CSE/STAT 416

Dimensionality Reduction & Recommender Systems Intro

Lecture 16

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Questions? Raise hand or **sli.do #cs416**
Listening to: Still Woozy

Announcements

-HW6 due tomorrow

-Final

- In person, pen and paper, June 3
- Time: 6:30 to 8:20 PM
- 1 hr 50 minutes

- Cheat Sheet

A4, two-sided

Dimensionality Reduction

Large Dimensionality

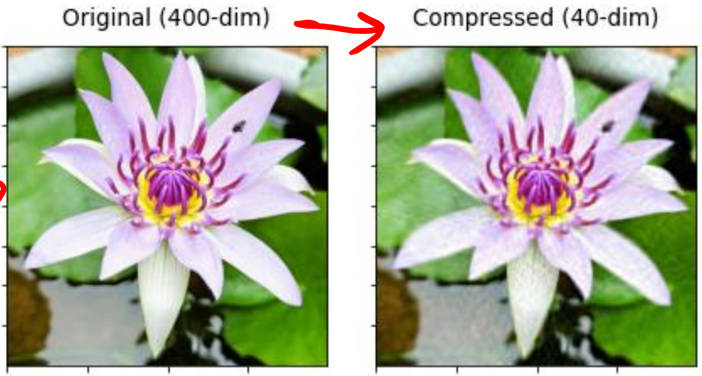
Input data might have thousands or millions of dimensions!

- **Images**: 200x200 image is 120,000 features!
- **Text**: # features = # n-grams 😮
- **Course Success**: dozen(s) of features
- **User Ratings**: 100s of ratings (one per rate-able item)

	Area Abbreviation	Area Code	Area	Item Code	Item	Element Code	Element	Unit	latitude	longitude	...	Y2004
0	AF	2	Afghanistan	2511	Wheat and products	5142	Food	1000 tonnes	33.94	67.71	...	3249.0
1	AF	2	Afghanistan	2805	Rice (Milled Equivalent)	5142	Food	1000 tonnes	33.94	67.71	...	419.0
2	AF	2	Afghanistan	2513	Barley and products	5521	Feed	1000 tonnes	33.94	67.71	...	58.0
3	AF	2	Afghanistan	2513	Barley and products	5142	Food	1000 tonnes	33.94	67.71	...	185.0
4	AF	2	Afghanistan	2514	Maize and products	5521	Feed	1000 tonnes	33.94	67.71	...	120.0

Issues with Too Many Dimensions

- **Visualization**: Hard to visualize more than 3D.
- **Overfitting**: Greater risk of overfitting with more features/dimensions
- **Scalability**: some ML approaches (e.g., k-nn, k-means) perform poorly in high-dimensional spaces (curse of dimensionality)
- **Redundancy**: high-dimensional data often occupies a lower-dimensional subspace.
  - Most pixels in MNIST (digit recognition) are white – are they necessary?
    - Image Compression



Dimensionality Reduction

<u>**Dimensionality Reduction**</u> is the the task of representing the data with a fewer number of dimensions, while keeping meaningful relations between data

