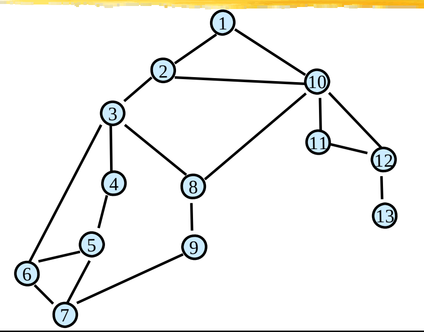
CSE 417: Algorithms and Computational Complexity

Winter 2001

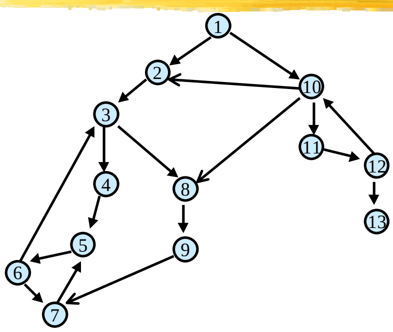
Lecture 9

Instructor: Paul Beame

Undirected Graphs



Directed Graphs

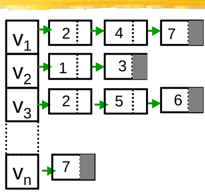


Representing Graph G=(V,E) n vertices, m edges

- Vertex set $V=\{v_1,\ldots v_n\}$
- Adjacency Matrix A
  - $A[i,j]=1$ iff $(v_i,v_j)\in E$
  - Space is n^2 bits
- Advantages:
  - $O(1)$ test for presence or absence of edges.
  - compact in packed binary form for large m
- Disadvantages: inefficient for sparse graphs

Representing Graph G=(V,E) n vertices, m edges

- Adjacency List:
  - $O(n+m)$ words
  - $O(\log n)$ bits each



- Advantages:
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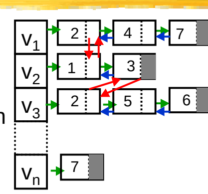

- Back pointers and cross pointers allow easier traversal and deletion of edges
  - usually assume this format

Graph Traversal

- Learn the basic structure of a graph
- Walk from a fixed starting vertex v to find all vertices reachable from v
- Three states of vertices
  - **undiscovered**
  - **discovered**
  - **completely-explored**

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Breadth-First Search

- Completely explore the vertices in order of their distance from v
- Naturally implemented using a queue

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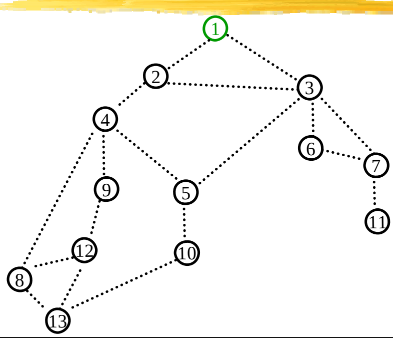
BFS(v)



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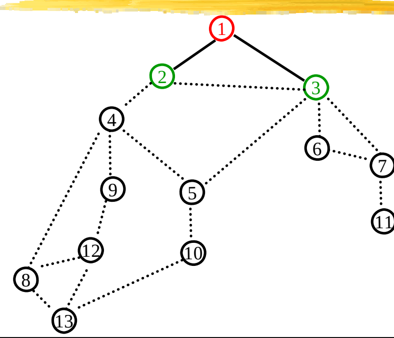

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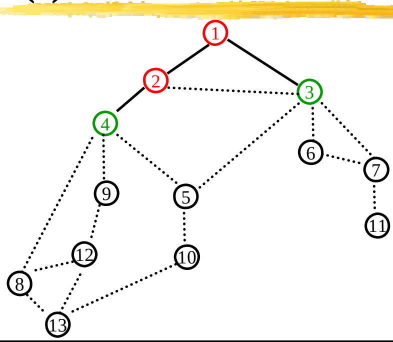
BFS(v)



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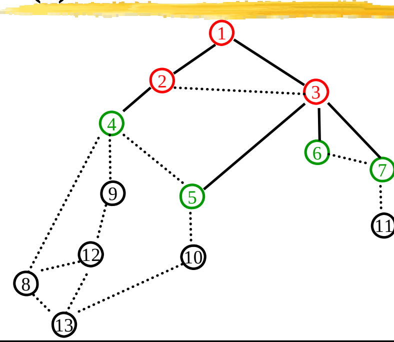
BFS(v)



