A lot of the buzz about ML recently has come from recent advancements in **deep learning**.

When people talk about “deep learning” they are generally talking about a class of models called **neural networks** that are a loose approximation of how our brains work.
Recall: Linear Classifier

Remember the linear classifier based on score

\[
\text{Score}(x) = w_0 + w_1 x[1] + w_2 x[2] + \ldots + w_d x[d] = 0
\]

\[
\text{Score}(x) > 0 \quad \text{Score}(x) < 0
\]

0
A perceptron is a type of single-layer neural network that classifies input data into categories. It works by calculating a linear combination of the input values weighted by connection weights, and then applying a threshold function to the result.

Mathematically, the perceptron model can be represented as:

\[ g(Score(x)) = \begin{cases} 
1, & \text{if } \sum_{j=1}^{d} w_j x[j] > 0 \\
0, & \text{otherwise}
\end{cases} \]

where \( g(x) \) is the activation function, \( Score(x) = \sum_{j=1}^{d} w_j x[j] \) is the weighted sum of the inputs, and \( w_j \) are the weights associated with each input feature. The perceptron learns by adjusting the weights based on the error in its predictions.
The perceptron can learn most boolean functions, but XOR always has to ruin the fun.

This data is not **linearly separable**, therefore can’t be learned with the perceptron

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$y$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
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</table>
Multi-Layer Perceptron (Neural Network)

Idea: Combine these perceptrons in layers to learn more complex functions.
- Since the inputs are the same, typically we combine them in the diagram, with multiple arrows coming out.
- We don’t explicitly show the sum and activation function – that is implicitly a part of each node.
- Oftentimes, the bias is not explicitly shown as another input, and instead written on top of a node.
- You will see both types of diagrams in this course.
Notice that we can represent
\[ x[1] \text{ XOR } x[2] = (x[1] \text{ AND } \overline{x}[2]) \text{ OR } (\overline{x}[1] \text{ AND } x[2]) \]
XOR

This is a 2-layer neural network

\[ y = x[1] \text{ XOR } x[2] = (x[1] \text{ AND } !x[2]) \text{ OR } (!x[1] \text{ AND } x[2]) \]

\[ v[1] = (x[1] \text{ AND } !x[2]) \]
\[ = g(-0.5 + x[1] - x[2]) \]

\[ v[2] = (!x[1] \text{ AND } x[2]) \]
\[ = g(-0.5 - x[1] + x[2]) \]

\[ y = v[1] \text{ OR } v[2] \]
\[ = g(-0.5 + v[1] + v[2]) \]
Neural Network

Two layer neural network (alt. one hidden-layer neural network)

Single

\[ \text{out}(x) = g \left( w_0 + \sum_j w_j x[j] \right) \]

1-hidden layer

\[ \text{out}(x) = g \left( w_0 + \sum_k w_k g \left( w_0^{(k)} + \sum_j w_j^{(k)} x[j] \right) \right) \]
A surprising fact is that a 2-layer network can represent any function, if we allow enough nodes in hidden layer.

For this example, consider regression function with one input.

See more here:
Aside: Missing Data
**Missing Data: Idea 1**

- **Idea 1**: Remove rows (datapoints) with missing values.

<table>
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<tr>
<th>Credit</th>
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<th>Loan Safety</th>
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<tr>
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<td>3 yrs</td>
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**Missing Data: Idea 2**

- **Idea 2**: Remove columns (features) with missing values.

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**Missing Data: Idea 3**

- **Idea 3**: Treat missing values as a separate value of the feature (only Decision Trees)

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### Idea 4: Replace missing values with a reasonable statistic (Imputation)

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(Most Commonly Used!)
Introduction to Neural Networks
1. Housing Prices - Regression
   - Regression Model
   - Assessing Performance
   - Ridge Regression
   - LASSO

2. Sentiment Analysis – Classification
   - Classification Overview
   - Logistic Regression
   - Naïve Bayes
   - Decision Trees
   - Ensemble Methods

3. Neural Networks – Image Classification
   - Neural Networks
   - Convolutional Neural Networks
History of Neural Networks

Generally layers and layers of linear models and non-linearities (activation functions).

Have been around for about 50 years

Fell in “disfavor” in the 90s when simpler models were doing well

In the last decade(s), have had a huge resurgence

Impressive accuracy on several benchmark problems

Have risen in popularity due to huge datasets, GPUs, and improvements to
Convolutional Neural Networks (CNNs) are commonly used in Computer Vision. We’ll learn about these on Wed!
Recurrent Neural Networks (RNNs) are commonly used in Natural Language Processing, where the model must remember context from earlier in the text.
Train two networks together:

- **Generator Network**: generate fake images
- **Discriminator Network**: given a real image and a fake image, determine which is fake

https://thispersondoesnotexist.com/
Neural Network Details
This is a 2-layer neural network

\[ y = x[1] \text{ XOR } x[2] = (x[1] \text{ AND } !x[2]) \text{ OR } (!x[1] \text{ AND } x[2]) \]

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\[ = g(-0.5 + v[1] + v[2]) \]
Think
2 mins

Compute the output for input (0, 1). There is a sign activation function on the hidden layers and output layer.

```
Input: (0, 1)
Hidden Layer 1: 2
Hidden Layer 2: 1
Output Layer: -1
```

Weights and biases:
- Input to Hidden Layer 1: 2
- Hidden Layer 1 to Hidden Layer 2: 3
- Hidden Layer 2 to Output Layer: 2
- Bias for Hidden Layer 1: -2
- Bias for Hidden Layer 2: 1
- Bias for Output Layer: 3

Activation function: Sign function
Compute the output for input (0, 1). There is a sign activation function on the hidden layers and output layer.
Before, we were using the sign activation function.

