CSE 417: Algorithms and Computational Complexity

Winter 2001

Lecture 10

Instructor: Paul Beame

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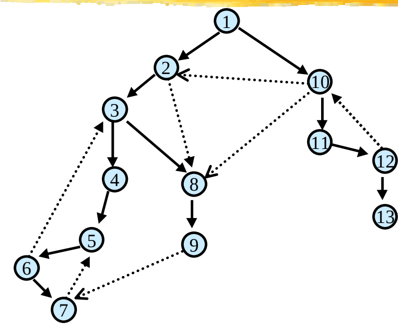
DFS(v) for a directed graph



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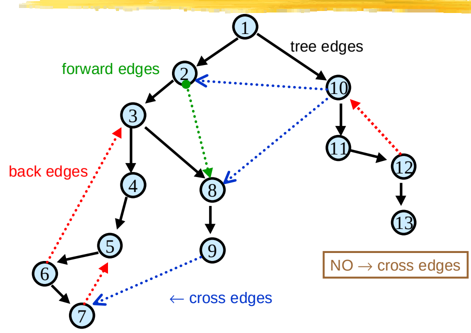
DFS(v)



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Strongly-connected components

- In directed graph if there is a path from a to b there might not be one from b to a
- a and b are **strongly connected** iff there is a path in both directions (i.e. a directed cycle containing both a and b
- Breaks graph into components

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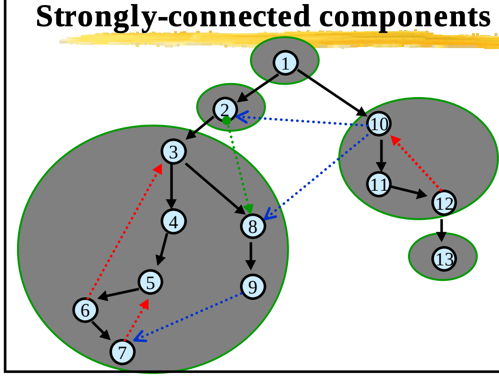
Strongly-connected components



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Uses for SCC's

- Optimizing compilers need to find loops, which are SCC's in the program flow graph.
- If (u,v) means process u is waiting for process v, SCC's show deadlocks.

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Directed Acyclic Graphs

- If we collapse each SCC to a single vertex we get a directed graph with no cycles
  - a **directed acyclic graph** or **DAG**
- Many problems on directed graphs can be solved as follows:
  - Compute SCC's and resulting DAG
  - Do one computation on each SCC
  - Do another computation on the overall DAG

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Simple SCC Algorithm

- u,v in same SCC iff there are paths $u \to v$ & $v \to u$
- DFS from every u, v: $O(nm) = O(n^3)$

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Better method

- Can compute all the SCC's while doing a single DFS! $O(n+m)$ time
- We won't do the full algorithm but will give some ideas

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Definition

The **root** of an SCC is the first vertex in it visited by DFS.

Equivalently, the root is the vertex in the SCC with the smallest number in DFS ordering.

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Subgoal

- All members of an SCC are descendants of its root.
- Can we identify some root?
- How about the root of the first SCC completely explored by DFS?
- Key idea: no exit from first SCC
  (first SCC is leftmost "leaf" in collapsed DAG)

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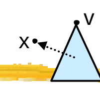
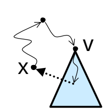
Definition



x is an *exit* from v (from v's subtree) if
- x is not a descendant of v, but
- x is the head of a (cross- or back-) edge from a descendant of v (including v itself)

- Any non-root vertex v has an exit



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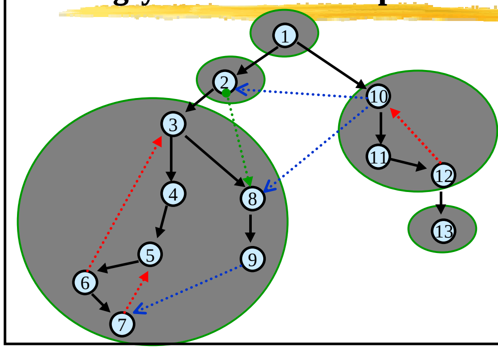
Finding SCC's

- A root nodes v sometimes have exits
  - only via a cross-edge to a node x that is not in a component with a root above v, e.g. vertex 10 in the example.