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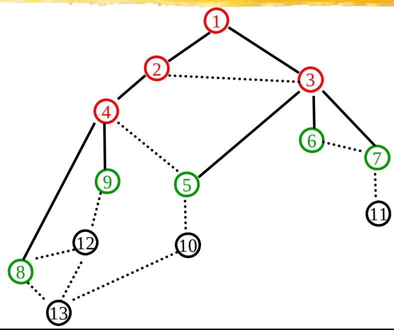
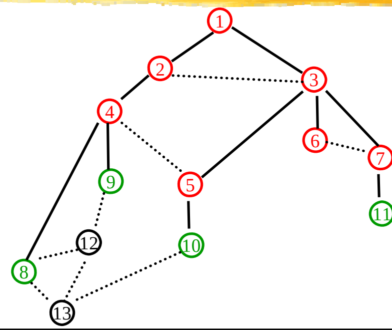
BFS(v)



BFS(v)



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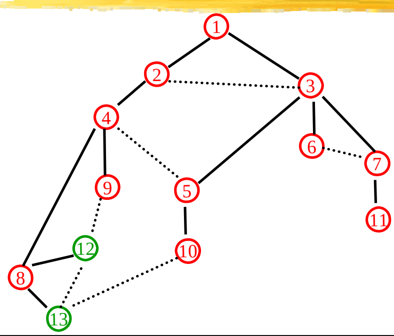
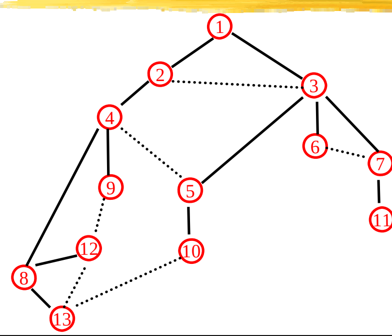


BFS(v)



BFS analysis

- Each edge is explored once from each end-point (at most)
- Each vertex is discovered by following a different edge
- Total cost O(m) where m=# of edges

Graph Search Application: Connected Components

- Want data structure that allows one to answer questions of the form:
  - given vertices u and v is there a path from u to v?
- Idea : create array A such that
  A[u] = smallest numbered vertex that is connected to u
- question reduces to whether A[u]=A[v]?

Graph Search Application: Connected Components

- **for** v=1 **to** n **do**
  - **if** state(v)!=fully-explored **then**
    - state(v)←discovered
    - BFS(v): **setting** A[u] ←v **for each** u **found**
  - **endif**
- **endfor**

- Total cost: O(n+m)
  - each vertex an each edge is touched a constant number of times
  - works also with DFS

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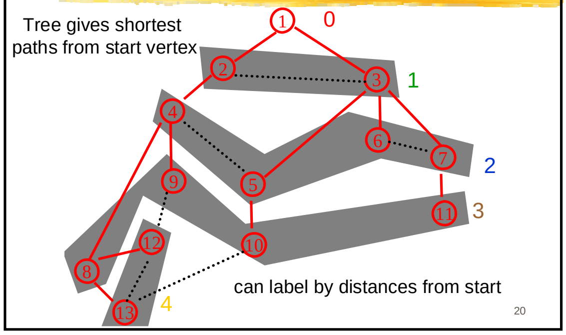
BFS Application: Shortest Paths



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Depth-First Search

- Follow the first path you find as far as you can go, recording all the vertices you will need to explore further as you go.

- Naturally implemented using recursive calls or a stack

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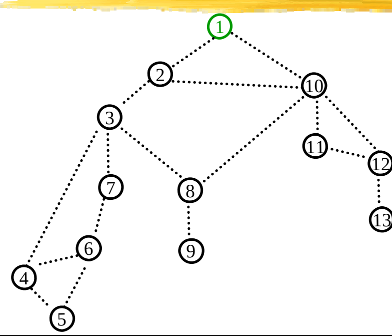
DFS(v)



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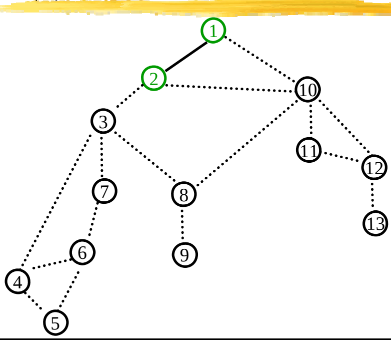
DFS(v)



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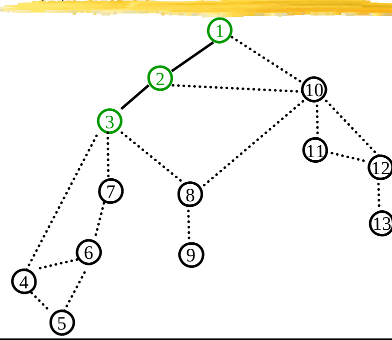