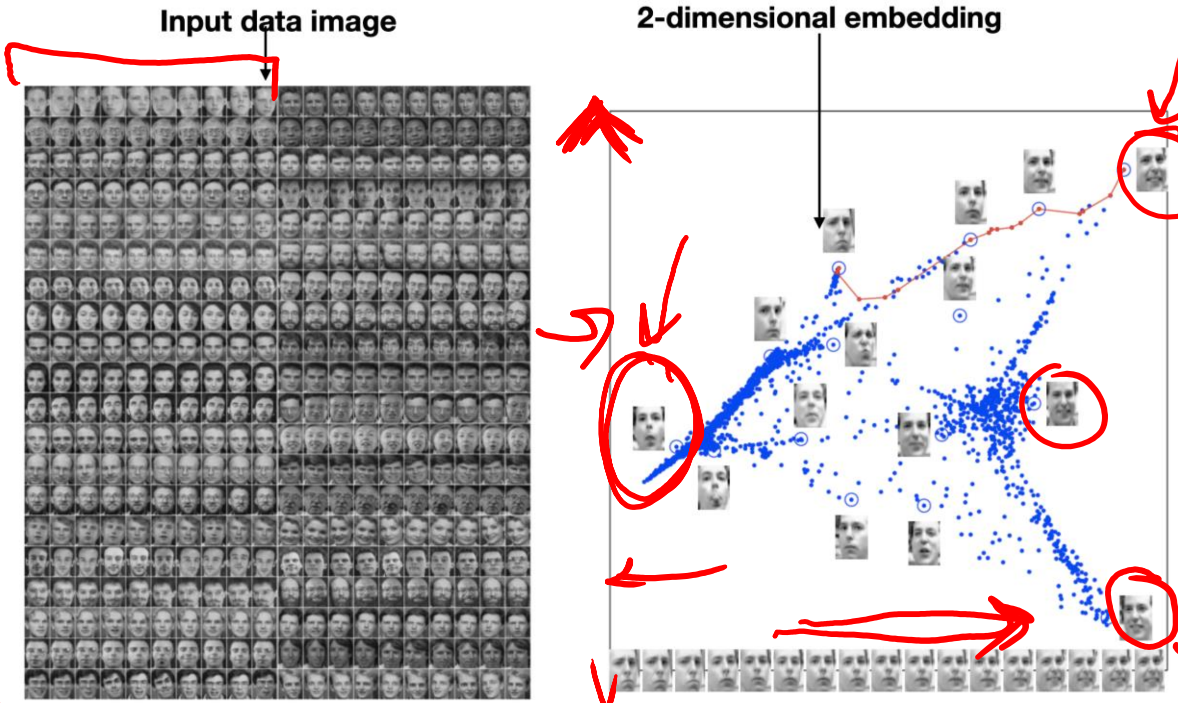
$200 \longrightarrow 10$

$200 \longrightarrow \boxed{10^8}$

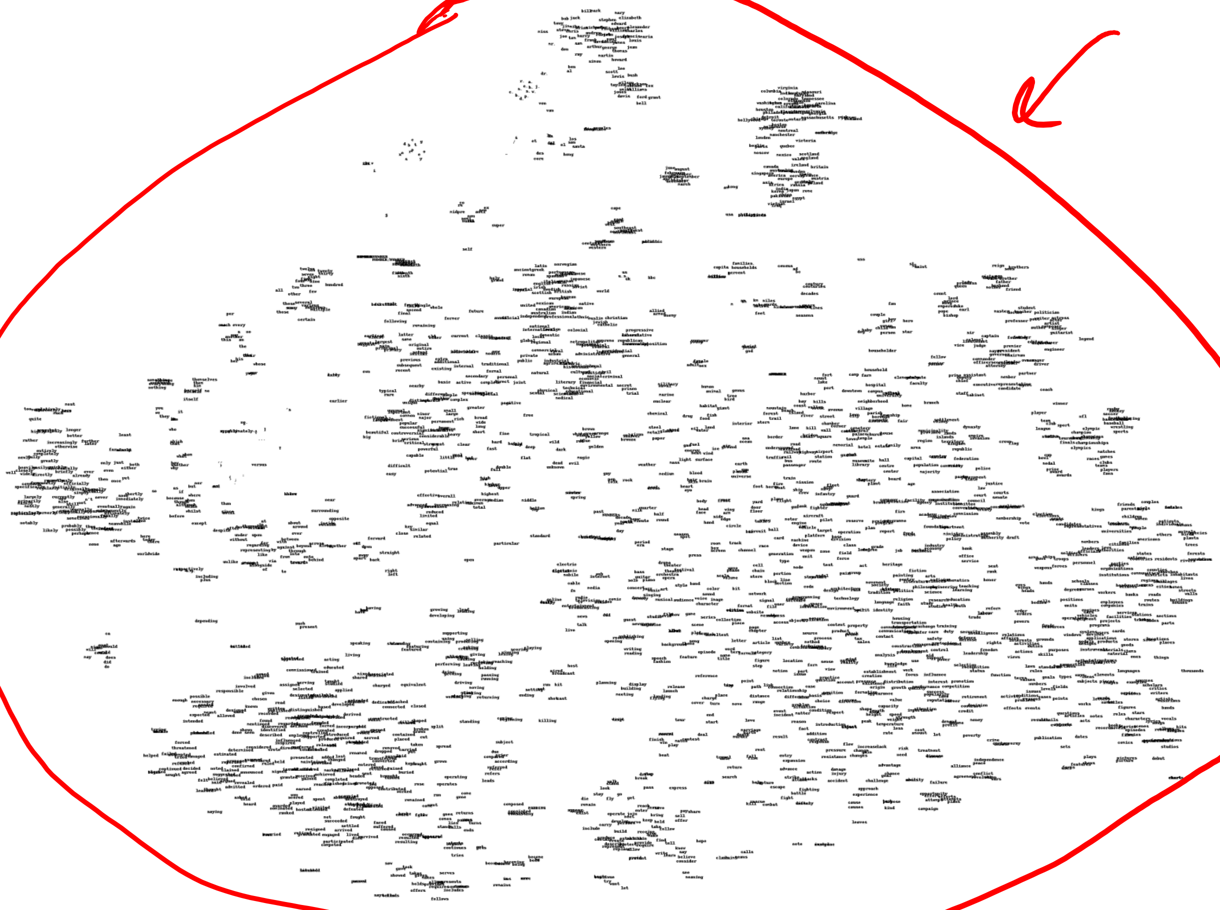
Example: Embedding Pictures

Example: Embed high dimensional data in low dimensions to visualize the data

- Goal: Similar images should be near each other.



Example: Embedding Words



Example: Embedding Words

Principal Component Analysis (PCA)

$d \rightarrow k$

error

One very popular dimensionality reduction algorithm is called **Principal Component Analysis (PCA)**.

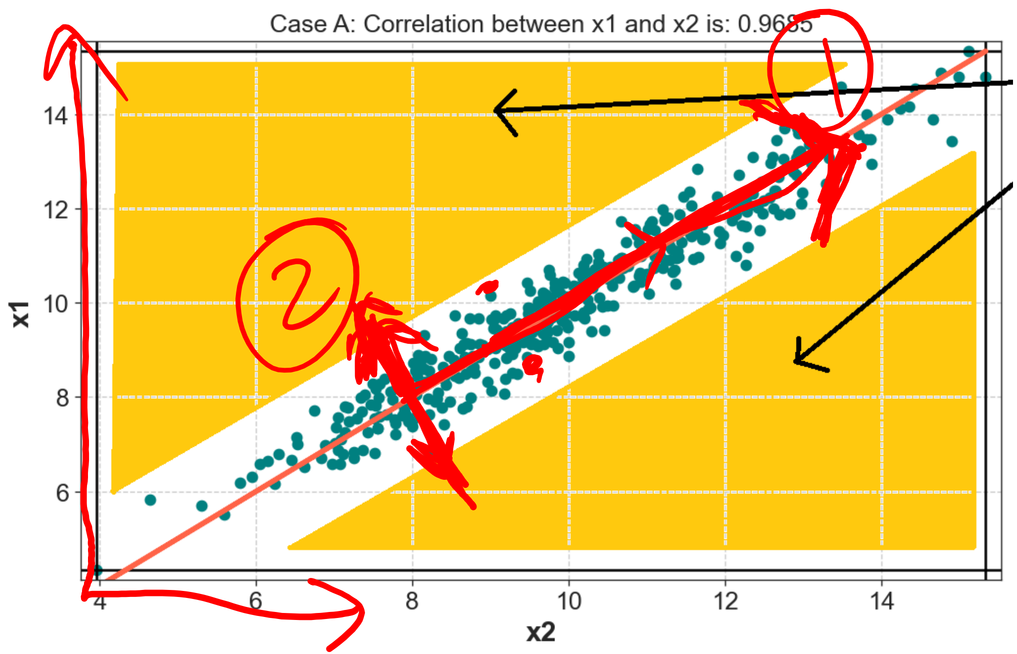
Idea: Use a linear projection from d-dimensional data to k-dimensional data

- E.g. 1000 dimension word vectors to 3 dimensions

Choose the projection that minimizes **reconstruction error**

- Idea: The information lost if you were to "undo" the projection

Principal Component Analysis (PCA)

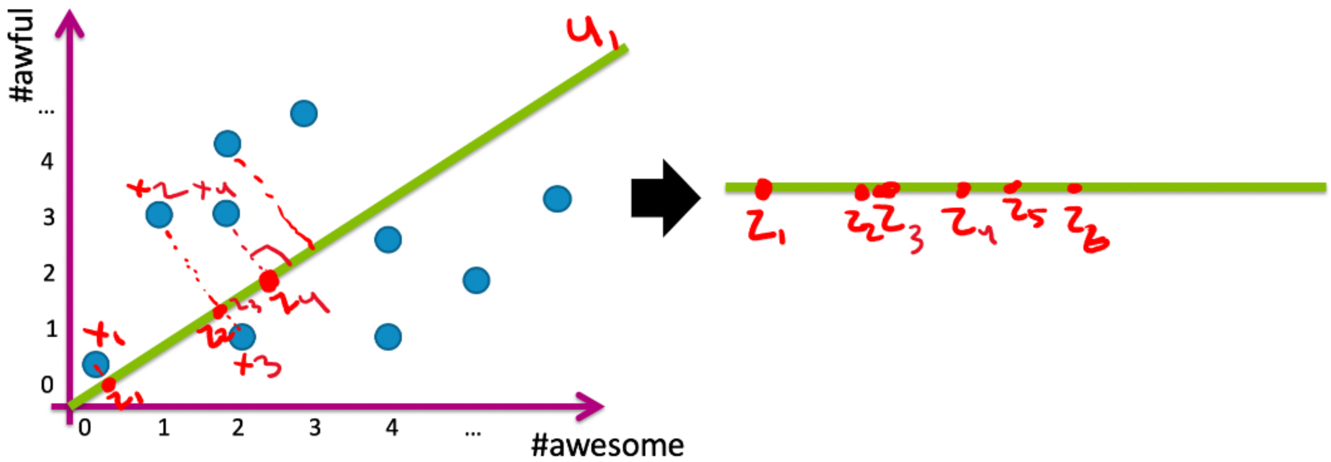


Regions with no data. Data exists close to a lower-dimensional subspace.

