CSE/STAT 416

Dimensionality Reduction & Recommender Systems Intro

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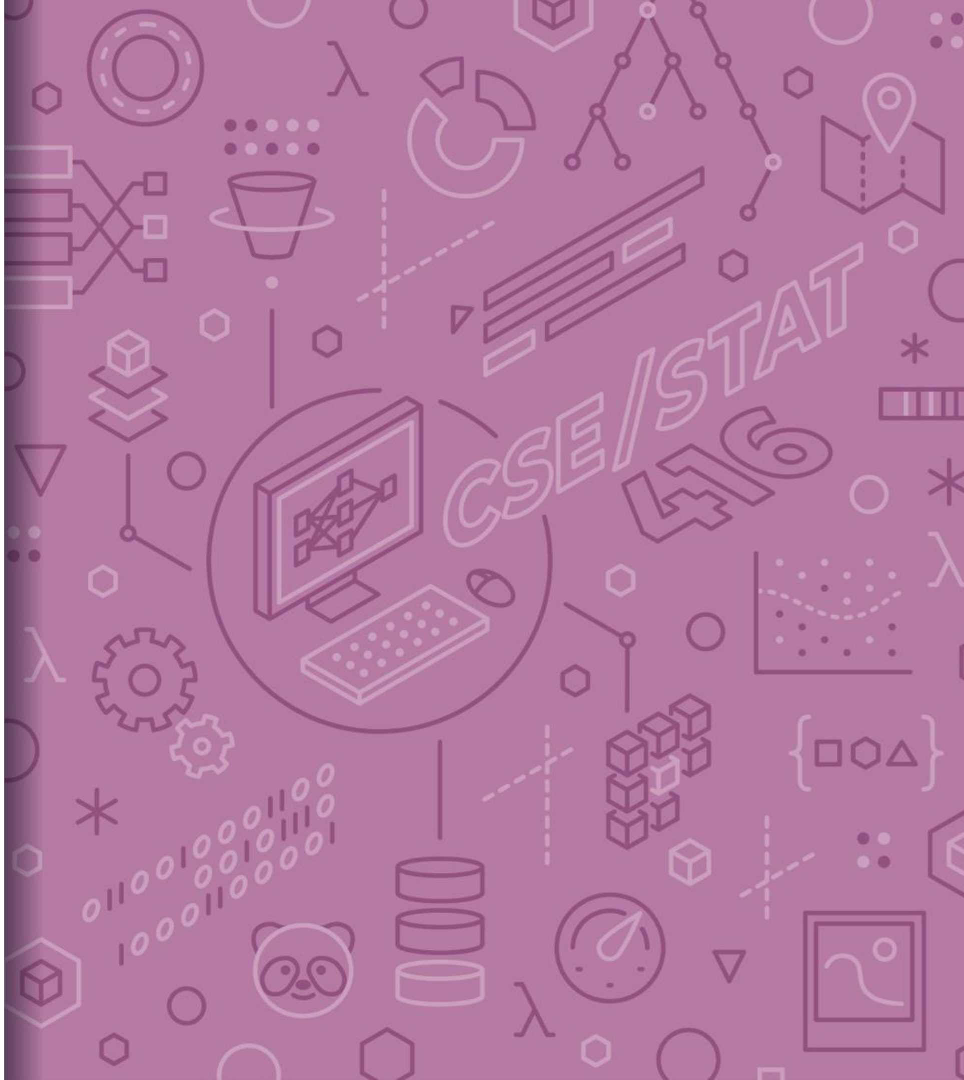
Aug 8, 2022

Adapted from Hunter Schafer's slides



Administrivia

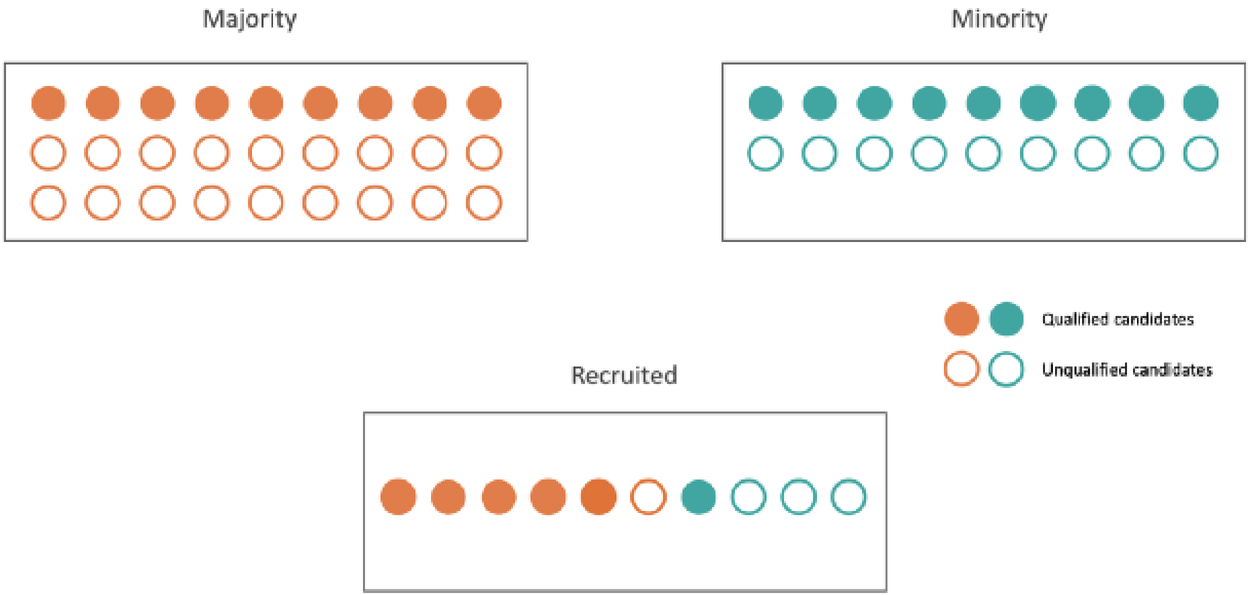
- **This Week**: Dimensionality Reduction, Recommender Systems
- **Next Week**: Course Wrap-Up & Guest Speakers
- Deadlines:
  - HW6 due TOMORROW, Tues 8/9 11:59PM
    - Submit Concept on Gradescope
    - Submit Programming on EdSTEM
  - HW7 (final HW) released Wed 8/10
    - Due Tues 8/16 11:59PM, **NO LATE DAYS**
  - LR 8 due Fri 8/12 11:59PM
  - **Extra Credit** Guest Panel Mon 8/15 during lecture.
  - Take-Home Final Exam: Wed 8/17 – Thurs 8/18