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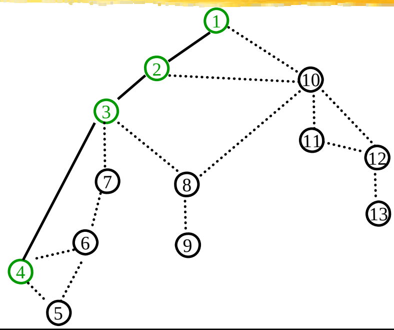

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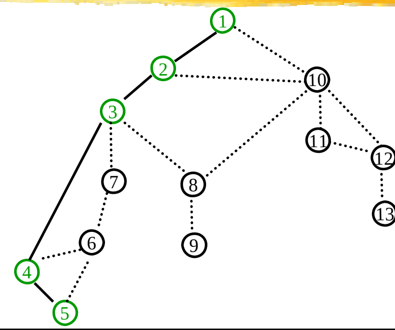

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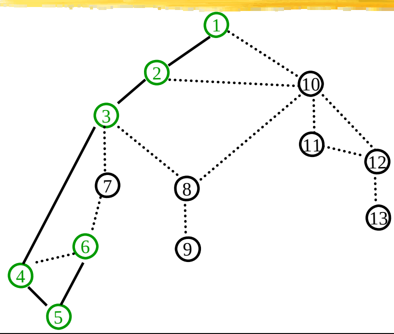
DFS(v)



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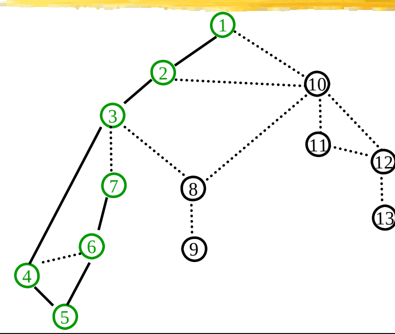

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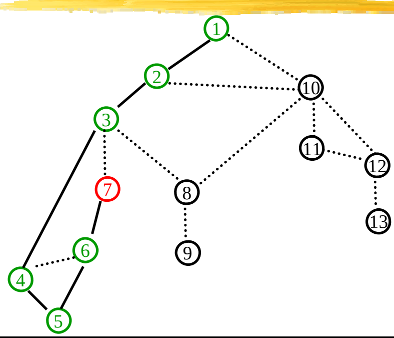

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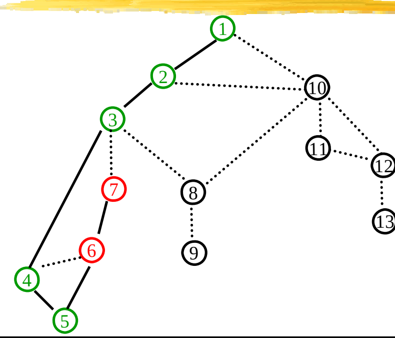

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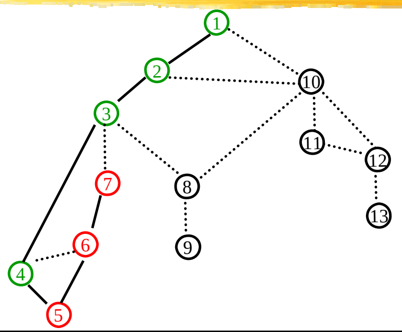
DFS(v)



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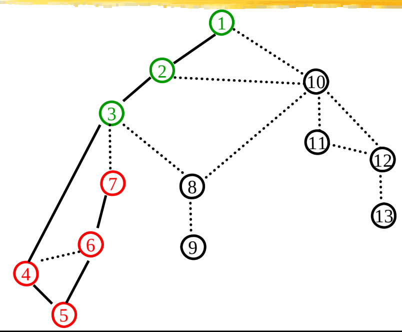
DFS(v)



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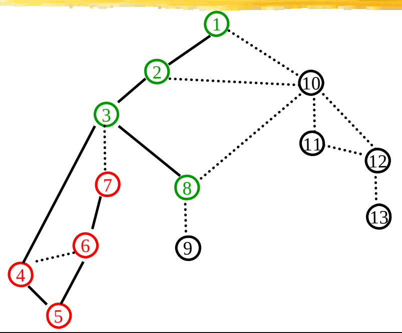
DFS(v)



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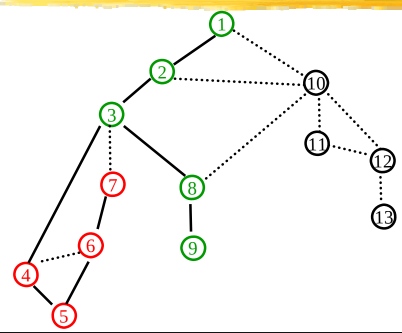
DFS(v)