Linear Projection

Linear Projection of $\vec{x}_1$ onto $\vec{u}_1$ is the point on $\vec{u}_1$ that is closest to $\vec{x}_1$

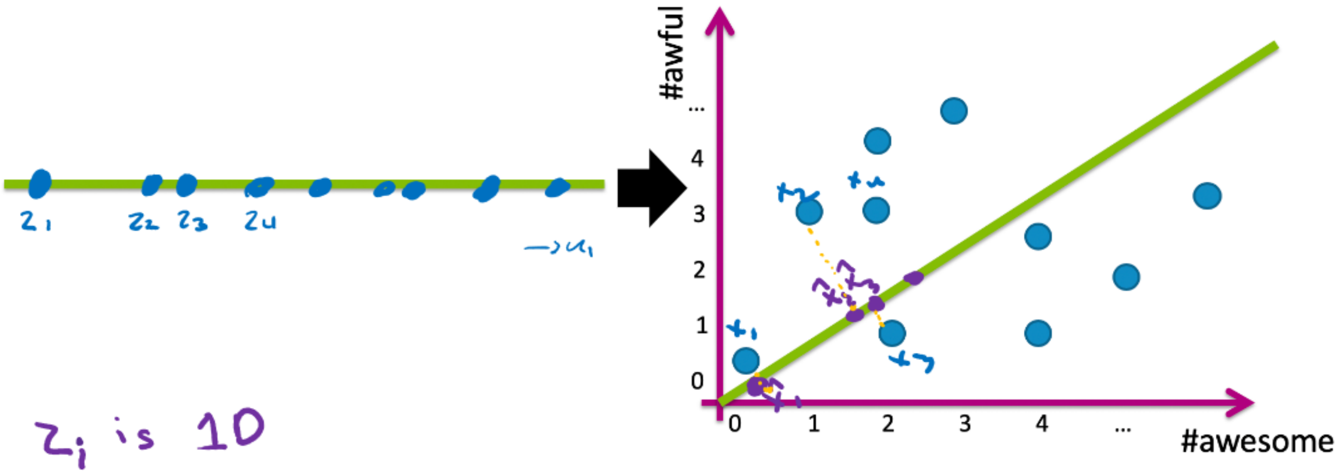
Project data into 1 dimension along a line



$$z_i = u_i^T x_i = \sum_{j=1}^{D} u_i[j] \cdot x_i[j]$$

Reconstruction

Reconstruct original data only knowing the projection



z_i is 1D

$\vec{u}_1$ is 2D vector

$\hat{x}_i = z_i u_i$

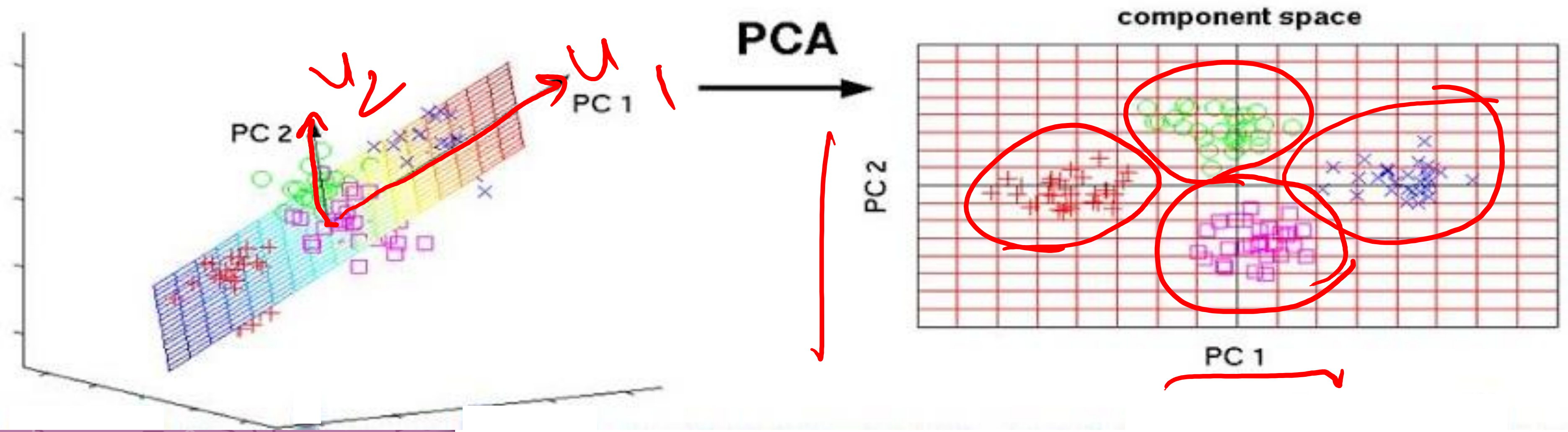
Reconstruction Error

$\|\hat{x}_i - x_i\|_2^2$

Linear Projection in Higher Dimensions

Think of PCA as giving each datapoint a new "address."

- Earlier, you could find the datapoint by going to the location (x, y, z).
- Now, if you are just moving in the projection plane, you can (approximately) find the datapoint by going to the location (u_1, u_2)



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Group

2 min

$x_i \in \mathbb{R}^5$

$z_i \in \mathbb{R}^2$

$z_{i,1} = \sum_{j=1}^{D} u_1[j] \cdot x_i[j]$

$z_{i,2} = \sum_{j=1}^{D} u_2[j] \cdot x_i[j]$

Compute the 2D coordinates of the following point. Then compute its reconstruction error.

Note that $u_1 \cdot u_2 = 0$

- $x_i = [0, -7, 3, 2, 5]$
- $u_1 = [-0.5, 0, 0.5, -0.5, 0.5]$
- $u_2 = [0.5, 0, 0.5, -0.5, -0.5]$
- $z_i = ??$ $(\underline{z_{i,1}}, \underline{z_{i,2}})$
- $\hat{x}_i = ??$ $(\_, \_, \_, \_, \_)$
- $\|\hat{x}_i - x_i\|_2^2 = ??$

slido

Group

2 min

$$z_{i,1} = \sum_{j=1}^{5} u_1[j]\cdot x_i[j] = -\tfrac{1}{2}\cdot 0 + 0\cdot(-7) + \tfrac{1}{2}\cdot 3 - \tfrac{1}{2}\cdot 2 + \tfrac{1}{2}\cdot 5 = 3$$

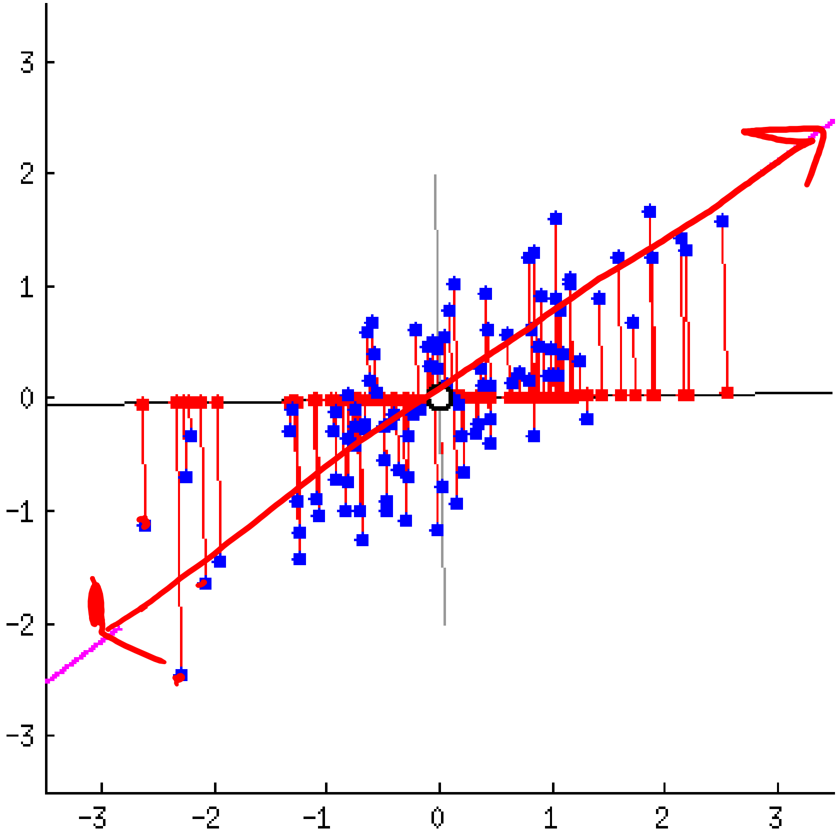
$$z_{i,2} = \sum_{j=1}^{5} u_2[j]\cdot x_i[j] = \tfrac{1}{2}\cdot 0 + 0\cdot(-7) + \tfrac{1}{2}\cdot 3 - \tfrac{1}{2}\cdot 2 - \tfrac{1}{2}\cdot 5 = -2$$

