CSE 417: Algorithms and Computational Complexity

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Lecture 20

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Another undecidable problem

- 1's problem: Given the code of a program M does M output 1 on input 1? If so, answer 1 else answer 0.
- Claim: the 1's problem is undecidable
- Proof: by reduction from the Halting Problem

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What we want for the reduction

- Halting problem takes as input a pair <P,x>
- 1's problem takes as input <M>
- Given <P,x> can we create an <M> so that M outputs 1 on input 1 exactly when P halts on input x?

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Yes

- Here is all that we need to do to create M
  - modify the code of P so that instead of reading x, x is hard-coded as the input to P and get rid of all output statements in P
  - add a new statement at the end of P that outputs 1.
- We can write another program T that can do this transformation from <P,x> to <M>

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How we might do the hard-coding if the code were in C?

- Include an assignment at the start that would place the characters in string x in some array A.
- Replace all scanf's in P with calls to a new function scanA that simulates scanf but gets its data from array A.
- Replace all printf's in P by printB which doesn't actually do anything.

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Finishing things off

- Therefore we get a reduction
  - Halting Problem ≤ 1's problem
- Since there is no program solving the Halting Problem there must be no program solving the 1's problem.

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Why the name reduction?

- Weird: it maps an easier problem into a harder one
- Same sense as saying Maxwell reduced the problem of analyzing electricity & magnetism to solving partial differential equations
  - solving partial differential equations in general is a much harder problem than solving E&M problems

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A geek joke

- An engineer
  - is led in a kitchen with an empty kettle on the table and told to boil water; she fills the kettle with water, puts it on the stove, turns on the gas and boils water.
  - she is next confronted with a kettle full of water sitting on the counter and told to boil water; she puts it on the stove, turns on the gas and boils water.
- A mathematician
  - is led in a kitchen with an empty kettle on the table and told to boil water; he fills the kettle with water, puts it on the stove, turns on the gas and boils water.
  - he is next confronted with a kettle full of water sitting on the counter and told to boil water: he empties the kettle in the sink, places the empty kettle on the table and says, "I've reduced this to an already solved problem".

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A general phenomenon: Can't tell a book by its cover

- Suppose you have a problem A that asks given program code <P>, to determine some property of the input-output behavior of P, answering 1 if P has the property and 0 if P doesn't have the property.
- Rice's Theorem: If A's answer isn't always the same then there is no program deciding A

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Even harder problems

- Recall that with the halting problem, we could always get at least one of the two answers correct
  - if it halted we could always answer 1 (and this would cover precisely all 1's we need to do) but we can't be sure about answering 0
- There are natural problems where you can't even do that!
  - e.g. Given the codes of two programs, P and Q, answer 1 if they compute the same function and 0 if they compute different functions

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Quick lessons

- Don't rely on the idea of improved compilers and programming languages to eliminate major programming errors
  - truly safe languages can't possibly do general computation
- Document your code!!!!
  - there is no way you can expect someone else to figure out what your program does with just your code ....since....in general it is provably impossible to do this!

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