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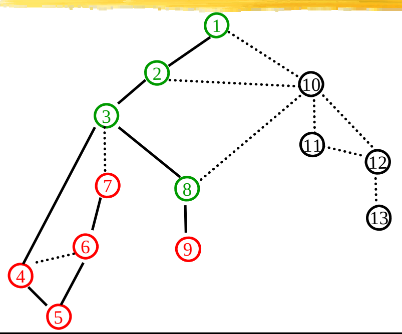
DFS(v)



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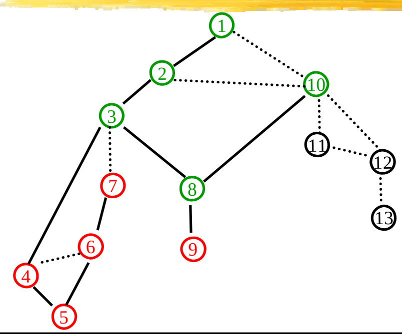
DFS(v)



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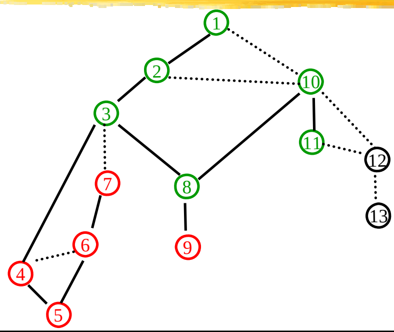
DFS(v)



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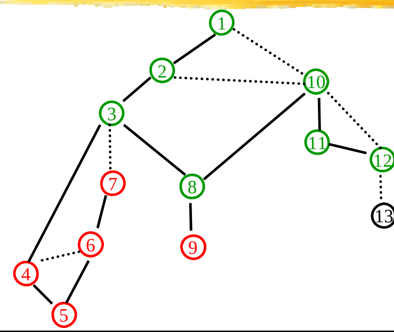
DFS(v)



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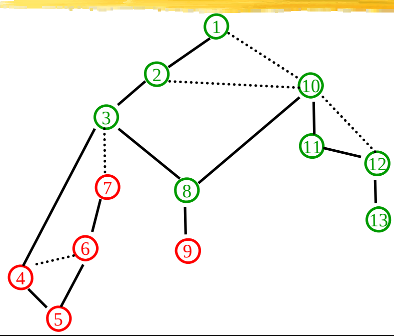
DFS(v)



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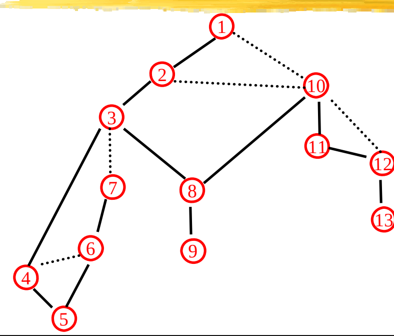

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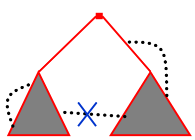
Non-tree edges

- All non-tree edges join a vertex and its descendent in the DFS tree
- No cross edges



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Application: Articulation Points

- A node in an undirected graph is an **articulation point** iff removing it disconnects the graph
- articulation points represent vulnerabilities in a network

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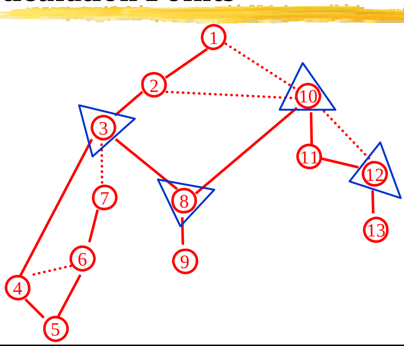
Articulation Points



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Articulation Points from DFS

- Every interior vertex of a tree is an articulation point
  - Non-tree edges eliminate articulation points
- Root nodes are articulation points iff they have more than one child

non-leaf, non-root node u is an articulation point ⇔ no non-tree edges going from sub-tree below some child of u to above u in the tree

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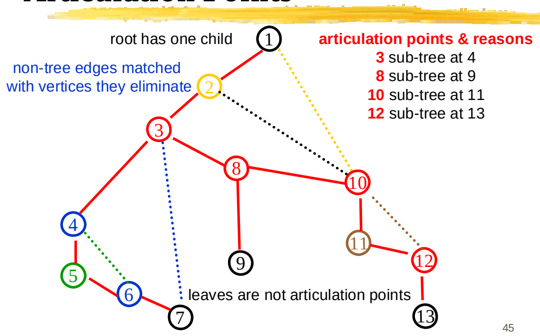
DFS Application: Articulation Points



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