Compute the 2D coordinates of the following point. Then compute its reconstruction error.

- $x_i = [0, -7, 3, 2, 5]$
- $u_1 = [-0.5, 0, 0.5, -0.5, 0.5]$
- $u_2 = [0.5, 0, 0.5, -0.5, -0.5]$
- $z_i = ??$ $[3, -2]$
- $\hat{x}_i = ??$ $[-2.5, 0, 0.5, -0.5, 2.5]$
- $\|\hat{x}_i - x_i\|_2^2 = ??$ $(-2.5-0)^2 + (0+7)^2 + (0.5-3)^2 + (-0.5-2)^2 + (2.5-5)^2 = 74$

$$\hat{x}_i = z_{i,1}\cdot u_1 + z_{i,2}\cdot u_2 = \left[-\tfrac{3}{2}, 0, \tfrac{3}{2}, -\tfrac{3}{2}, \tfrac{3}{2}\right] + [-1, 0, -1, 1, 1] = [-2.5, 0, 0.5, -0.5, 2.5]$$

How do we find the best projection vector(s)?



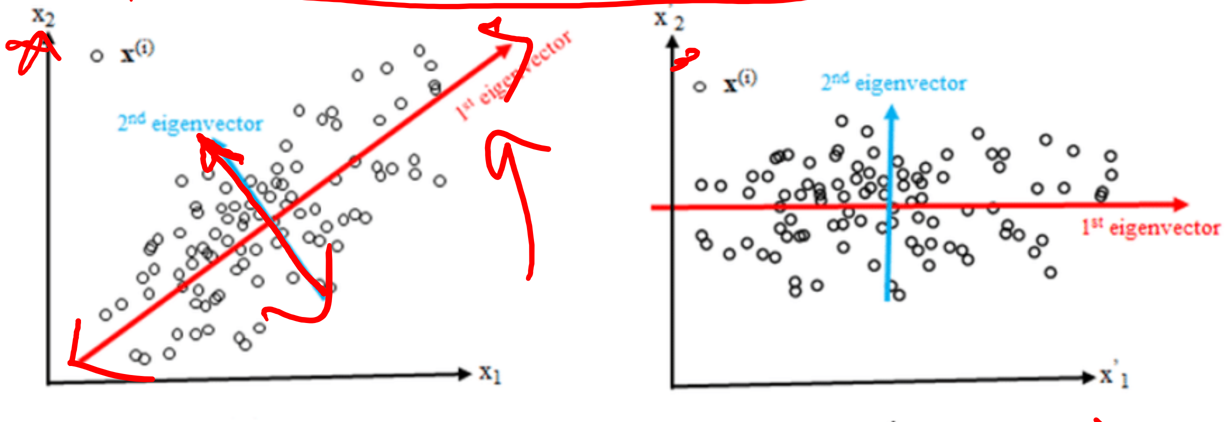
Pick the vector(s) along which the datapoints have the most variation!

Eigenvectors

num dimensions

= num. of total eigenvalues

- There is a quantity in linear algebra that does exactly that!
- The **eigenvectors** of a d-dimensional dataset* are a collection of d *perpendicular* vectors that point in the directions of greatest variation amongst the points in the dataset.



a b

- Eigenvectors rotate the axes of the d dimensional space.

* (caveat) the eigenvectors are actually associated with the *covariance matrix* of the dataset

$d \rightarrow K$ PCA transform

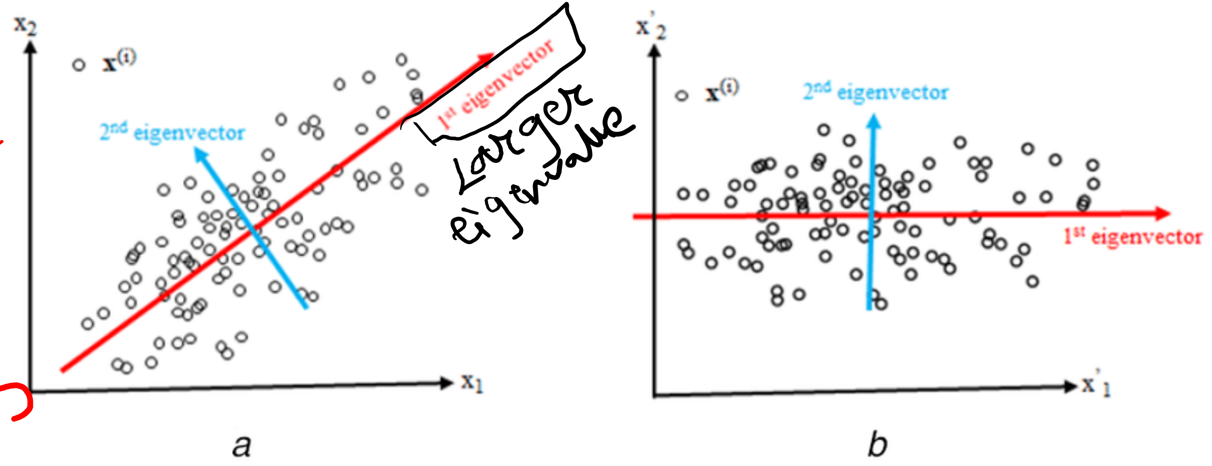
Eigenvalues

is $K=5$, $d=10$

10 eigenvalues

pick highest 5 eigenvalues

- Each eigenvector has a corresponding **eigenvalue**, indicating how much the dataset varies in that direction.
- Greater eigenvalue ➔ greater variance

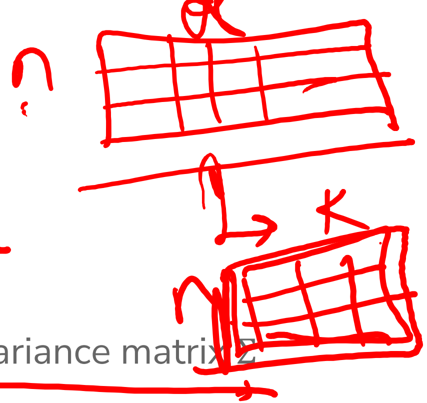


- **PCA**: Take the k eigenvectors with greatest eigenvalues.