This is not generally used in practice.
- Not differentiable
- No notion of confidence

What if we use the logistic function instead?

$$g(w_0 + \sum_{j} w_j x[j]) = \frac{1}{1 + e^{-(w_0 + \sum_{j} w_j x[j])}}$$

$$g = \frac{1}{1 + e^{-\text{Score}(x)}}$$
Activation Functions

- **Sigmoid**
  - Historically popular, but (mostly) fallen out of favor
  - Neuron’s activation saturates
    (weights get very large -> gradients get small)
  - Not zero-centered -> other issues in the gradient steps
  - When put on the output layer, called “softmax” because interpreted as class probability (soft assignment)

- **Hyperbolic tangent** \( g(x) = \tanh(x) \)
  - Saturates like sigmoid unit, but zero-centered

- **Rectified linear unit (ReLU)** \( g(x) = x^+ = \max(0,x) \)
  - Most popular choice these days
  - Fragile during training and neurons can “die off”… be careful about learning rates
  - ”Noisy” or “leaky” variants

- **Softplus** \( g(x) = \log(1+\exp(x)) \)
  - Smooth approximation to rectifier activation
Classification or Regression

You can use neural networks for classification and regression!

**Regression**
The output layer will generally have one node that is the output (outputs a single number). Don’t apply activation to the last layer.

**Classification**
The output layer will have one node per class. Usually take the node with the highest score as the prediction for an example. Can also use the logistic function (softmax) to turn scores into probabilities!
Are NNs likely to overfit? **YES**.

Consequence of being able to fit any function!

How to avoid overfitting?

Get more training data

Few hidden nodes / better architecture

- **Rule of thumb**: 3-layer NNs outperform 2-layer NNs, but going deeper only helps if you are very careful (different story next time with convolutional neural networks)

Regularization

- Dropout

Early stopping
Brain Break
Application to Computer Vision
Features in computer vision are local detectors
Combine features to make prediction

In reality, these features are much more low level (e.g. Corner?)
A popular approach to computer vision was to make hand-crafted features for object detection.

Relies on coming up with these features by hand (yuck!)
Neural Networks implicitly find these low level features for us!

Each layer learns more and more complex features

[Zeiler & Fergus ‘13]
Training
Neural
Networks
So the idea of neural networks might make sense, but how do we actually go about learning the coefficients in the layers?

First we need to define a quality metric or cost function

   - For regression, generally use MSE or RMSE
   - For classification, generally use something call the Cross Entropy loss.

Can we use gradient descent here? Actually yes!

   - How do we take the derivative of a network?
   - Are there convergence guarantees?
What does gradient descent do in general? Have the model make predictions and update the model in a special way such that the new weights have lower error.

To do gradient descent with neural networks, we generally use **backpropagation**.

1. Do a forward pass of the data through the network to get predictions
2. Compare predictions to true values
3. Backpropagate errors so the weights make better predictions
It’s pretty expensive to do this update for the entire dataset at once, so it’s common to break it up into small batches to process individually.

However, processing each batch only once isn’t enough. You generally have to repeatedly update the model parameters. We call an iteration that goes over every batch once an epoch.

```python
for i in range(num_epochs):
    for batch in batches(training_data):
        preds = model.predict(batch.data)  # Forward pass
        diffs = compare(preds, batch.labels)  # Compare
        model.backprop(diffs)  # Backpropagation
```
In general, loss functions with neural networks are not convex. This means the backprop algorithm for gradient descent will only converge to a local optima. This means that how you initialize the weights is really important and can impact the final result. How should you initialize weights? Usually people do random initialization. People also use adaptive ways of changing the learning rate to reduce the empirical likelihood of getting stuck in local minima.
Consider the below neural network, used for regression (hence, no activation on the last layer).

The input, prediction, and actual label are shown.

To move the prediction slightly closer to the label, would you (increase / decrease) $w_1$?
Backpropogation
Intuition on
Multiple Layers
Hyper-parameter Tuning
Training NN

Neural Networks have MANY hyperparameters

- How many hidden layers and hidden neurons?
- What activation function?
- What is the learning rate for gradient descent?
- What is the batch size?
- How many epochs to train?
- And much much more!

How do you decide these values should be? ¯\_(ツ)_/¯

The most frustrating thing is that we don’t have a great grasp on how these things impact performance, so you generally have to try them all.
How do we choose hyperparameters to train and evaluate?

Grid search:

Hyperparameters on 2d uniform grid
How do we choose hyperparameters to train and evaluate?

Grid search:
- Hyperparameters on 2d uniform grid

Random search:
- Hyperparameters randomly chosen
Hyperparameter Optimization

How do we choose hyperparameters to train and evaluate?

Grid search:
- Hyperparameters on 2d uniform grid

Random search:
- Hyperparameters randomly chosen

Bayesian Optimization:
- Hyperparameters adaptively chosen
Recent work attempts to speed up hyperparameter evaluation by stopping poor performing settings before they are fully trained.

Tips on Hyperparameter Optimization

In general, hyperparameter optimization is a non-convex optimization problem where we know very little about how the function behaves.

Your time is valuable and compute time is cheap. Write your code to be modular so you can use compute time to try a range of values.

Tools for different purposes

- Very few evaluations: use random search (and pray)
- Few evaluations and long-run computations: See last slide
- Moderate number of evaluations: Bayesian optimization
- Many evaluations possible: Use random search. Why overthink it?
Recap

**Theme:** Details of neural networks and how to train them

**Ideas:**

- Perceptron (Single-Layer Neural Network)
- Neural Networks
- Activation functions
- Neural Networks and Overfitting
- Backpropagation idea
- NN Hyperparameters
- Hyperparameter optimization
- NN Convergence guarantees