Addressing LR Questions

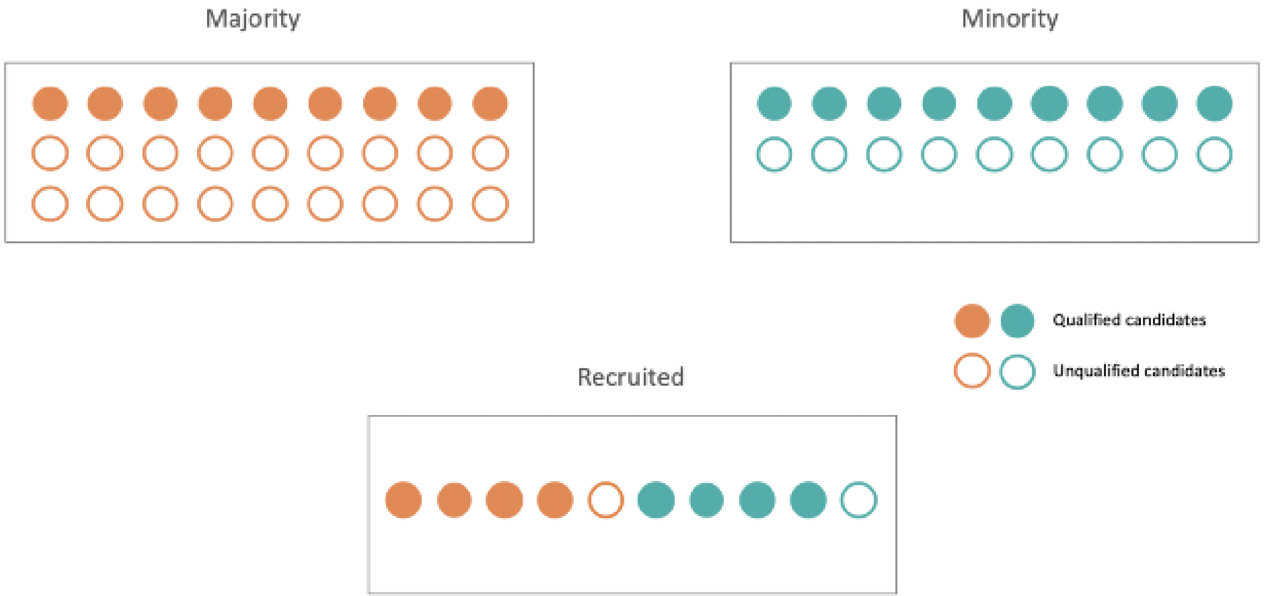
Fairness Definitions

1. "Fairness through Unawareness"
    1. To avoid unfair decisions, prevent the model from every looking at protected attribute (e.g., race, gender).
    2. **Doesn't work in practice**
2. Statistical Parity
    1. Idea: Equal performance across groups.
       $$\Pr(\hat{Y} = +|\, A = \blacksquare) = \Pr(\hat{Y} = +|\, A = \bigcirc)$$
    2. Also phrased as matching demographic statistics (e.g., if 33% of population are Circles, 33% of those admitted should be Circles).
3. Equal Opportunity
    1. Idea: True positive rate should be equal across groups
       $$\Pr(\hat{Y} = +|\, A = \blacksquare, Y = +) = \Pr(\hat{Y} = +|\, A = \bigcirc, Y = +)$$

Statistical Parity Example



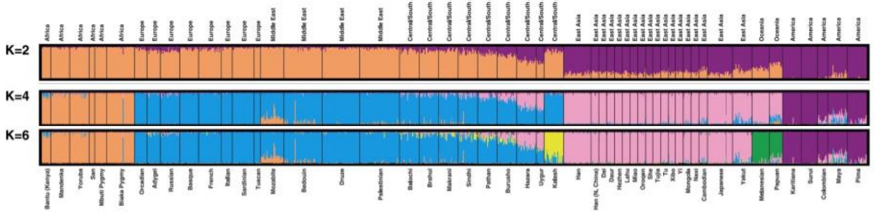
Equality of Opportunity Example



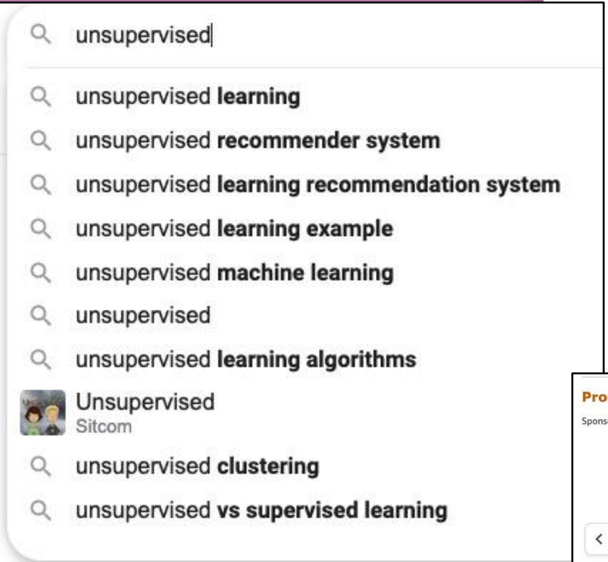
Unsupervised Learning

- A type of machine learning that detects underlying patterns in **unlabeled** data.
- Examples of unsupervised learning tasks:
  - Cluster similar articles together.

  - Cluster gene sequences.

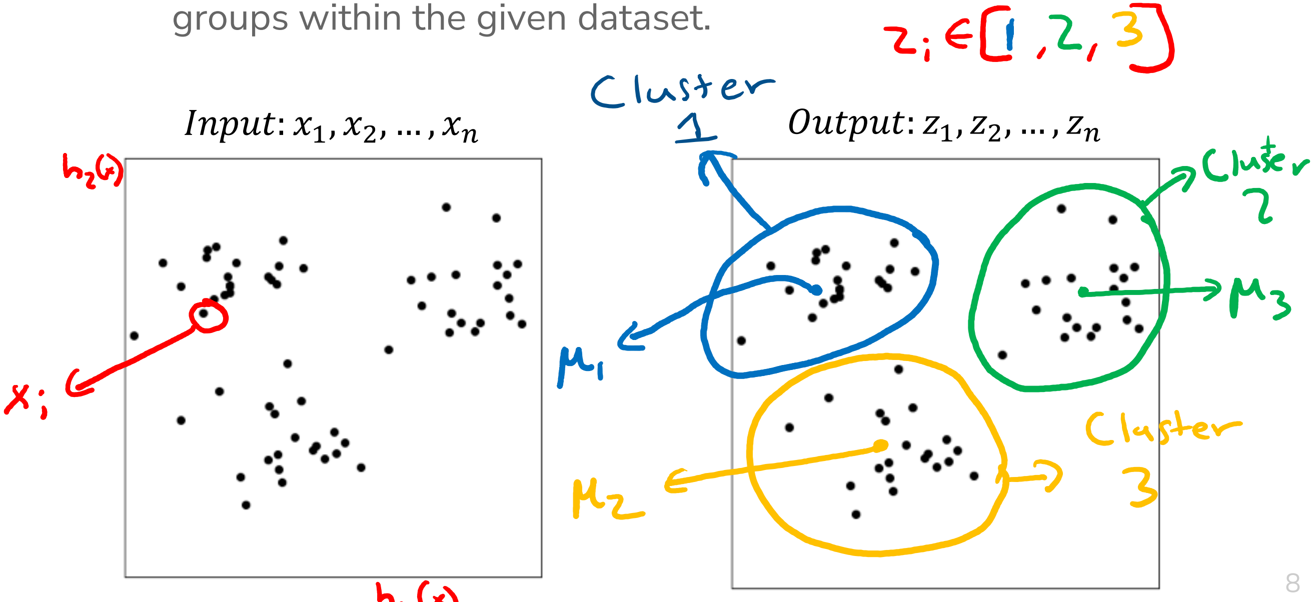


  - Recommend items, searches, movies, etc.



Unlabeled Data

- In many real world contexts, there aren't clearly defined labels so we won't be able to do classification
- We will need to come up with methods that uncover structure from the (unlabeled) input data X.
- **Clustering** is an automatic process of trying to find related groups within the given dataset.



Poll Everywhere

Think

2 min

- Which word(s) have the largest IDF? Which word(s) have the smallest IDF?

Red = low

Green = high

Review
“Sushi was great, the food was awesome, but the service was terrible”
“Terrible food; the sushi was rancid.”

Note that if we divide the Bag of Words embedding by the num words in the document, we get the TF!

Sushi	was	great	the	food	awesome	but	service	terrible	rancid
1	3	1	2	1	1	1	1	1	0
1	1	0	1	1	0	0	0	1	1

Coordinate Descent

k-means is trying to minimize the heterogeneity objective

$$\underset{z,\mu}{\operatorname{argmin}} \sum_{j=1}^{k} \sum_{i=1}^{n} \mathbf{1}\{z_i = j\} \left|\left|\mu_j - x_i\right|\right|_2^2$$

Step 0: Initialize cluster centers

Repeat until convergence:

fix μ, minimize z

- Step 1: Assign each example to its closest cluster centroid
- Step 2: Update the centroids to be the mean of all the points assigned to that cluster

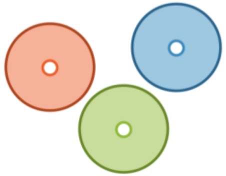
fix z, minimize μ

Coordinate Descent alternates how it updates parameters to find minima. On each of iteration of Step 1 and Step 2, heterogeneity decreases or stays the same.

