CSE/STAT 416
Recommender Systems: Matrix Factorization
Pre-Class Videos

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Matrix Completion

Want to recommend movies based on user ratings for movies.

**Challenge**: Users have rated relatively few of the entire catalog.

Can think of this as a matrix of users and ratings with missing data!
Assume that each item has \( k \) (unknown) features.
   e.g., \( k \) possible genres of movies (action, romance, sci-fi, etc.)

Then, we can describe an item \( v \) with feature vector \( R_v \)
   How much is the movie action, romance, sci-fi, ...
   e.g., \( R_v = [0.3, \ 0.01, \ 1.5, \ ...] \)

We can also describe each user \( u \) with a feature vector \( L_u \)
   How much they like action, romance, sci-fi, ....
   Example: \( L_u = [2.3, \ 0, \ 0.7, \ ...] \)

Estimate rating for user \( u \) and movie \( v \) as
\[
\text{Rating}(u, v) = L_u \cdot R_v = 2.3 \cdot 0.3 + 0 \cdot 0.01 + 0.7 \cdot 1.5 + ...
\]
Example

Suppose we have learned the following user and movie features using 2 features

<table>
<thead>
<tr>
<th>User ID</th>
<th>Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2, 0)</td>
</tr>
<tr>
<td>2</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>3</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>4</td>
<td>(2, 1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Movie ID</th>
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</tr>
<tr>
<td>3</td>
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</tr>
</tbody>
</table>

Then we can predict what each user would rate each movie

\[ L \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} R^T \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 4 \\ 4 & 3 & 3 \\ 1 & 2 & 1 \\ 7 & 4 & 5 \end{bmatrix} \]
Goal: Find $L_u$ and $R_v$ that when multiplied, achieve predicted ratings that are close to the values that we have data for.

Our quality metric will be (could use others)

$$\hat{L}, \hat{R} = \arg\min_{L,R} \sum_{u,v: \text{rating unknown}} (L_u \cdot R_v - r_{uv})^2$$
Is this problem well posed? Unfortunately, there is not a unique solution 😞

For example, assume we had a solution

\[
\begin{bmatrix}
6 & 2 & 4 \\
4 & 3 & 3 \\
1 & 2 & 1 \\
7 & 4 & 5
\end{bmatrix}
\]

Then doubling everything in \( L \) and halving everything in \( R \) is also a valid solution. The same is true for all constant multiples.

\[
\begin{bmatrix}
6 & 2 & 4 \\
4 & 3 & 3 \\
1 & 2 & 1 \\
7 & 4 & 5
\end{bmatrix}
\begin{bmatrix}
2 & 0 \\
1 & 1 \\
0 & 1 \\
2 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
3 & 1 & 2 \\
1 & 2 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
6 & 2 & 4 \\
4 & 3 & 3 \\
1 & 2 & 1 \\
7 & 4 & 5
\end{bmatrix}
\begin{bmatrix}
4 & 0 \\
2 & 2 \\
0 & 2 \\
4 & 2
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.5 & 0.5 & 1.0 \\
0.5 & 1.0 & 0.5
\end{bmatrix}
\]
You have $n$ users and $m$ items in your system
- Typically, $n \gg m$. E.g., Youtube: 2.6B users, 800M videos

Based on the content, we have a way of measuring user preference. This data is put together into a **user-item interaction matrix**.

**Task:** Given a user $u_i$ or item $v_j$, predict one or more items to recommend.
Solution 0: Popularity

**Simplest Approach:** Recommend whatever is popular
Rank by global popularity (i.e., Squid Game)
Solution 1: Nearest User (User-User)

User-User Recommendation:
Given a user \( u_i \), compute their \( k \) nearest neighbors.
Recommend the items that are most popular amongst the nearest neighbors.
Solution 2: “People Who Bought This Also Bought...” (Item-Item)

**Item-Item Recommendation:**

Create a co-occurrence matrix $C \in \mathbb{R}^{m \times m}$ ($m$ is the number of items). $C_{ij} =$ # of users who bought both item $i$ and $j$.

For item $i$, predict the top-k items that are bought together.}

![Co-occurrence Matrix](image)
**Problem:** popular items drown out the rest!