PCA Algorithm

Input Data: An $n \times d$ data matrix X

- Each row is an example



1. **Center Data**: Subtract mean from each row

$$X_c \leftarrow X - \bar{X}[1{:}d]$$

2. **Compute spread/orientation**: Compute covariance matrix Σ

$$\boxed{\Sigma[t,s] = \frac{1}{n}\sum_{i=1}^{n} x_{c,i}[t]x_{c,i}[s]}$$

3. **Find basis for orientation**: Compute eigenvectors of Σ
   - Select k eigenvectors $u_1, \ldots, u_k$ with largest eigenvalues

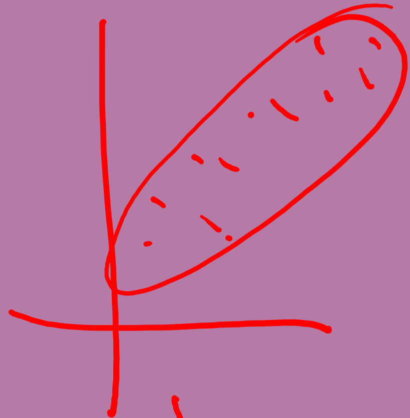
4. **Project Data**: Project data onto principal components

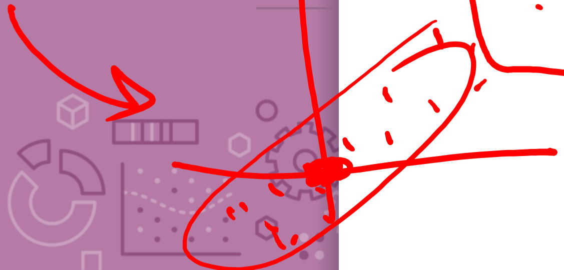
$$z_i[1] = u_1^T x_{c,i} = u_1[1]x_{c,i}[1] + \cdots + u_1[d]x_{c,i}[d]$$

$$\ldots$$

$$z_i[k] = u_k^T x_{c,i} = u_k[1]x_{c,i}[1] + \cdots + u_k[d]x_{c,i}[d]$$

K





Reconstructing Data

Using principal components and the projected data, you can reconstruct the data in the original domain.

$$\hat{x}_i[1:d] = \bar{X}[1:d] + \sum_{j=1}^{k} z_i[j]\ u_j$$

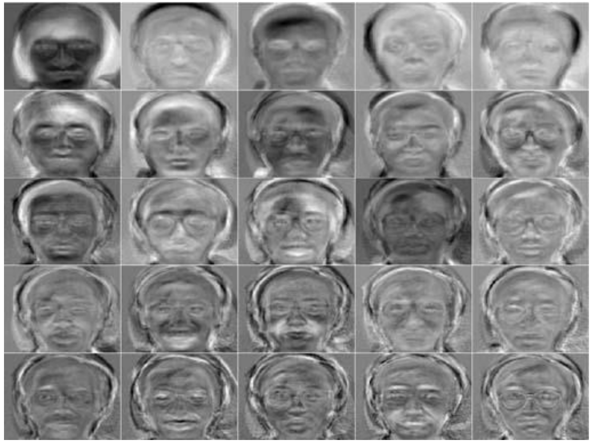
Example: Eigenfaces

Apply PCA to face data

Input Data

25

Principal Components



Reconstructing Faces

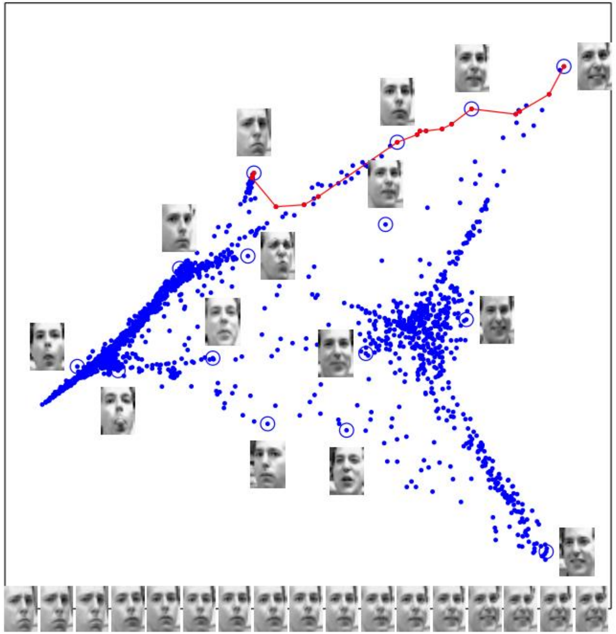
Depending on context, it may make sense to look at either original data or projected data.

In this case, let's see how the original data looks after using more and more principal components for reconstruction.

- Each image shows additional 8 principal components

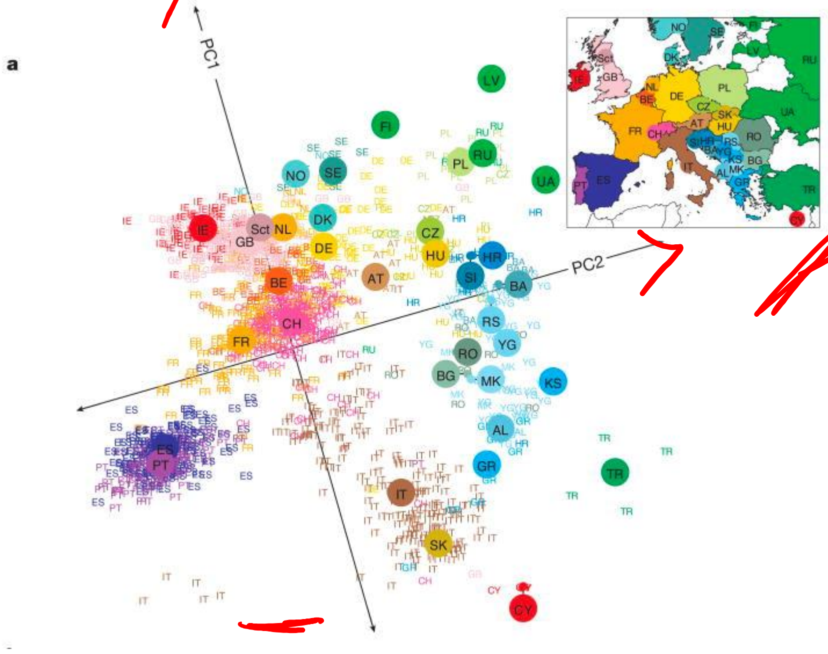
Embedding Images

Other times, it does make sense to look at the data in the projected space! (Usually if $k \leq 3$)



Example: Genes

Dataset of genes of Europeans (3192 people; 500,568 loci) and their country of origin, ran PCA on the data and plotted 2 principal components.