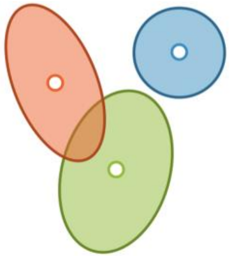
=> Will converge in finite time

Finding Shapes

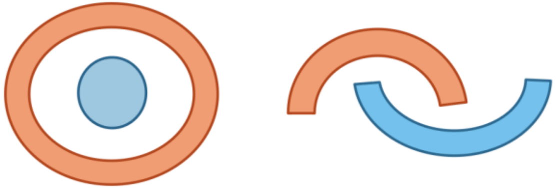
k-means



Mixture Models



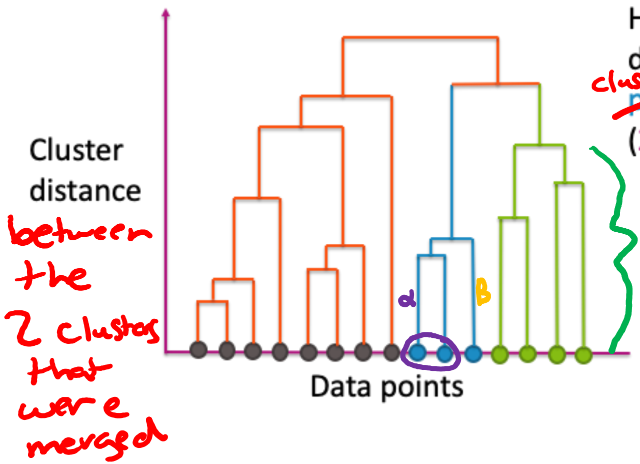
Hierarchical Clustering



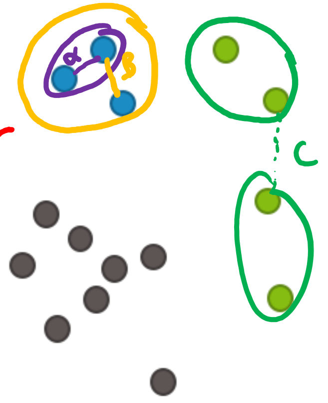
Dendrogram

x-axis shows the datapoints (arranged in a very particular order)

y-axis shows distance between pairs of clusters



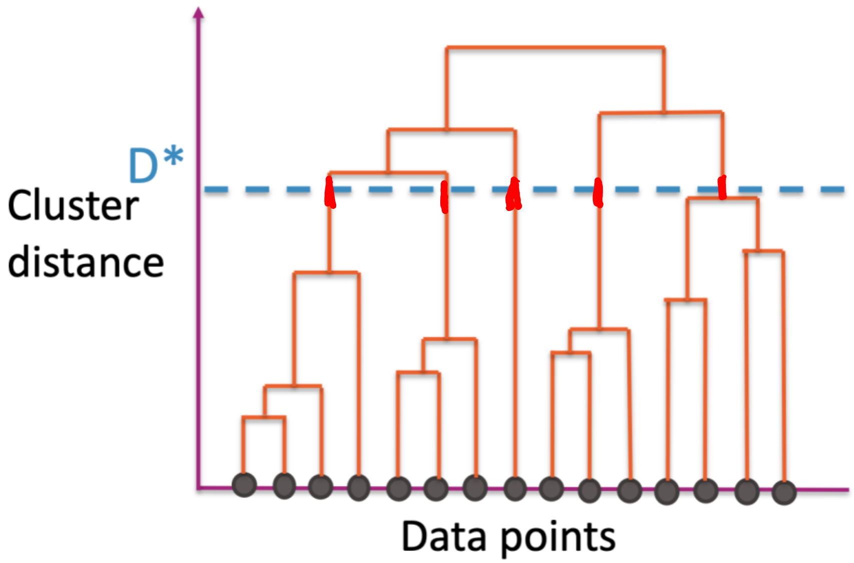
Height here indicates min distance between blue ~~pts~~ cluster and green ~~pts~~ cluster (2 clusters)



Cut Dendrogram

Choose a distance D^* to "cut" the dendrogram

- Use the largest clusters with distance < D^*
- Usually ignore the idea of the nested clusters after cutting



Dimensionality Reduction

Large Dimensionality

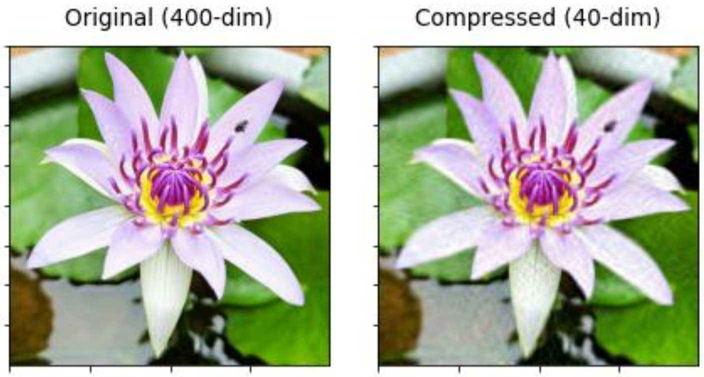
Input data might have thousands or millions of dimensions!

- **Images**: 200x200 image is 120,000 features!
- **Text**: # features = # n-grams 😮
- **Course Success**: dozen(s) of features
- **User Ratings**: 100s of ratings (one per rate-able item)

	Area Abbreviation	Area Code	Area	Item Code	Item	Element Code	Element	Unit	latitude	longitude	...	Y2004
0	AF	2	Afghanistan	2511	Wheat and products	5142	Food	1000 tonnes	33.94	67.71	...	3249.0
1	AF	2	Afghanistan	2805	Rice (Milled Equivalent)	5142	Food	1000 tonnes	33.94	67.71	...	419.0
2	AF	2	Afghanistan	2513	Barley and products	5521	Feed	1000 tonnes	33.94	67.71	...	58.0
3	AF	2	Afghanistan	2513	Barley and products	5142	Food	1000 tonnes	33.94	67.71	...	185.0
4	AF	2	Afghanistan	2514	Maize and products	5521	Feed	1000 tonnes	33.94	67.71	...	120.0

Issues with Too Many Dimensions

- **Visualization**: Hard to visualize more than 3D.
- **Overfitting**: Greater risk of overfitting with more features/dimensions
- **Scalability**: some ML approaches (e.g., k-nn, k-means) perform poorly in high-dimensional spaces (curse of dimensionality)
- **Redundancy**: high-dimensional data often occupies a lower-dimensional subspace.
  - Most pixels in MNIST (digit recognition) are white – are they necessary?
    - Image Compression

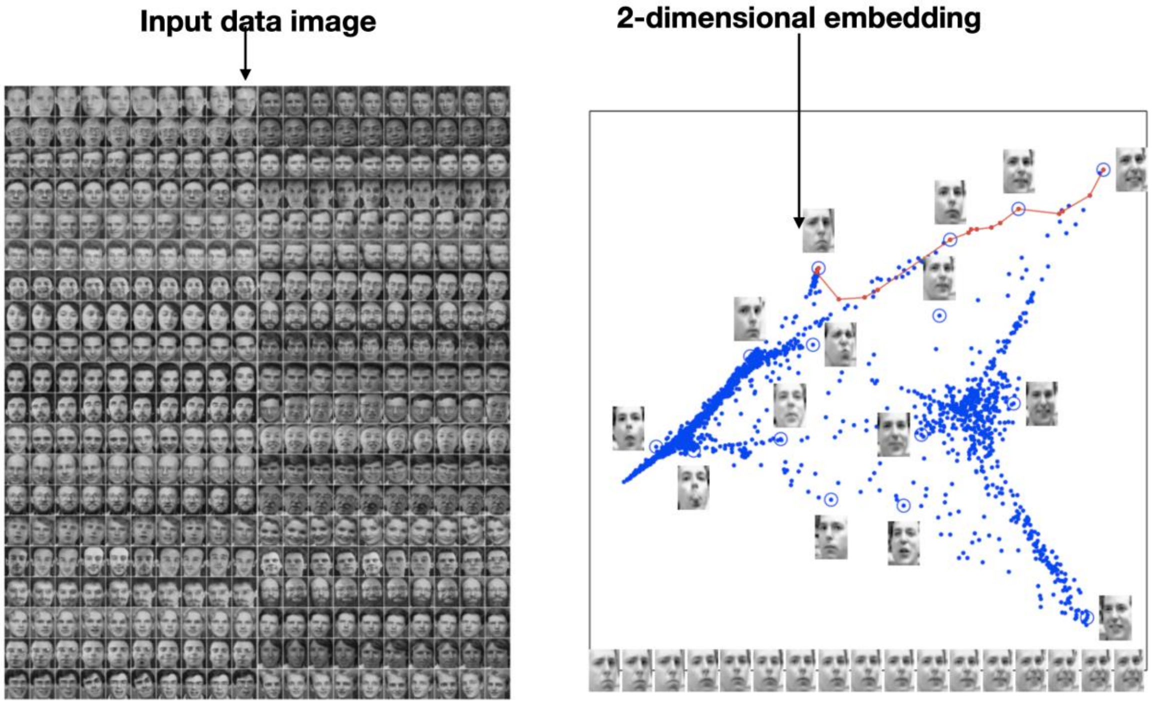


<u>Dimensionality Reduction</u> is the the task of representing the data with a fewer number of dimensions, while keeping meaningful relations between data