**Solution:** Normalizing using Jaccard Similarity.

\[
S_{ij} = \frac{\text{# purchased } i \text{ and } j}{\text{# purchased } i \text{ or } j} = \frac{C_{ij}}{C_{ii} + C_{jj} - C_{ij}}
\]

<table>
<thead>
<tr>
<th></th>
<th>Sunglasses</th>
<th>Baby Bottle</th>
<th>...</th>
<th>Diapers</th>
<th>Swim Trunks</th>
<th>Baby Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunglasses</td>
<td>1.00</td>
<td>0.03</td>
<td>...</td>
<td>0.02</td>
<td>0.23</td>
<td>0.04</td>
</tr>
<tr>
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<td>0.03</td>
<td>1.00</td>
<td>...</td>
<td>0.09</td>
<td>0.04</td>
<td>0.12</td>
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Solution 3:
Feature-Based
Solution 3: Feature-Based

What if we know what factors lead users to like an item?

Idea: Create a feature vector for each item. Learn a regression model!

<table>
<thead>
<tr>
<th>Genre</th>
<th>Year</th>
<th>Director</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>1994</td>
<td>Quentin Tarantino</td>
<td>...</td>
</tr>
<tr>
<td>Sci-Fi</td>
<td>1977</td>
<td>George Lucas</td>
<td>...</td>
</tr>
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Define weights on these features for all users (global)

\[ w_G \in \mathbb{R}^d \]

Fit linear model
Solution 3: Feature-Based

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Define weights on these features for **all users** (global)

\[ w_G \in \mathbb{R}^d \]

Fit linear model

\[
\hat{r}_{uv} = w_G^T h(v) = \sum_i w_{G,i} h_i(v)
\]

\[
\hat{w}_G = \text{argmin}_w \frac{1}{\# \text{ratings}} \sum_{u,v:r_{uv}\neq?} (w_G^T h(v) - r_{uv})^2 + \lambda \|w_G\|
\]
Personalization: Option A

Add user-specific features to the feature vector!

<table>
<thead>
<tr>
<th>Genre</th>
<th>Year</th>
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<th>Age</th>
</tr>
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<td>Quentin Tarantino</td>
<td>F</td>
<td>25</td>
</tr>
<tr>
<td>Sci-Fi</td>
<td>1977</td>
<td>George Lucas</td>
<td>M</td>
<td>42</td>
</tr>
</tbody>
</table>
Include a user-specified deviation from the global model.

\[ \hat{r}_{uv} = (\hat{w}_G + \hat{w}_u)^T h(v) \]

Start a new user at \( \hat{w}_u = 0 \), update over time.

- OLS on the residuals of the global model
- Bayesian Update (start with a probability distribution over user-specific deviations, update as you get more data)
Solution 3
(Feature-Based) Pros / Cons

**Pros:**
- No cold-start issue!
  - Even if a new user/item has no purchase history, you know features about them.
- Personalizes to the user and item.
- Scalable (only need to store weights per feature)
- Can add arbitrary features (e.g., time of day)

**Cons:**
- Hand-crafting features is very tedious and unscalable 😞
Solution 4: Matrix Factorization

Can we learn the features of items?
Assume that each item has $k$ (unknown) features.
   e.g., $k$ possible genres of movies (action, romance, sci-fi, etc.)

Then, we can describe an item $v$ with feature vector $R_v$
   How much is the movie action, romance, sci-fi, ...
   e.g., $R_v = [0.3, \quad 0.01, \quad 1.5, \quad ...]$

We can also describe each user $u$ with a feature vector $L_u$
   How much they like action, romance, sci-fi, ....
   Example: $L_u = [2.3, \quad 0, \quad 0.7, \quad ...]$

Estimate rating for user $u$ and movie $v$ as
   $\text{Rating}(u,v) = L_u \cdot R_v = 2.3 \cdot 0.3 + 0 \cdot 0.01 + 0.7 \cdot 1.5 + ...$
**Goal:** Find $L_u$ and $R_v$ that when multiplied, achieve predicted ratings that are close to the values that we have data for.

Our quality metric will be (could use others)

$$\hat{L}, \hat{R} = \arg\min_{L,R} \frac{1}{\# ratings} \sum_{u,v: r_{uv} \neq ?} (L_u \cdot R_v - r_{uv})^2$$

This is the MSE, but we are learning both “weights” (how much the user likes each feature) and item features!
Why Is It Called Matrix Factorization?

\[
\text{Rating} = L \approx R'
\]

Also called **Matrix Completion**, because this technique can be used to fill in missing values of a matrix.
Suppose we have learned the following user and movie features using 2 features:

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<td>3</td>
<td>(2, 1)</td>
</tr>
<tr>
<td>4</td>
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<td></td>
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What is the predicted rating user 1 will have of movie 2?