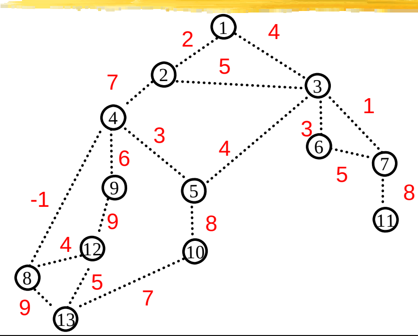
Strongly-connected components



Minimum Spanning Trees (Forests)

- Given an undirected graph G=(V,E) with each edge e having a weight w(e)
- Find a subgraph T of G of minimum total weight s.t. every pair of vertices connected in G are also connected in T
  - if G is connected then T is a tree otherwise it is a forest

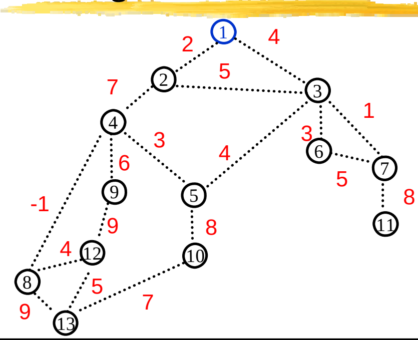
Weighted Undirected Graph



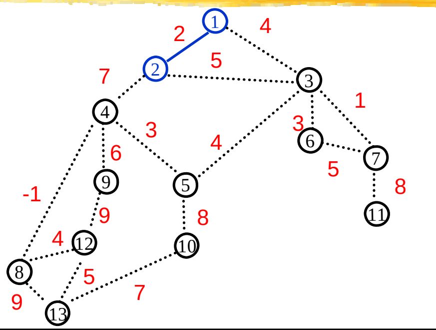
First Greedy Algorithm

- Prim's Algorithm:
  - start at a vertex v
  - add the cheapest edge adjacent to v
  - repeatedly add the cheapest edge that joins the vertices explored so far to the rest of the graph.
- We'll show it works later

Prim's Algorithm



Prim’s Algorithm



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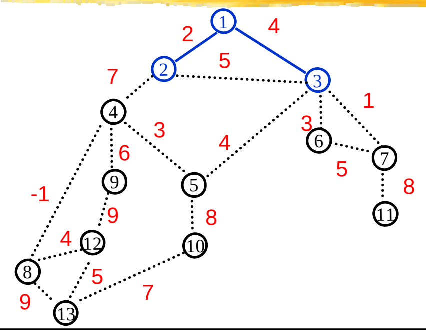
Prim’s Algorithm



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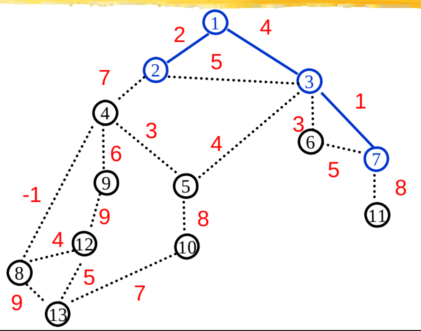
Prim’s Algorithm



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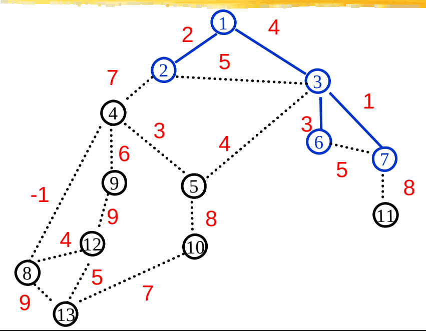
Prim’s Algorithm