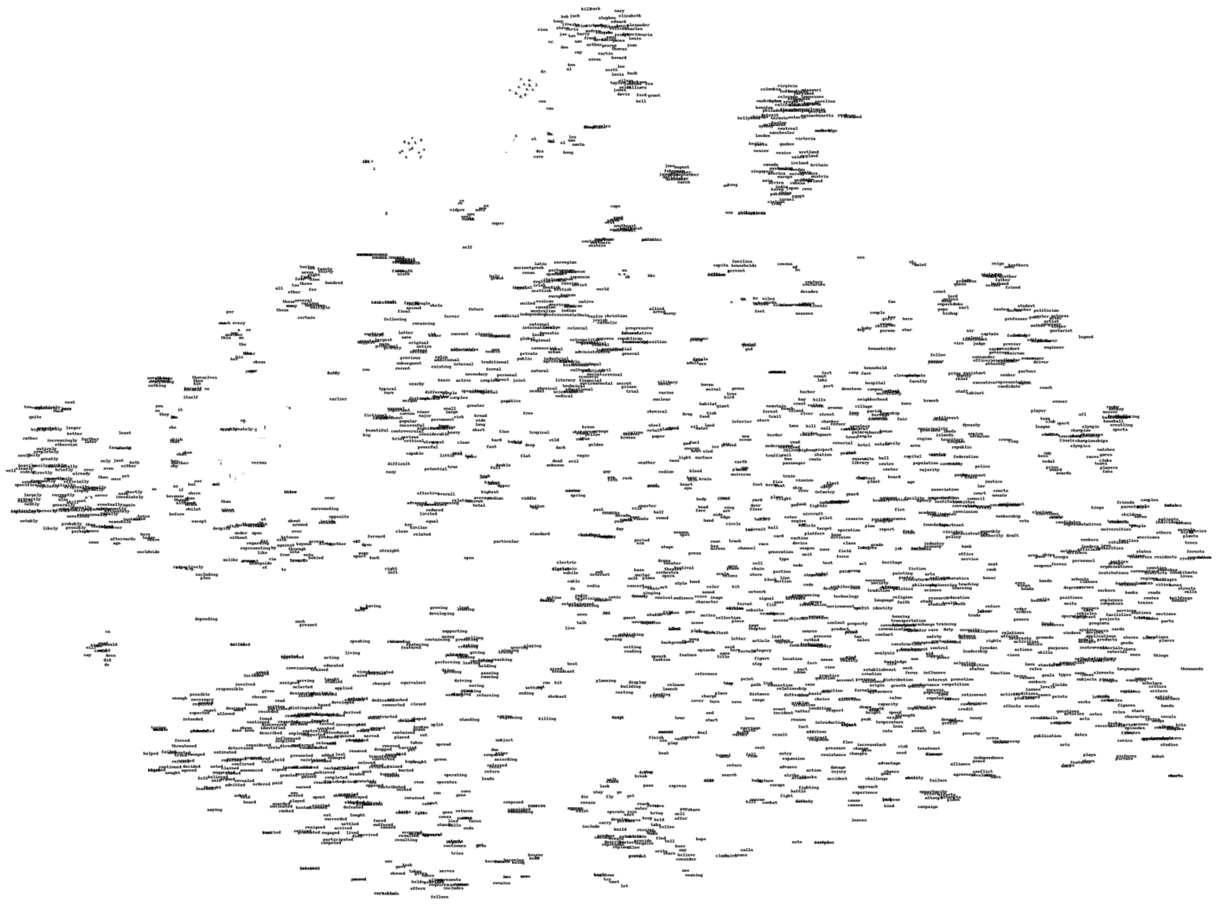
Example: Embedding Pictures

Example: Embed high dimensional data in low dimensions to visualize the data

- Goal: Similar images should be near each other.



Example: Embedding Words



Example: Embedding Words

Principal Component Analysis (PCA)

One very popular dimensionality reduction algorithm is called **Principal Component Analysis (PCA)**.

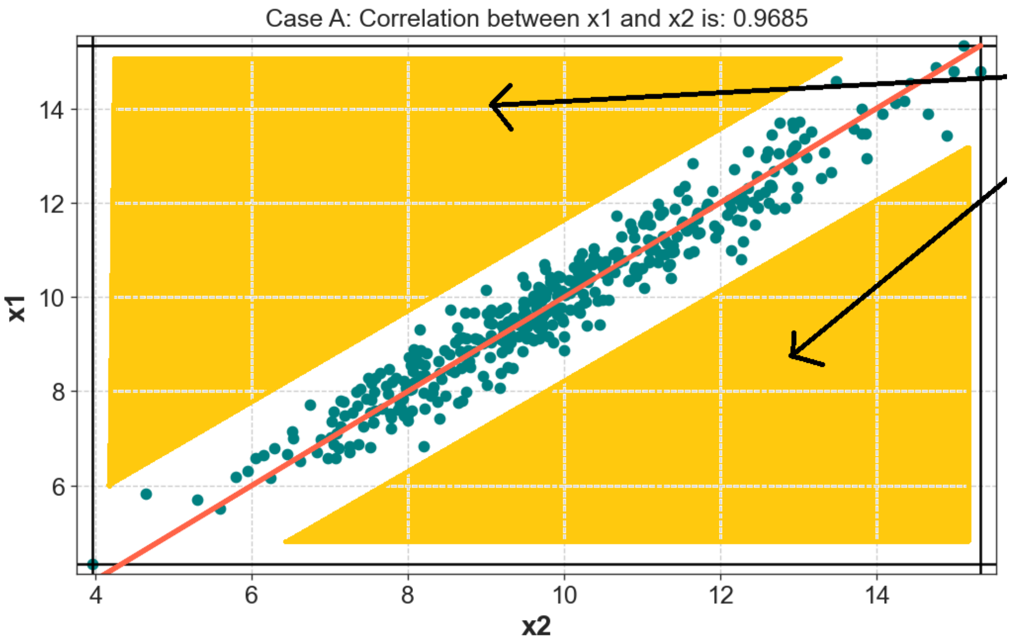
Idea: Use a linear projection from d-dimensional data to k-dimensional data

- E.g. 1000 dimension word vectors to 3 dimensions

Choose the projection that minimizes **reconstruction error**

- Idea: The information lost if you were to ”undo” the projection

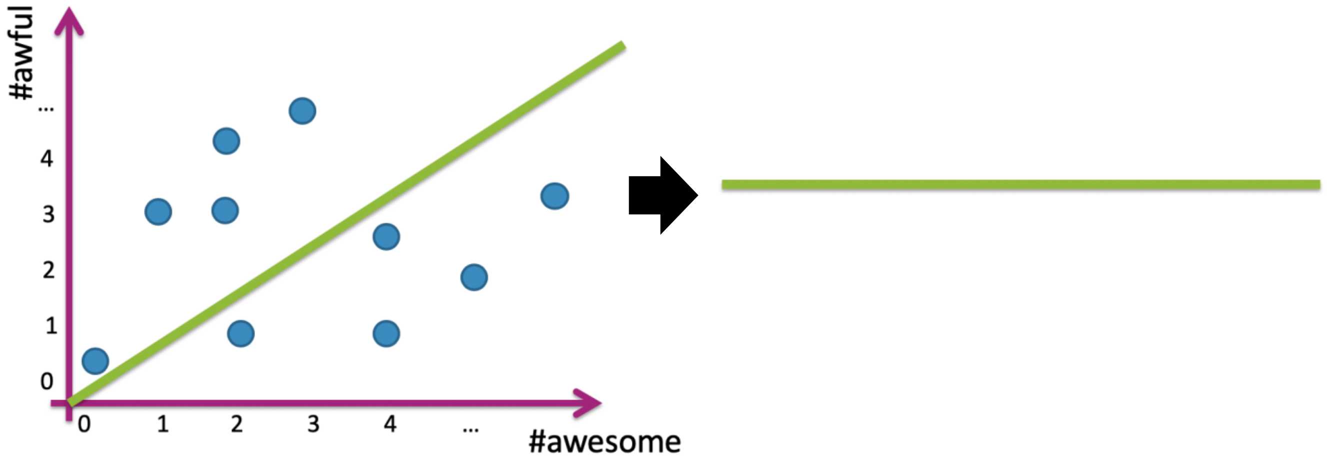
Principal Component Analysis (PCA)



Regions with no data. Data exists close to a lower-dimensional subspace.