What is the highest predicted rating from this matrix factorization model? Which user made the prediction, for which movie?
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</table>

Then we can predict what each user would rate each movie

\[
L = \begin{pmatrix}
2 & 0 \\
1 & 1 \\
0 & 1 \\
2 & 1 \\
\end{pmatrix}
\quad R^T = \begin{pmatrix}
3 & 1 & 2 \\
1 & 2 & 1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
6 & 2 & 4 \\
4 & 3 & 3 \\
1 & 2 & 1 \\
7 & 4 & 5 \\
\end{pmatrix}
\]
Coordinate Descent
Find $\hat{L} \& \hat{R}$

Remember, our quality metric is

$$\hat{L}, \hat{R} = \text{argmin}_{L,R} \frac{1}{\# \text{ratings}} \sum_{u,v : r_{uv} \neq ?} (L_u \cdot R_v - r_{uv})^2$$

Gradient descent is not used in practice to optimize this, since it is much easier to implement coordinate descent (i.e., Alternating Least Squares) to find $\hat{L}$ and $\hat{R}$
**Coordinate Descent**

**Goal:** Minimize some function \( g(w) = g(w_0, w_1, \ldots, w_D) \)

Instead of finding optima for all coordinates, do it for one coordinate at a time!

*Initialize* \( \hat{w} = 0 \) (or smartly)

*while not converged:*

pick a coordinate \( j \)

\[
\hat{w}_j = \arg\min_{w_j} g(\hat{w}_0, \ldots, \hat{w}_{j-1}, w_j, \hat{w}_{j+1}, \ldots, \hat{w}_D)
\]

To pick coordinate, can do round robin or pick at random each time.

Guaranteed to find an optimal solution under some constraints
Coordinate Descent for Matrix Factorization

\[
\hat{L}, \hat{R} = \arg\min_{L,R} \frac{1}{\#\text{ratings}} \sum_{u,v: r_{uv} \neq ?} (L_u \cdot R_v - r_{uv})^2
\]

Fix movie factors \( R \) and optimize for \( L \)

\[
\hat{L} = \arg\min_{L} \frac{1}{\#\text{ratings}} \sum_{u,v: r_{uv} \neq ?} (L_u \cdot R_v - r_{uv})^2
\]

First key insight: users are independent!

\[
\hat{L}_u = \arg\min_{L_u} \frac{1}{\#\text{ratings for } u} \sum_{v: r_{uv} \neq ?} (L_u \cdot R_v - r_{uv})^2
\]
Coordinate Descent for Matrix Factorization

\[
\hat{L}_u = \arg\min_{L_u} \frac{1}{\# \text{ratings for } u} \sum_{v: r_{uv} \neq ?} (L_u \cdot R_v - r_{uv})^2
\]

**Second key insight:** this looks a lot like linear regression!

\[
\hat{w} = \arg\min_{w} \frac{1}{n} \sum_{i=1}^{n} (w \cdot h(x_i) - y_i)^2
\]

**Takeaway:** For a fixed \( R \), we can learn \( L \) using linear regression, separately per user.

Conversely, for a fixed \( L \), we can learn \( R \) using linear regression, separately per movie.
Want to optimize

\[ \hat{L}, \hat{R} = \arg\min_{L,R} \frac{1}{\# \text{ratings}} \sum_{u,v : r_{uv} \neq 0} (L_u \cdot R_v - r_{uv})^2 \]

Fix movie factors \( R \), and optimize for user factors separately

**Step 1:** Independent least squares for each user

\[ \hat{L}_u = \arg\min_{L_u} \frac{1}{\# \text{ratings for } u} \sum_{v \in V_u} (L_u \cdot R_v - r_{uv})^2 \]

Fix user factors, and optimize for movie factors separately

**Step 2:** Independent least squares for each movie

\[ \hat{R}_v = \arg\min_{R_v} \frac{1}{\# \text{ratings for } v} \sum_{u \in U_v} (L_u \cdot R_v - r_{uv})^2 \]

Repeatedly do these steps until convergence (to local optima)

System might be underdetermined: Use regularization
Consider we had the ratings matrix

<table>
<thead>
<tr>
<th></th>
<th>Movie 1</th>
<th>Movie 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>User 1</td>
<td>4</td>
<td>?</td>
</tr>
<tr>
<td>User 2</td>
<td>?</td>
<td>2</td>
</tr>
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During one step of optimization, user and movie factors are

<table>
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</tr>
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<tr>
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</tr>
<tr>
<td>User 2</td>
<td>[1, 1, 0]</td>
</tr>
<tr>
<td>Movie 1</td>
<td>[2, 1, 0]</td>
</tr>
<tr>
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</table>

Two questions:

What is the current MSE loss? (number)

Assume the next step of coordinate descent updates the user factors. Which factors would change?

