(x) = 4+3x+2x^2+2x^3+x^4+x^5$. Proposition 2 from lecture showed how to efficiently evaluate a polynomial of degree $N-1$ at N points that have Property S. Evaluate $a(x)$ at the 6 points $(1, 2, 3, -1, -2, -3)$ by the algorithm given in Proposition 2. Show your work.