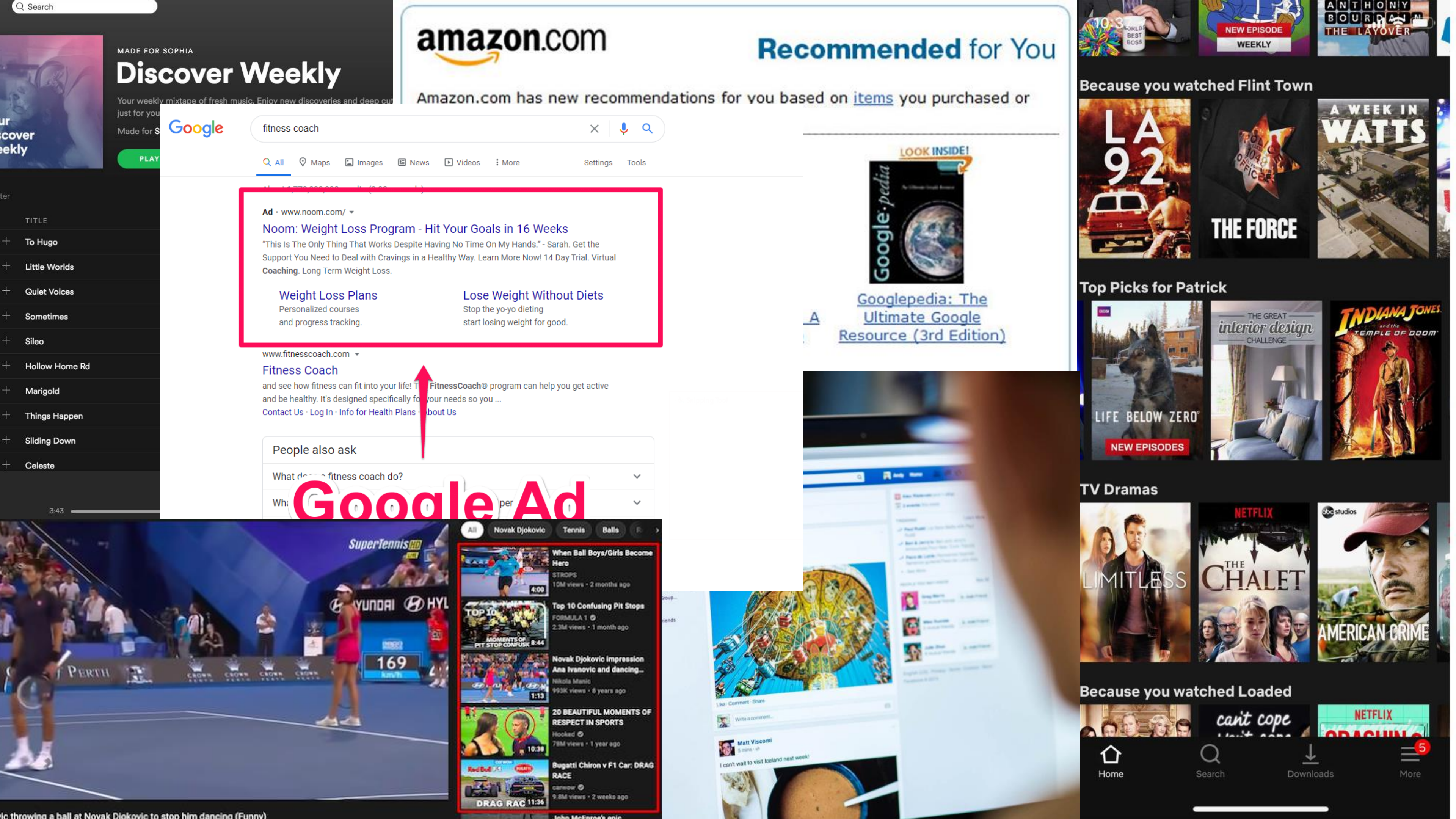
Brain Break

General Steps to Take as an ML Practitioner

Given a new dataset:

- Split into train and test sets.
- Understand the dataset:
  - Understand the feature/label types and values
  - Visualize the data: scatterplot, boxplot, PCA, clustering
- Use that intuition to decide:
  - What features to use, and what transformations to apply to them (pre-processing).
  - What model(s) to train.
- Train the models, evaluate them using a validation set or cross-validation.
- Deploy the best model.

Intro to Recommender Systems



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amazon.com
Recommended for You
Amazon.com has new recommendations for you based on items you purchased or
Because you watched Flint Town
Top Picks for Patrick
TV Dramas
Because you watched Loaded
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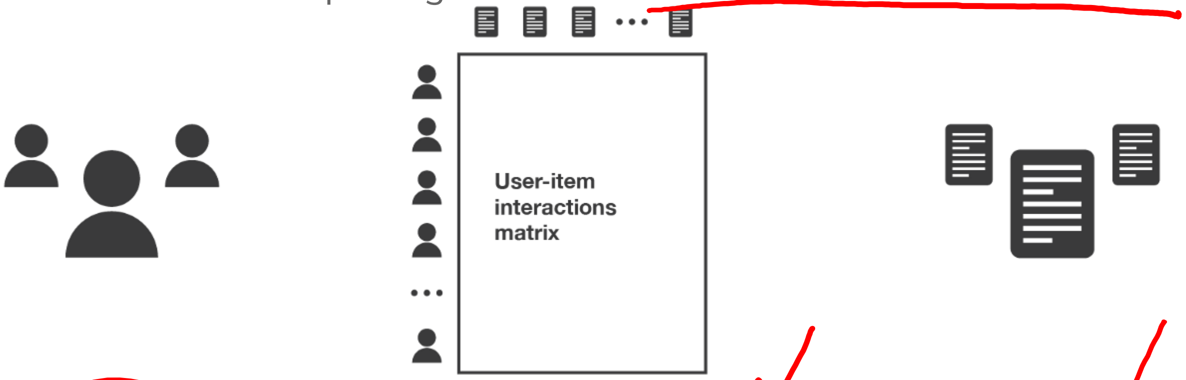
Recommender Systems Setup

- You have n users and m items in your system
  - Typically, $n \gg m$. E.g., Youtube: 2.6B users, 800M videos
- Based on the content, we have a way of measuring user preference.
- This data is put together into a **user-item interaction matrix**.



Users	User-item interactions matrix	Items
suscribers	rating given by a user to a movie (integer)	movies
readers	time spent by a reader on an article (float)	articles
buyers	product clicked or not when suggested (boolean)	products

• • •

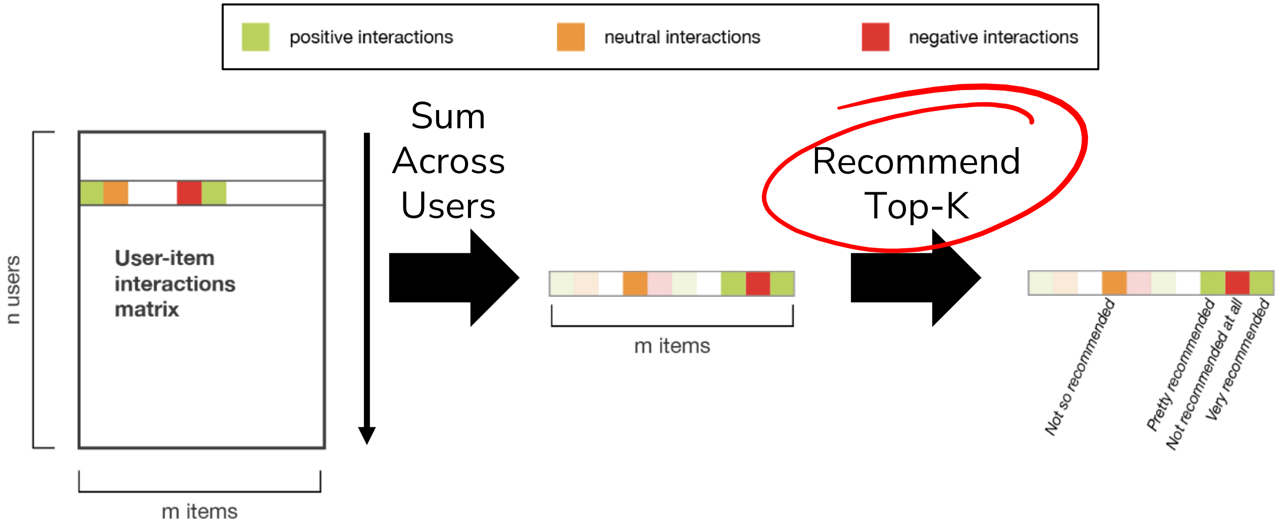
- **Task**: Given a user u_i or item v_j, predict one or more items to recommend.