- User 1
- User 2
- User 1 and 2
- None
Consider we had the ratings matrix

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**What is the current MSE loss?** (number)

**Assume the next step of coordinate descent updates the user factors. Which factors would change?**

- User 1
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- User 1 and 2
- None
Brain Break
Matrix Factorization is a very versatile technique!

Learns a latent space of topics that are most predictive of user preferences.

Learns the “topics” that exist in movie $v$: $\hat{R}_v$
Learns the “topic preferences” for user $u$: $\hat{L}_u$

Predict how much a user $u$ will like a movie $v$

$$\hat{\text{Rating}}(u, v) = \hat{L}_u \cdot \hat{R}_v$$

$$\text{Rating} = \begin{bmatrix} \text{known} \newline \text{unknown} \end{bmatrix} \approx \begin{bmatrix} \text{L} \newline \text{R'} \end{bmatrix}$$
Applications: Recommender Systems

Recommendations: (Semi-Supervised)

Use matrix factorization to predict user ratings on movies the user hasn’t watched

Recommend movies with highest predicted rating

<table>
<thead>
<tr>
<th>User</th>
<th>Movie</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>User 1</td>
<td><img src="image1.png" alt="Image 1" /></td>
<td><img src="image2.png" alt="Image 2" /></td>
</tr>
<tr>
<td>User 2</td>
<td><img src="image3.png" alt="Image 3" /></td>
<td><img src="image4.png" alt="Image 4" /></td>
</tr>
<tr>
<td>User 3</td>
<td><img src="image5.png" alt="Image 5" /></td>
<td><img src="image6.png" alt="Image 6" /></td>
</tr>
<tr>
<td>User 4</td>
<td><img src="image7.png" alt="Image 7" /></td>
<td><img src="image8.png" alt="Image 8" /></td>
</tr>
<tr>
<td>User 5</td>
<td><img src="image9.png" alt="Image 9" /></td>
<td><img src="image10.png" alt="Image 10" /></td>
</tr>
<tr>
<td>User 6</td>
<td><img src="image11.png" alt="Image 11" /></td>
<td><img src="image12.png" alt="Image 12" /></td>
</tr>
</tbody>
</table>

User 
Movie 
Rating
**Topic Modeling** (Unsupervised)

- Treat the TF-IDF matrix as the user-item matrix
  - Documents are "users"
  - Words are "items"

$L$ tells us which topics are present in each document

$R$ tells us what words each topic is composed of

Oftentimes, the topics are interpretable!

**HW7 Programming: Tweet Topic Modeling**
Solution 4 (Matrix Factorization) Pros / Cons

Pros:
- Personalizes to item and user!
- Learns latent features that are most predictive of user ratings.

Cons:
- Cold-Start Problem
  - What do you do about new users or items, with no data?
Common Issues with Recommender Systems

(and some solutions)
Recommender systems

- **Content based methods**
  Define a model for user-item interactions where users and/or items representations are given (explicit features).

- **Collaborative filtering methods**
  - **Model based**
    Define a model for user-item interactions where users and items representations have to be learned from interactions matrix.
  - **Memory based**
    Define no model for user-item interactions and rely on similarities between users or items in terms of observed interactions.

- **Hybrid methods**
  Mix content based and collaborative filtering approaches.
You are a software engineer at Spotify and have developed a matrix-factorization based recommendation system. The system works very well on existing users and songs, but does not work on new users or new songs.

How can you augment, extend, and/or modify your recommender system to handle new songs/users?
You are a software engineer at Spotify and have developed a matrix-factorization based recommendation system. The system works very well on existing users and songs, but does not work on new users or new songs.