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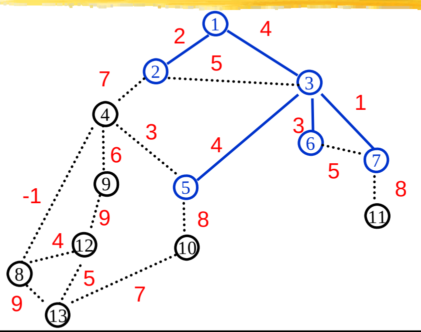
Prim’s Algorithm



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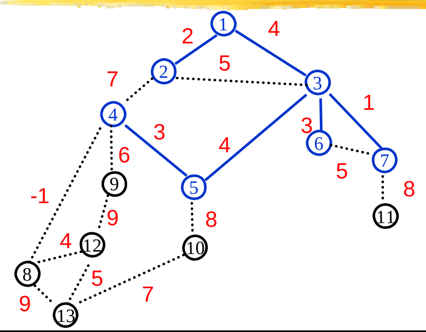
Prim’s Algorithm



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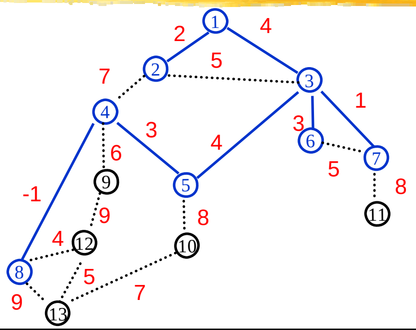
Prim’s Algorithm



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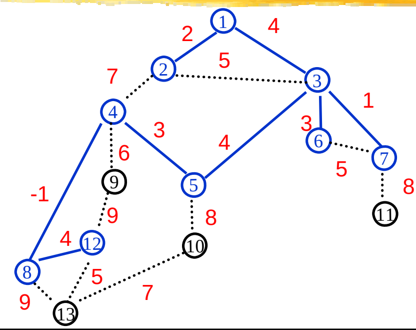
Prim’s Algorithm



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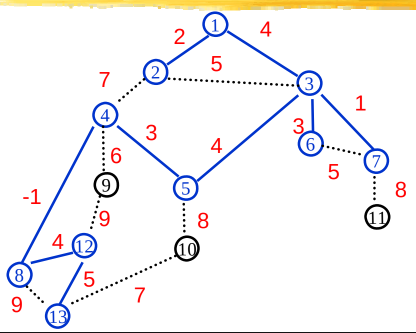
Prim’s Algorithm



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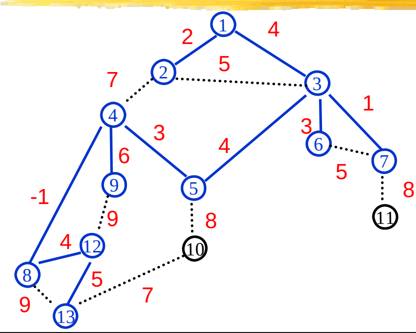
Prim’s Algorithm



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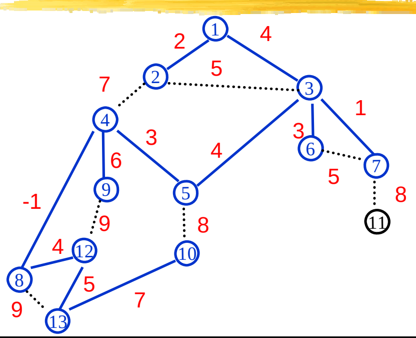
Prim’s Algorithm



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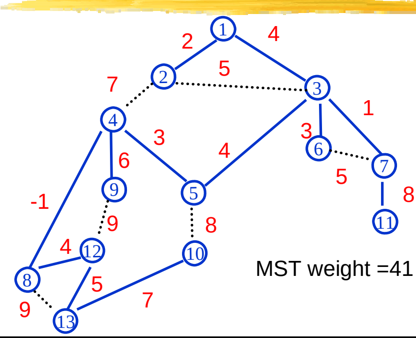

