CSE 417: Algorithms and Computational Complexity

Winter 2001 Lecture 18 Instructor: Paul Beame

Turing Machines

- Church-Turing Thesis
 - Any reasonable model of computation that includes all possible algorithms is equivalent in power to a Turing machine

Evidence

Huge numbers of equivalent models to TM's based on radically different ideas

Universal Turing Machine

- A Turing machine interpreter U
 - I On input the code of a program (or Turing machine) P and an input x, U outputs the same thing as P does on input x
 - Basis for modern stored-program computer

Notation:

We'll write <P> for the code of program P and <P,x> for the pair of the program code and input

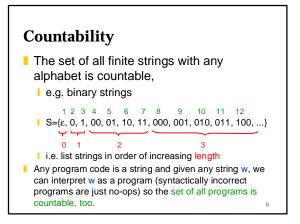
Halting Problem

- Given: the code of a program P and an input x for P, i.e. given <P,x>
- Output: 1 if P halts on input x and 0 if P does not halt on input x
- Theorem (Turing): There is no program that solves the halting problem "The halting problem is undecidable"

Proof ideas: Countability (Cantor 1875)

- Defn: A set S is countable iff there is a function mapping the natural numbers N onto S.
 - I i.e. we can write $S = \{s_1, s_2, s_3, ...\}$, i.e. $f(i) = s_i$
- All finite sets are countable.
- The natural numbers are countable
- The integers are countable
 - Z={0,1,-1,2,-2,3,-3,4,-4,5,-5,...}

1234567891011...



Uncountability

- The set of all functions f from the natural numbers N to {0,1} is not countable.
- Suppose it were and we had a list of all such functions {f₁, f₂, f₃,...}
- We build an infinite table of these functions

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	f_2	1	1	0	1	0	1	1	0	1	1	1	0		
	f_3	1	0	1	0	0	0	0	0	0	0	1	1		
	f_4	0	1	1	0	1	0	1	1	0	1	0	1		
_	f_5	0	1	0	0	1	1	1	0	0	0	1	1		
function	f_6	1	1	0	1	1	1	1	0	1	1	1	0		
^o	f_7	1	0	1	0	0	1	0	0	0	0	1	1		
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	f ₂	1	1	0	1	0	1	1	0	1	1	1	0		
	f ₃	1	0	1	0	0	0	0	0	0	0	1	1		
		0	1	1	0	1	0	1	1	0	1	0	1		
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	f_4	0	1	1	1	1	0	1	1	0	1	0	1.				
L _	f_5	0	1	0	0	0	1	1	0	0	0	1	1.				
function	f_6	1	1	0	1	1	0	1	0	1	1	1	0.	• • •			
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Diagonalization

- Define function D from N to {0,1} such that
 D(i)=1-f_i(i)
 - I i.e. we flipped the diagonal elements
- D must be different from every function in the list since D differs from f_i on input i
- Contradicts our assumption that the list had all such functions!
- Corollary: There is some function f from N to {0,1} not computed by any program
 more functions than programs!

Undecidability of the Halting Problem

- Suppose that there is a program H that computes the answer to the Halting Problem
- We'll build a similar table with all the possible programs down one side and all the possible inputs along the other and do a diagonal flip to produce a contradiction

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