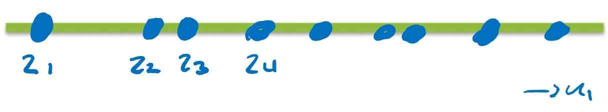
Linear Projection

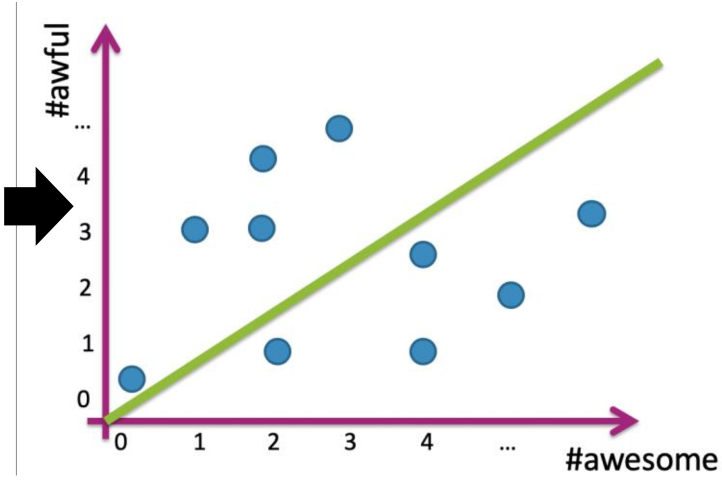
Project data into 1 dimension along a line



Reconstruction

Reconstruct original data only knowing the projection





Poll Everywhere

Think

1 min

- Compute the 2D coordinates of the following point. Then compute its reconstruction error.
  - $x_i = [0, -7, 3, 2, 5]$
  - $u_1 = [-0.5, 0, 0.5, -0.5, 0.5]$
  - $u_2 = [0.5, 0, 0.5, -0.5, -0.5]$
  - $z_i = ??$
  - $\hat{x}_i = ??$
  - $\|\hat{x}_i - x_i\|_2^2 = ??$

Poll Everywhere

Group

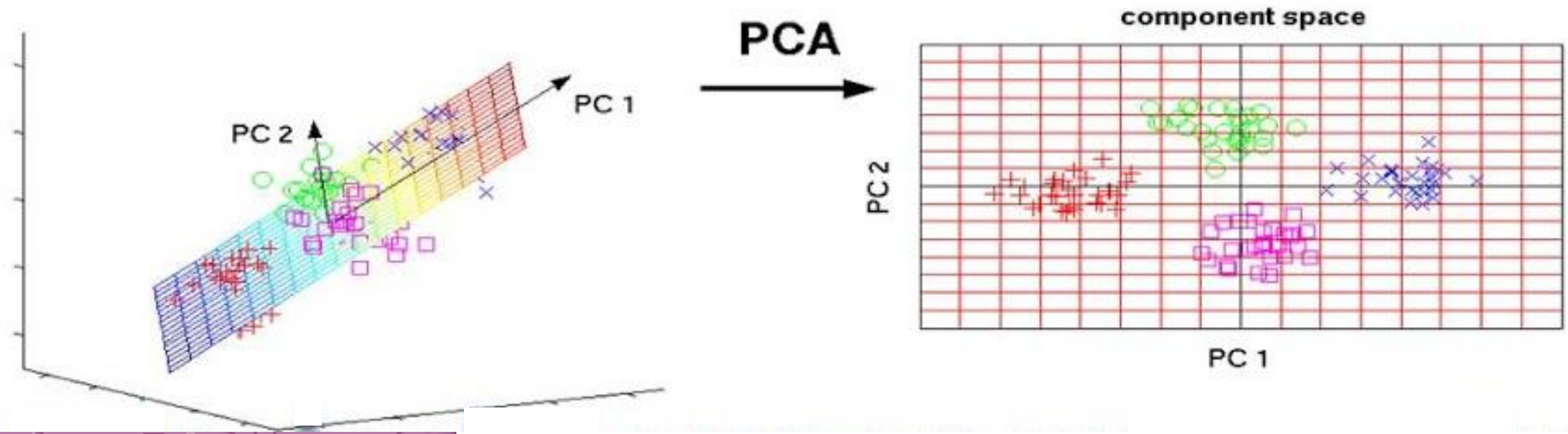
2 min

- Compute the 2D coordinates of the following point. Then compute its reconstruction error.
  - $x_i = [0, -7, 3, 2, 5]$
  - $u_1 = [-0.5, 0, 0.5, -0.5, 0.5]$
  - $u_2 = [0.5, 0, 0.5, -0.5, -0.5]$
  - $z_i = ??$
  - $\hat{x}_i = ??$
  - $\|\hat{x}_i - x_i\|_2^2 = ??$

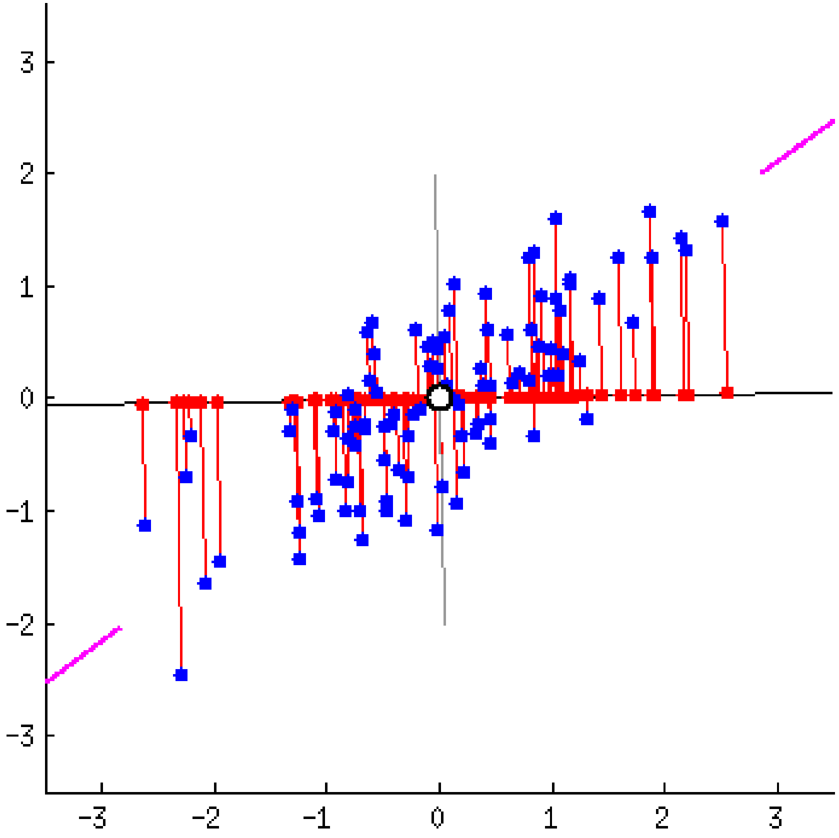
Linear Projection in Higher Dimensions

Think of PCA as giving each datapoint a new "address."

- Earlier, you could find the datapoint by going to the location (x, y, z).
- Now, if you are just moving in the projection plane, you can (approximately) find the datapoint by going to the location (u_1, u_2)



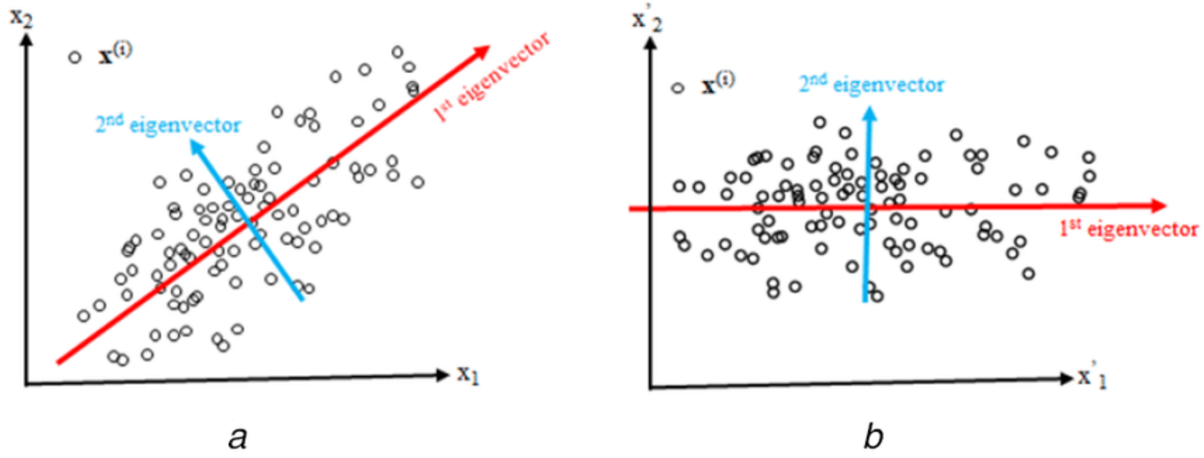
How do we find the best projection vector(s)?