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Second Greedy Algorithm

- Kruskal's Algorithm
  - Start with the vertices and no edges
  - Repeatedly add the cheapest edge that joins two different components. i.e. that doesn't create a cycle
- Again we save the proof of correctness for later

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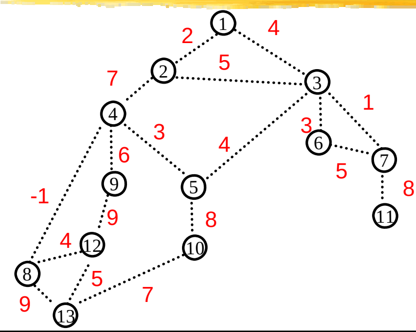
Kruskal's Algorithm



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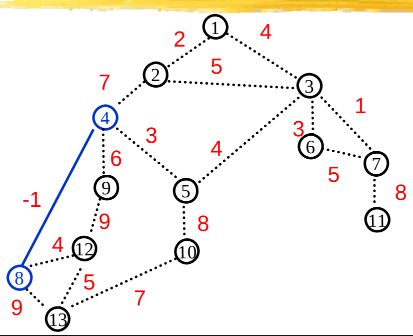
Kruskal's Algorithm



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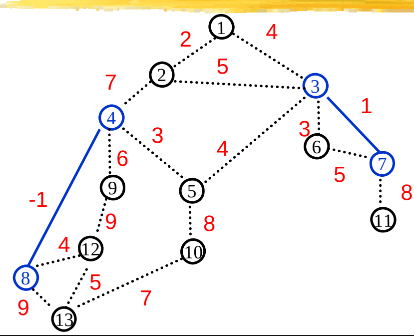
Kruskal's Algorithm



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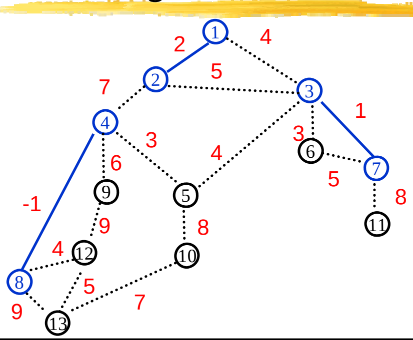
Kruskal's Algorithm



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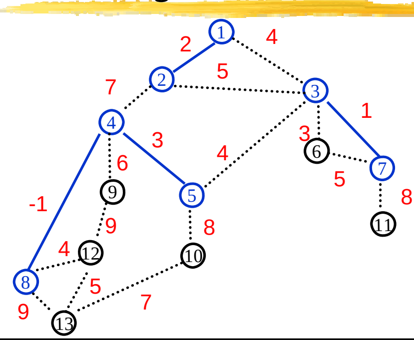
Kruskal's Algorithm



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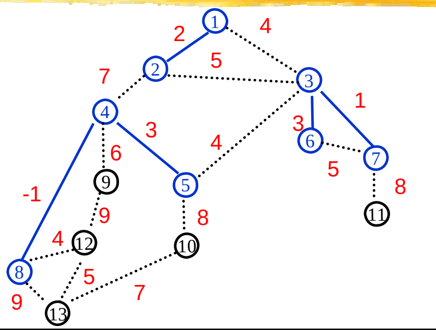

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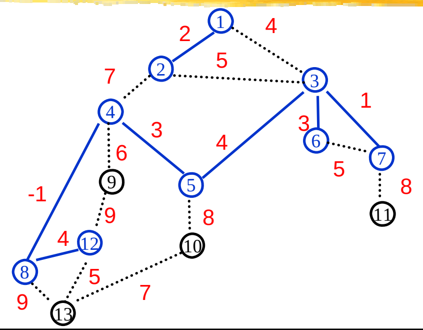