How can you augment, extend, and/or modify your recommender system to handle new songs/users?
Comparing Recommender Systems

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<tr>
<th></th>
<th>Efficiency (Space, Deploy)</th>
<th>Efficiency (Time, Deploy)</th>
<th>Addresses Cold-Start?</th>
<th>Personalizes to User?</th>
<th>Discovers Latent Features?</th>
</tr>
</thead>
<tbody>
<tr>
<td>User-User</td>
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<td>Item-Item</td>
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<td>Feature-Based</td>
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<td>Matrix Factorization</td>
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<tr>
<td>Hybrid (Feature-Based + Matrix Factorization)</td>
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Featurized Matrix Factorization

**Feature-based approach**
- Feature representation of user and movie fixed
- Can address cold start problem

**Matrix factorization approach**
- Suffers from cold start problem
- User & Movie features are learned from data

**A unified model**
Cold-Start Problem

When a new user comes in, we don’t know what items they like! When a new item comes into our system, we don’t know who likes it! This is called the cold start problem.

Addressing the cold-start problem (for new users):

- Give random predictions to a new user.
- Give the globally popular recommendations to a new user.
- Require users to rate items before using the service.
- Use a feature-based model (or a hybrid between feature-based and matrix factorization) for new users.
Top-K versus Diverse Recommendations

Top-k recommendations might be very redundant

Someone who likes Rocky I also will likely enjoy Rocky II, Rocky III, Rocky IV, Rocky V

Diverse Recommendations

Users are multi-faceted & we want to hedge our bets

Maybe recommend: Rocky II, Always Sunny in Philadelphia, Robin Hood

Solution: Maximal Marginal Relevance

Pick recommendations one-at-a-time.

Select the item that the user is most likely to like and that is most dissimilar from existing recommendations.

- Hyperparameter $\lambda$ to trade-off between those objectives.
Feedback Loops / Echo Chambers

Users always get recommended similar content and are unable to discover new content they might like.

Exploration-Exploitation Dilemma
- Common problem in “online learning” settings

Pure Exploration: show users random content
- Users can discover new interests, but will likely be unsatisfied

Pure Exploitation: show users content they’re likely to like
- Users can’t discover new interests.

Solution: Multi-Armed Bandit Algorithms (beyond the scope of 416)
In the real-world, recommender systems guide us along a path through the content in a service.

- If watch video 1, recommend video 2
- If watch video 2, recommend video 3

A 2019 study found that YouTube’s algorithms lead users to more and more radical content.

- “Intellectual Dark Web” ➔ Alt-Lite ➔ Alt-Right

See more: iSchool 2021 Spring Lecture on Algorithmic Bias & Governance

Youtube’s response has been whack-a-mole. (Remove the content, manually tweak the recommendations for that topic)
2021 experiment on time-to-seeing radical alt-right content

Source: https://www.tiktok.com/@tofology/video/7016081760643534085?lang=en
Evaluating Recommender Systems
It is possible to evaluate recommender systems using existing metrics we have learnt:
- MSE (if predicting ratings)
- Accuracy (if predicting like/dislike, or click/ignore)

However, we don’t really care about accurately predicting what a user won’t like.

Rather, we care about finding the few items they will like.

Instead, we focus on the following metrics:

How many of our recommendations did the user like?
How many of the items that the user liked did we recommend?

Sound familiar?
Precision - Recall

Precision and recall for recommender systems

\[
\text{precision} = \frac{\# \text{liked} \& \text{shown}}{\# \text{shown}}
\]

\[
\text{recall} = \frac{\# \text{liked} \& \text{shown}}{\# \text{liked}}
\]

What happens as we vary the number of recommendations we make?

Best Recommender System:

- **Top-1**: high precision, low recall
- **Top-k (large k)**: high precision, high recall

Average Recommender System:

- **Top-1**: average precision, low recall
- **Top-k (large k)**: low precision, high recall
Precision - Recall Curves
Comparing Recommender Systems

In general, it depends

What is true always is that for a given precision, we want recall to be as large as possible (and vice versa)

What target precision/recall depends on your application

One metric: area under the curve (AUC)

Another metric: Set desired recall and maximize precision (precision at k)
Recap

Now you know how to:

- Describe the input (observations, number of “topics”) and output (“topic” vectors, predicted values) of a matrix factorization model
- Implement a coordinate descent algorithm for optimizing the matrix factorization objective presented
- Compare different approaches to recommender systems
- Describe the cold-start problem and ways to handle it (e.g., incorporating features)
- Analyze performance of various recommender systems in terms of precision and recall
- Use AUC or precision-at-k to select amongst candidate algorithms