Pick the vector(s) along which the datapoints have the most variation!

Eigenvectors

- There is a quantity in linear algebra that does exactly that!
- The **eigenvectors** of a d-dimensional dataset* are a collection of d *perpendicular* vectors that point in the directions of greatest variation amongst the points in the dataset.

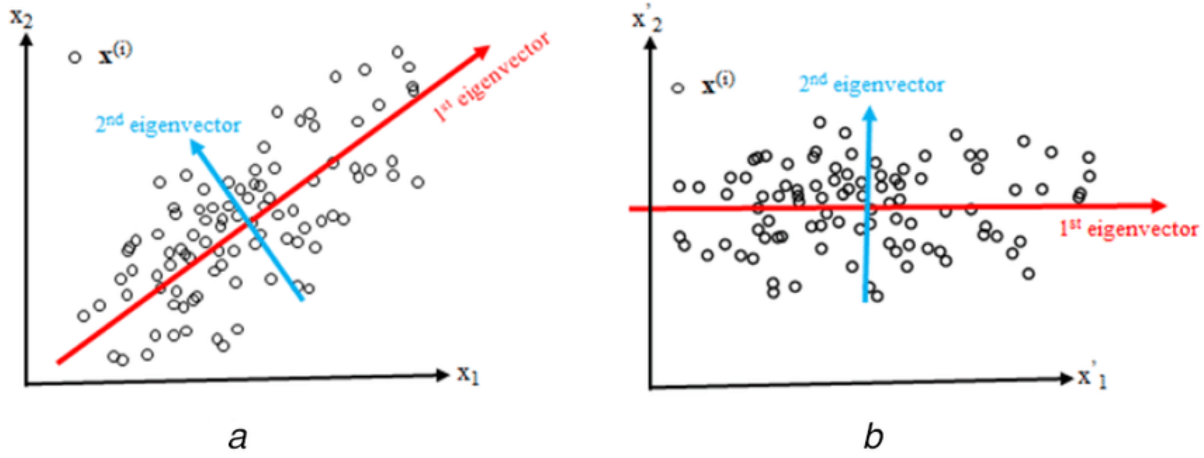


- Eigenvectors rotate the axes of the d dimensional space.

* (caveat) the eigenvectors are actually associated with the *covariance matrix* of the dataset

Eigenvalues

- Each eigenvector has a corresponding **eigenvalue**, indicating how much the dataset varies in that direction.
- Greater eigenvalue ➔ greater variance.



- **PCA**: Take the k eigenvectors with greatest eigenvalues.

PCA Algorithm

Input Data: An $\mathrm{n} \times d$ data matrix X

- Each row is an example

1. **Center Data**: Subtract mean from each row
$$X_c \leftarrow X \ - \bar{X}[1\colon d]$$
2. **Compute spread/orientation**: Compute covariance matrix Σ
$$\Sigma[t, s] = \frac{1}{n} \sum_{i=1}^{n} x_{c,i}[t] x_{c,i}[s]$$
3. **Find basis for orientation**: Compute eigenvectors of Σ
   - Select k eigenvectors $u_1, \ldots, u_k$ with largest eigenvalues
4. **Project Data**: Project data onto principal components
$$z_i[1] = u_1^T x_{c,i} \ = u_1[1] x_{c,i}[1] + \cdots + u_1[d] x_{c,i}[d]$$
$$\ldots$$
$$z_i[k] = u_k^T x_{c,i} \ = u_k[1] x_{c,i}[1] + \cdots + u_k[d] x_{c,i}[d]$$

Reconstructing Data

Using principal components and the projected data, you can reconstruct the data in the original domain.

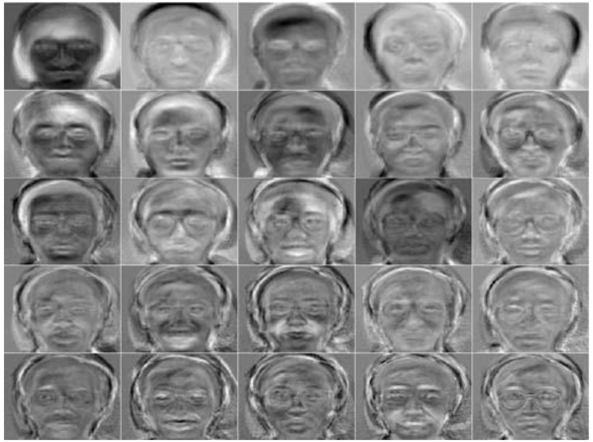
$$\hat{x}_i[1{:}d] = \bar{X}[1{:}d] + \sum_{j=1}^{k} z_i[j]\ u_j$$

Example: Eigenfaces

Apply PCA to face data

Input Data

Principal Components



Reconstructing Faces

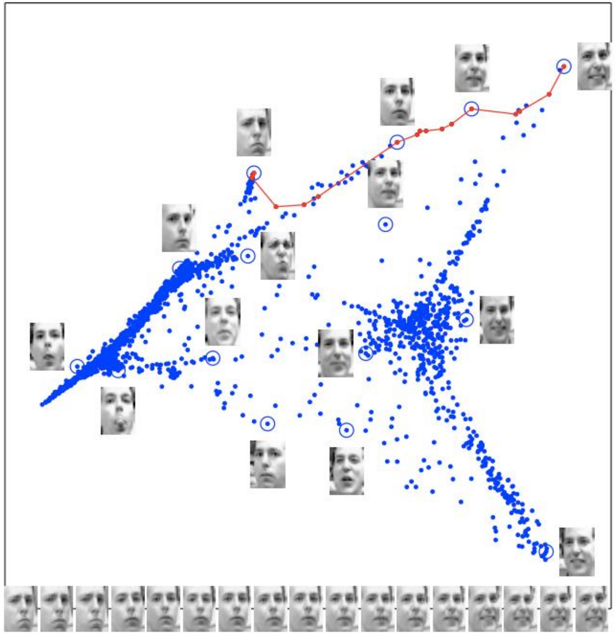
Depending on context, it may make sense to look at either original data or projected data.

In this case, let's see how the original data looks after using more and more principal components for reconstruction.

- Each image shows additional 8 principal components

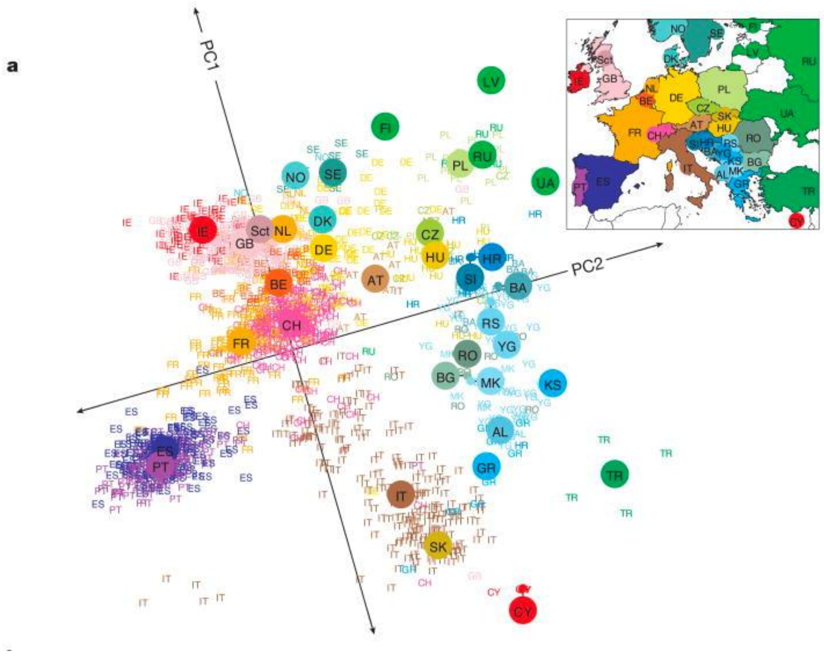
Embedding Images

Other times, it does make sense to look at the data in the projected space! (Usually if $k \leq 3$)