Solution 0: Popularity

Solution 0: Popularity

Simplest Approach: Recommend whatever is popular

- Rank by global popularity (i.e., Squid Game)



Solution 0 (Popularity) Pros / Cons

Pros:

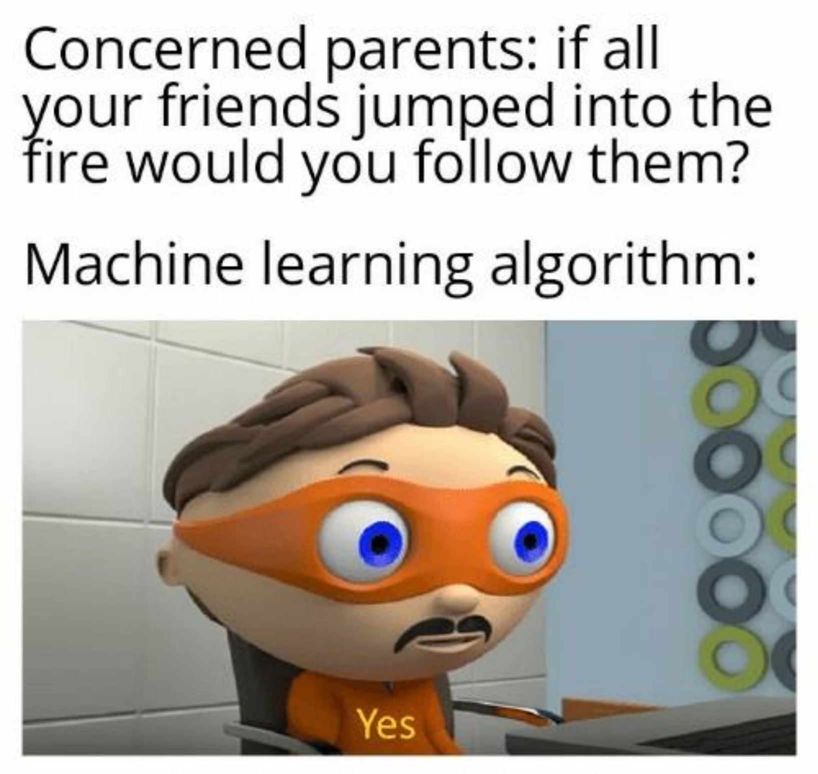
- Easy to implement

Cons:

- No Personalization
- Feedback Loops
- Top-K recommendations might be redundant
  - e.g., when a new Harry Potter movie is released, the others may also jump into top-k popularity.

Solution 1: Nearest User

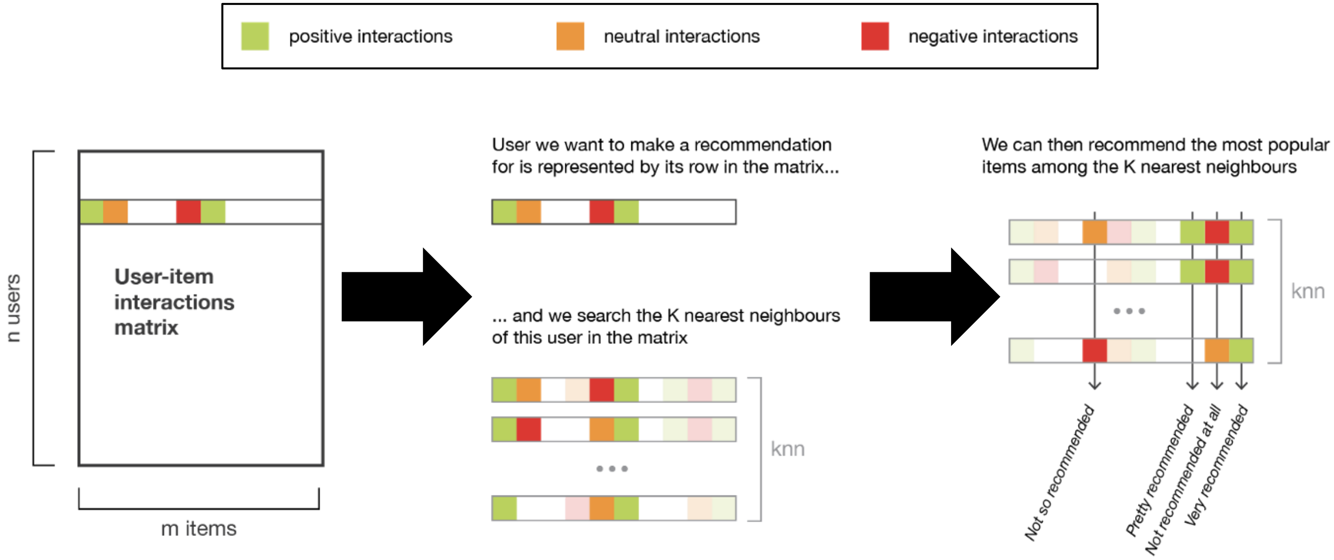
User-User



Solution 1: Nearest User (User-User)

User-User Recommendation:

- Given a user u_i, compute their k nearest neighbors.
- Recommend the items that are most popular amongst the nearest neighbors.



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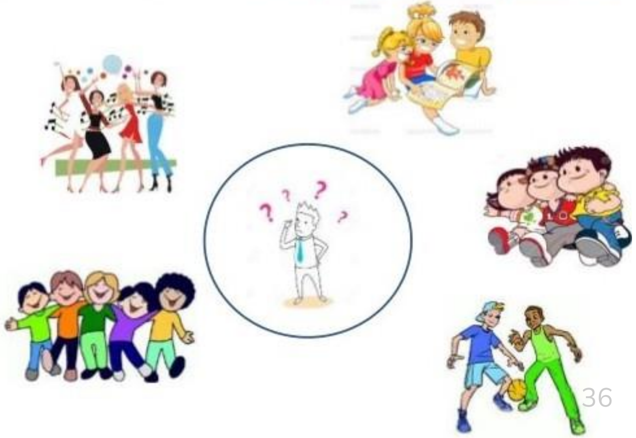
Group

2 min

- What do you see as pros / cons of the nearest user approach to recommendations?

Tell me about your friends(*who your neighbors are*) and *I will tell you who you are*.



Solution 1 (User-User) Pros / Cons

Pros:

- Personalized to the user.

Cons:

- Nearest Neighbors might be too similar
  - This approach only works if the nearest neighbors have interacted with items that the user hasn't.
- Feedback Loop (Echo Chambers)
- Scalability
  - Must store and search through entire user-item matrix
- Cold-Start Problem
  - What do you do about new users, with no data?

Solution 2: “People Who Bought This Also Bought…”

Item-Item

Solution 2: "People Who Bought This Also Bought…" (Item-Item)

Item-Item Recommendation:

- Create a **co-occurrence matrix** $C \in \mathbb{R}^{m \times m}$ (m is the number of items). C_{ij} = # of users who bought both item i and j.
- For item i, predict the top-k items that are bought together.