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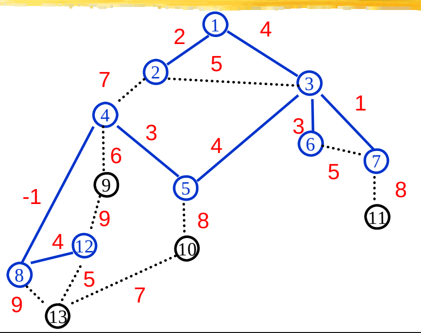

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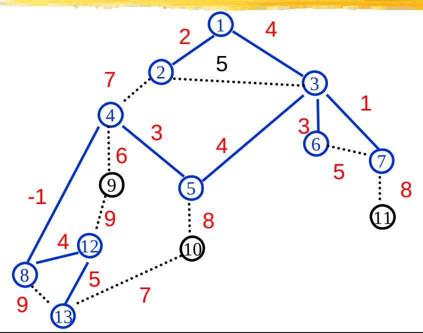

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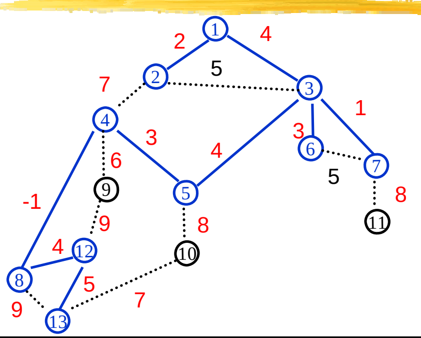

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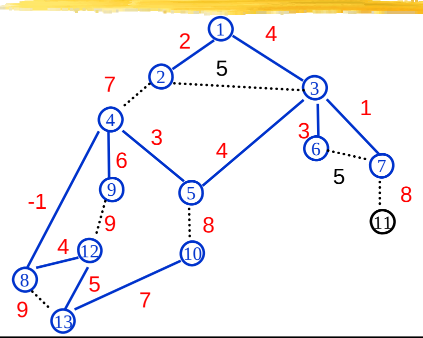
Kruskal's Algorithm



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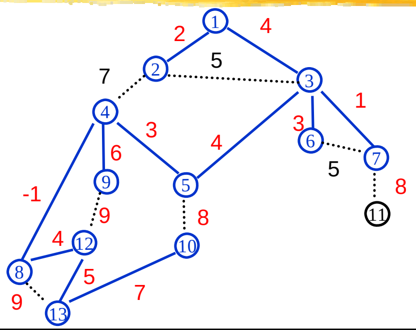

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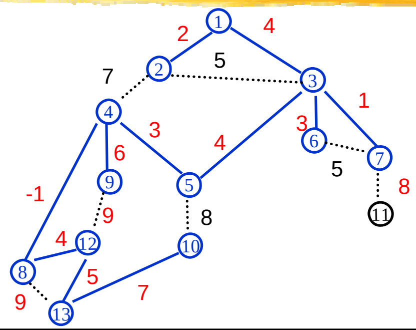

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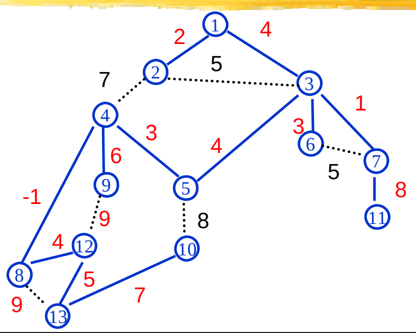

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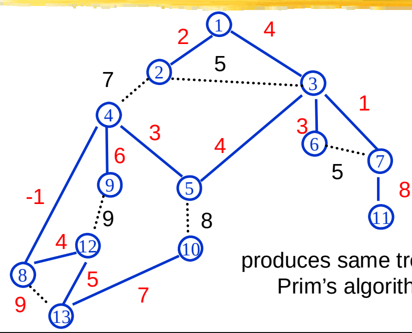
Kruskal's Algorithm



produces same tree as
Prim's algorithm

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