Example: Genes

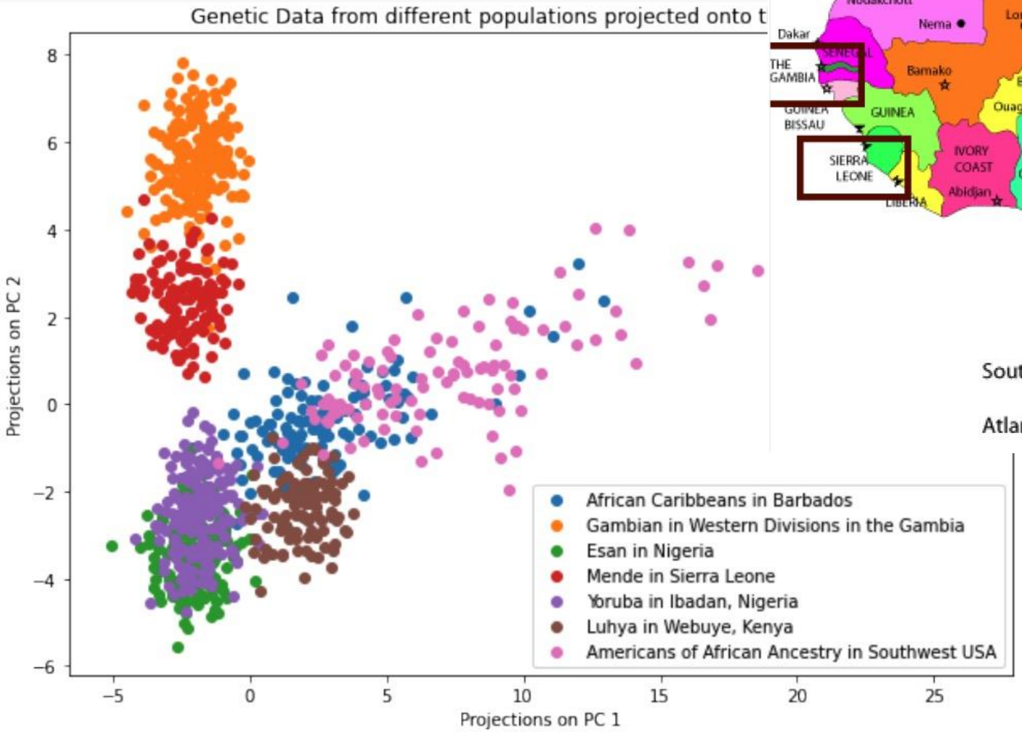
Dataset of genes of Europeans (3192 people; 500,568 loci) and their country of origin, ran PCA on the data and plotted 2 principal components.

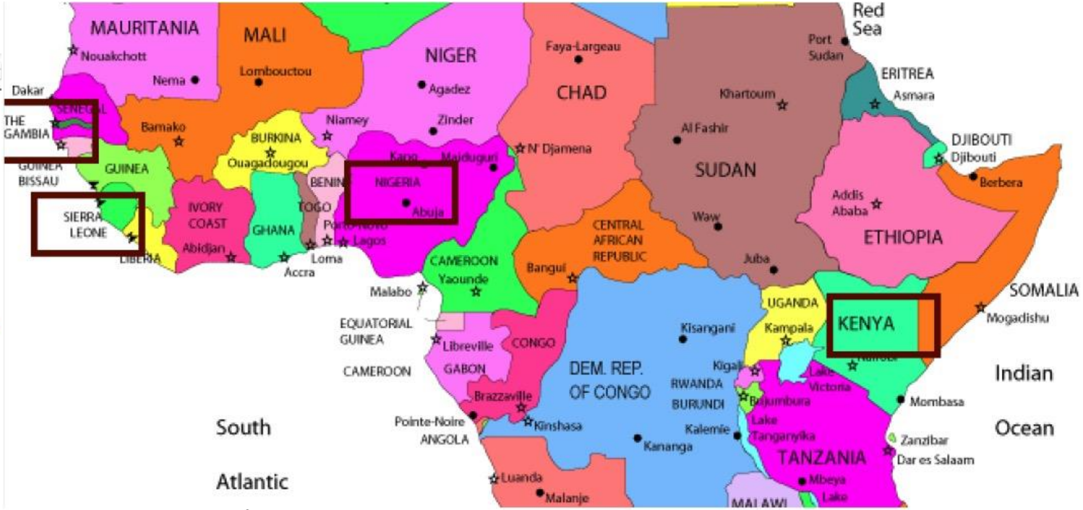


Poll Everywhere

Think

- Consider the genetic data projected onto its top two PCs. How would you interpret PC1 and PC2?

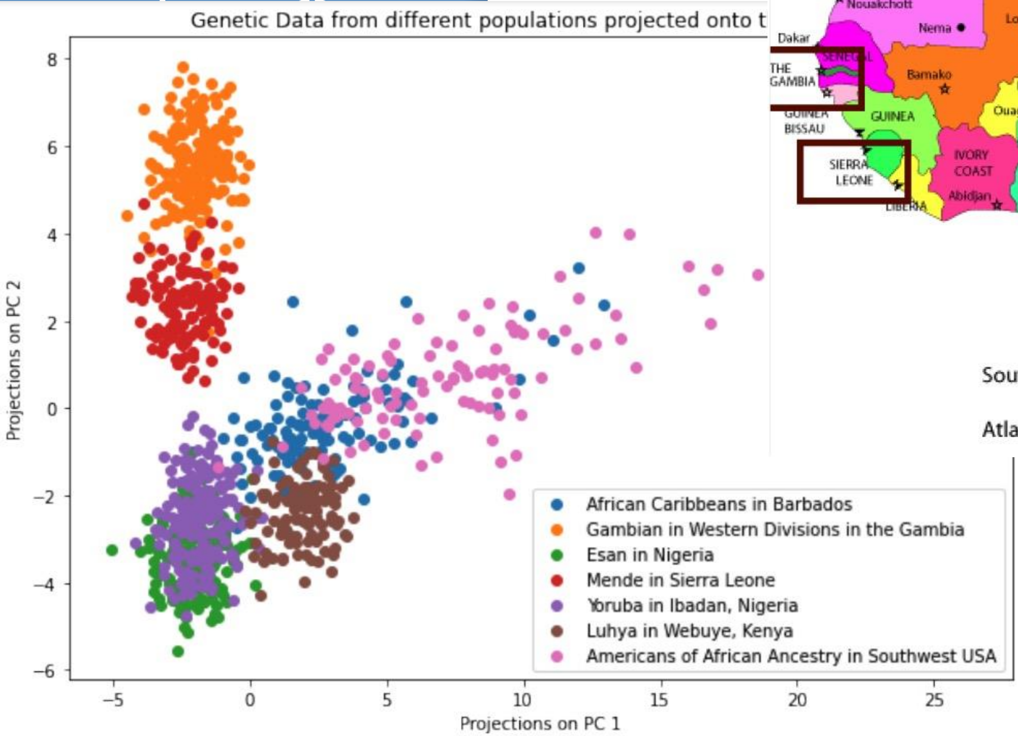


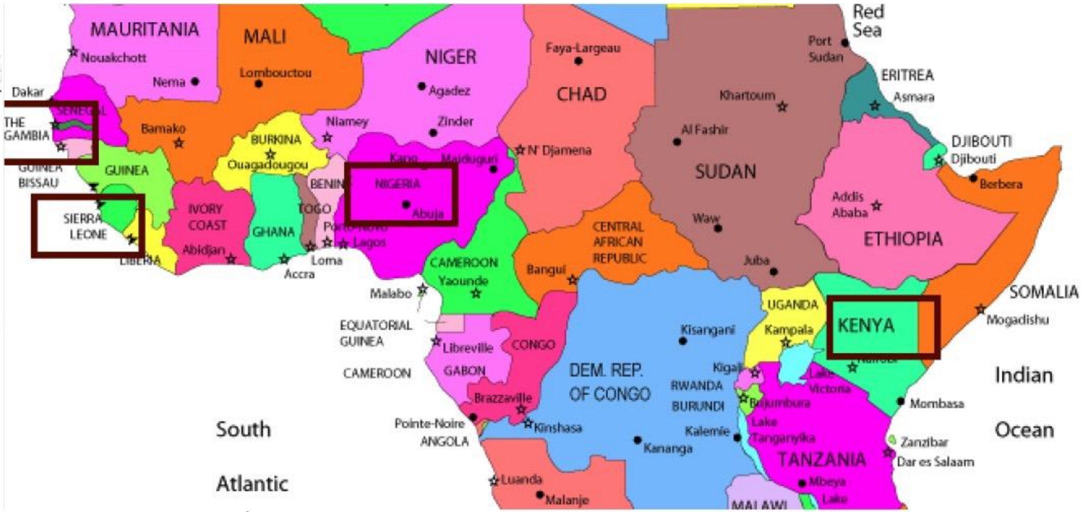


Poll Everywhere

Group

- Consider the genetic data projected onto its top two PCs. How would you interpret PC1 and PC2?





