CSE 417: Algorithms and Computational Complexity

4: Dynamic Programming, I Fibonacci

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Lecture 4

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A Possible Misunderstanding?

- We have looked at
  - type of complexity analysis
    - worst-, best-, average-case
  - types of function bounds
    - O, Ω, Θ
- These two considerations are independent of each other
  - one can do any type of function bound with any type of complexity analysis

Insertion Sort:

$\Omega(n^2)$ (worst case)

O(n) (best case)

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Another Possible Misunderstanding?

- Insertion sort is not the best sorting algorithm, unless n is < 10 or 20.

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Some Algorithm Design Techniques, I

- General overall idea
  - Reduce solving a problem to a smaller problem or problems of the same type
- Greedy algorithms
  - Used when one needs to build something a piece at a time
  - Repeatedly make the **greedy** choice - the one that looks the best right away
    - e.g. closest pair in TSP search
  - Usually fast if they work (but often don't)

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Some Algorithm Design Techniques, II

- Divide & Conquer
  - Reduce problem to one or more sub-problems of the same type
  - Typically, each sub-problem is at most a constant fraction of the size of the original problem
    - e.g. Mergesort, Binary Search, Strassen's Algorithm, Quicksort (kind of)

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Some Algorithm Design Techniques, III

- Dynamic Programming
  - Give a solution of a problem using smaller sub-problems, e.g. a recursive solution
  - Useful when the same sub-problems show up again and again in the solution

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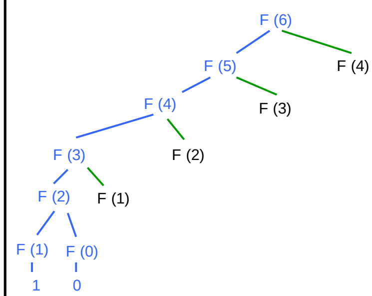
A simple case: Computing Fibonacci Numbers

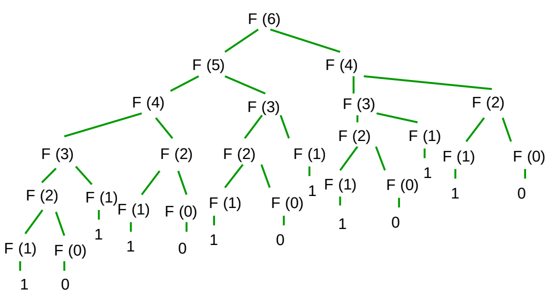
- Recall $F_n = F_{n-1} + F_{n-2}$ and $F_0 = 0$, $F_1 = 1$
- Recursive algorithm:
  - Fibo(n)

    if n=0 **then return**(0)
    else if n=1 **then return**(1)
    else return(Fibo(n-1)+Fibo(n-2))

Call tree - start



Full call tree



Memo-ization (Caching)

- Remember all values from previous recursive calls
- Before recursive call, test to see if value has already been computed
- Dynamic Programming
  - Convert memo-ized algorithm from a recursive one to an iterative one

Fibonacci - Dynamic Programming Version

- FiboDP(n):

  F[0]←0
  F[1] ←1
  for i=2 **to** n **do**
      F[i]=F[i-1]+F[i-1]
  endfor
  return(F[n])

Dynamic Programming

- Useful when
  - same recursive sub-problems occur repeatedly
  - Can anticipate the parameters of these recursive calls
  - The solution to whole problem can be figured out with knowing the internal details of how the sub-problems are solved
    - principle of optimality

List partition problem

- Given: a sequence of n positive integers $s_1,\ldots,s_n$ and a positive integer k
- Find: a partition of the list into up to k blocks:
  $s_1,\ldots,s_{i_1} | s_{i_1+1}\ldots s_{i_2} | s_{i_2+1}\ldots s_{i_{k-1}} | s_{i_{k-1}+1}\ldots s_n$
  so that the sum of the numbers in the largest block is as small as possible.
  i.e. find spots for up to $k-1$ dividers

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Greedy approach

- Ideal size would be $P=\sum_{i=1}^{n} s_i/k$
- Greedy: walk along until what you have so far adds up to P then insert a divider
- Problem: it may not exact (or correct)

  100 200 400 500 900 700 600 800 600

  - sum is 4800 so size must be at least 1600.
  - Greedy? Best?

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