	Sunglasses	Baby Bottle	…	Diapers	Swim Trunks	Baby Formula
Sunglasses	500	15	…	9	130	20
Baby Bottle	15	45	…	6	10	10
…	…	…	…	…	…	…
Diapers	9	6	…	30	9	6
Swim Trunks	130	10	…	9	200	8
Baby Formula	20	10	…	6	8	50

Normalizing Co-Occurence Matrices

Problem: popular items drown out the rest!

Solution: Normalizing using Jaccard Similarity.

$$S_{ij} = \frac{\text{\# purchased } i \text{ and } j}{\text{\# purchased } i \text{ or } j} = \frac{C_{ij}}{C_{ii} + C_{jj} - C_{ij}}$$

	Sunglasses	Baby Bottle	...	Diapers	Swim Trunks	Baby Formula
Sunglasses	1.00	0.03	...	0.02	0.23	0.04
Baby Bottle	0.03	1.00	...	0.09	0.04	0.12
...	...	...	...	...	...	...
Diapers	0.02	0.09	...	1.00	0.04	0.08
Swim Trunks	0.23	0.04	...	0.04	1.00	0.03
Baby Formula	0.04	0.12	...	0.08	0.03	1.00

Incorporating Purchase History

What if I know the user u has bought a baby bottle and formula?

Idea: Take the average similarity between items they have bought

$$Score(u, diapers) = \frac{S_{diapers,baby\ bottle} + S_{diapers,baby\ formula}}{2}$$

Could also weight them differently based on recency of purchase!

Then all we need to do is find the item with the highest average score!

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Group

2 min

- What do you see as pros / cons of the item-item approach to recommendations?

Solution 2 (Item-Item) Pros / Cons

Pros:

- Personalizes to item (incorporating purchase history also personalizes to the user)

Cons:

- Can still suffer from feedback loops
  - (As can all recommender systems – but in some cases it's worse than others)
- Scalability (must store entire item-item matrix)
- Cold-Start Problem
  - What do you do about new ***items***, with no data?

Solution 3: Feature-Based

Solution 3: Feature-Based

What if we know what factors lead users to like an item?

Idea: Create a feature vector for each item. Learn a regression model!

Genre	**Year**	**Director**	**...**
Action	1994	Quentin Tarantino	...
Sci-Fi	1977	George Lucas	...

Define weights on these features for **all users** (global)

$$w_G \in \mathbb{R}^d$$

Fit linear model

Solution 3: Feature-Based

What if we know what factors lead users to like an item?

Idea: Create a feature vector for each item. Learn a regression model!

Genre	**Year**	**Director**	**...**
Action	1994	Quentin Tarantino	...
Sci-Fi	1977	George Lucas	...

Define weights on these features for **all users** (global)

$$w_G \in \mathbb{R}^d$$

Fit linear model

$$\hat{r}_{uv} = w_G^T h(v) = \sum_i w_{G,i}\, h_i(v)$$

$$\hat{w}_G = argmin_w \frac{1}{\#\, ratings} \sum_{u,v: r_{uv} \neq ?} \left(w_G^T h(v) - r_{uv}\right)^2 + \lambda \|w_G\|$$

Personalization: Option A

Add user-specific features to the feature vector!

Genre	**Year**	**Director**	**...**	**Gender**	**Age**	**...**
Action	1994	Quentin Tarantino	...	F	25	...
Sci-Fi	1977	George Lucas	...	M	42	...

Personalization: Option B

Include a user-specified deviation from the global model.

$$\hat{r}_{uv} = (\hat{w}_G + \hat{w}_u)^T h(v)$$

Start a new user at $\hat{w}_u = 0$, update over time.

- OLS on the residuals of the global model
- Bayesian Update (start with a probability distribution over user-specific deviations, update as you get more data)

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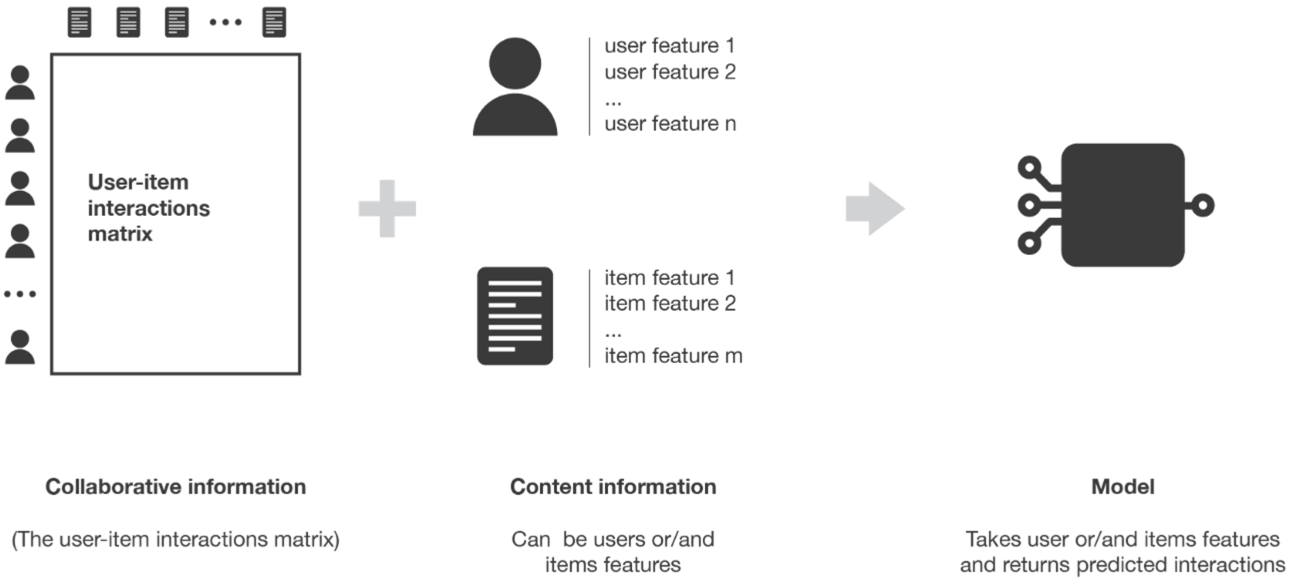
Group

2 min

- Will feature-based recommender systems suffer from the cold start problem? Why or why not?
- What about other pros/cons of feature-based?



Solution 3 (Feature-Based) Pros / Cons

Pros:

- No cold-start issue!
  - Even if a new user/item has no purchase history, you know features about them.
- Personalizes to the user and item.
- Scalable (only need to store weights per feature)
- Can add arbitrary features (e.g., time of day)

Cons:

- Hand-crafting features is very tedious and unscalable ☹

Recap

Dimensionality Reduction & PCA:

- Why and when it's important
- High level intuition for PCA
- Linear Projections & Reconstruction
- Eigenvectors / Eigenvalues

Recommender Systems:

- Sol 0: Popularity
- Sol 1: Nearest User (User-User)
- Sol 2: "People who bought this also bought" (item-item)
- Sol 3: Feature-Base

Next Time (Rec System Continued):

- Sol 4: Matrix Factorization
- Sol 5: Hybrid Model
- Addressing common issues with Recommender Systems
- Evaluating Recommender Systems