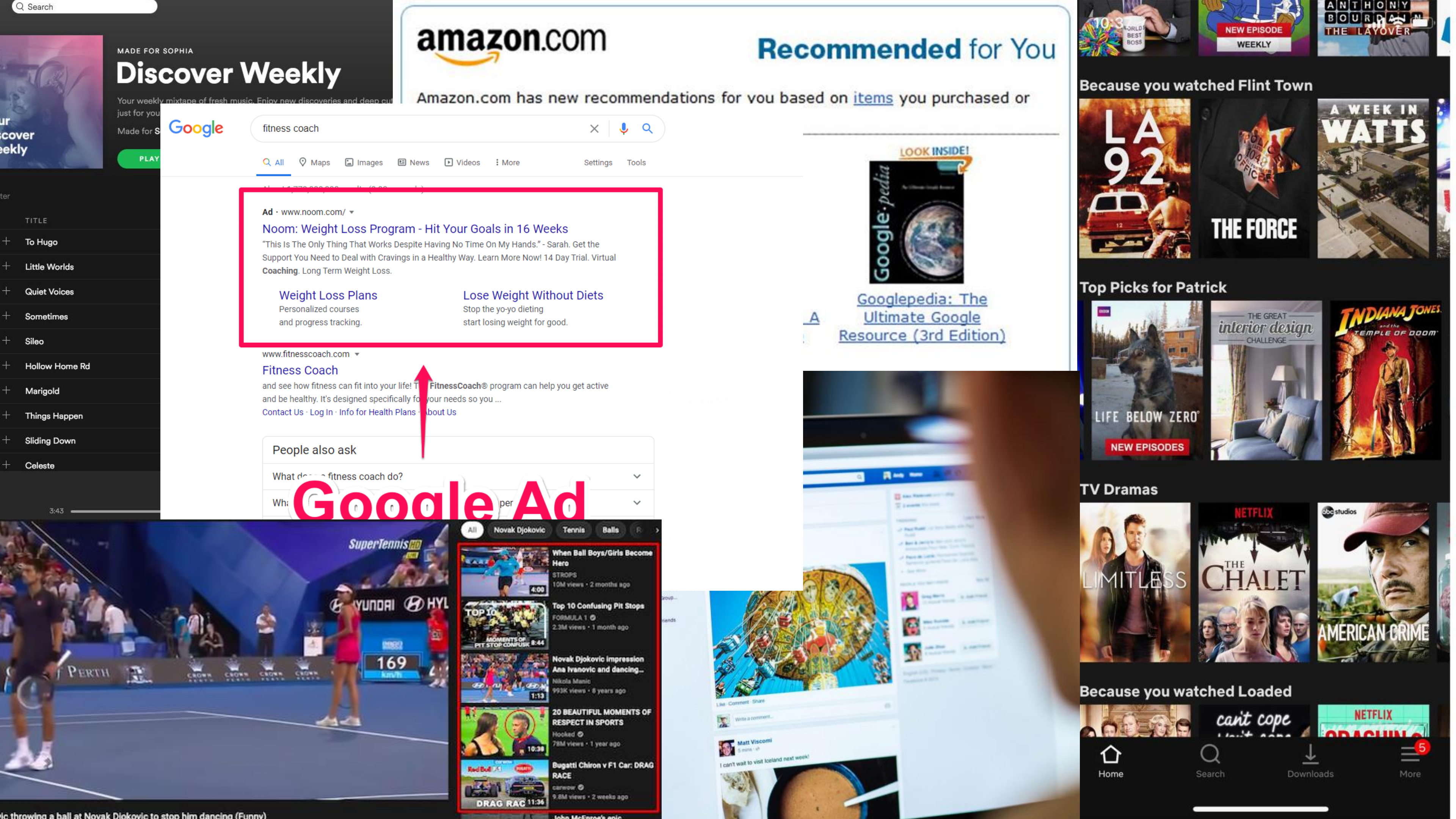
General Steps to Take as an ML Practitioner

Given a new dataset:

- Split into train and test sets.
- Understand the dataset:
  - Understand the feature/label types and values
  - Visualize the data: scatterplot, boxplot, PCA, clustering
- Use that intuition to decide:
  - What features to use, and what transformations to apply to them (pre-processing).
  - What model(s) to train.
- Train the models, evaluate them using a validation set or cross-validation.
- Deploy the best model.

Brain Break

Intro to Recommender Systems


Search
MADE FOR SOPHIA
Discover Weekly
Your weekly mixtape of fresh music. Enjoy new discoveries and deep cu
PLAY
TITLE
To Hugo
Little Worlds
Quiet Voices
Sometimes
Sileo
Hollow Home Rd
Marigold
Things Happen
Sliding Down
Celeste
3:43
amazon.com
Recommended for You
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Weight Loss Plans
Personalized courses and progress tracking.
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and see how fitness can fit into your life! The FitnessCoach® program can help you get active and be healthy. It's designed specifically for your needs so you you ...
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People also ask
What does a fitness coach do?
Google Ad
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A WEEK IN WATTS
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Top Picks for Patrick
LIFE BELOW ZERO
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INDIANA JONES and the TEMPLE OF DOOM
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THE CHALET
AMERICAN CRIME
Because you watched Loaded
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HYUNDAI
PERTH
169 km/h
All
Novak Djokovic
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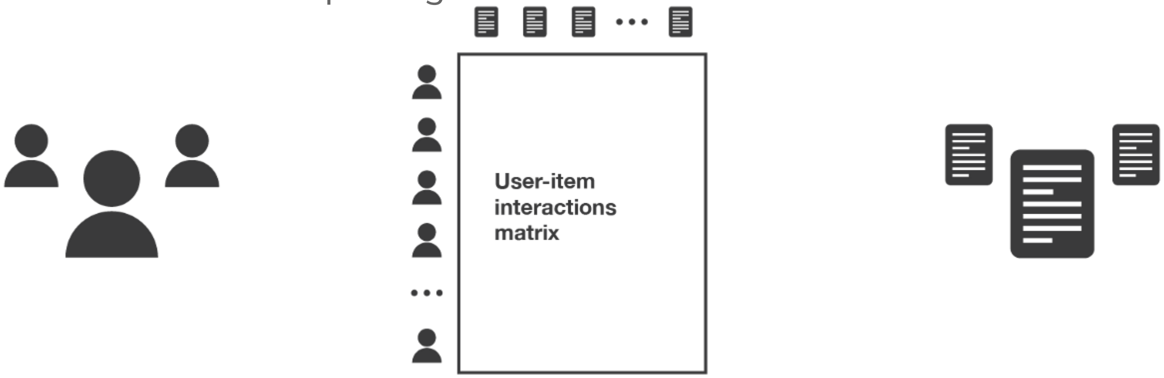
When Ball Boys/Girls Become Hero
STROPS
10M views · 2 months ago
Top 10 Confusing Pit Stops
FORMULA 1
2.3M views · 1 month ago
Novak Djokovic impression Ana Ivanovic and dancing...
Nikola Manic
993K views · 8 years ago
20 BEAUTIFUL MOMENTS OF RESPECT IN SPORTS
Hooked
78M views · 1 year ago
Bugatti Chiron v F1 Car: DRAG RACE
carwow
9.8M views · 2 weeks ago
Matt Viscomi
I can't wait to visit Iceland next week!

Recommender Systems Setup

- You have n users and m items in your system
  - Typically, $n \gg m$. E.g., Youtube: 2.6B users, 800M videos
- Based on the content, we have a way of measuring user preference.
- This data is put together into a **user-item interaction matrix**.



Users	User-item interactions matrix	Items
suscribers	rating given by a user to a movie (integer)	movies
readers	time spent by a reader on an article (float)	articles
buyers	product clicked or not when suggested (boolean)	products
	...	

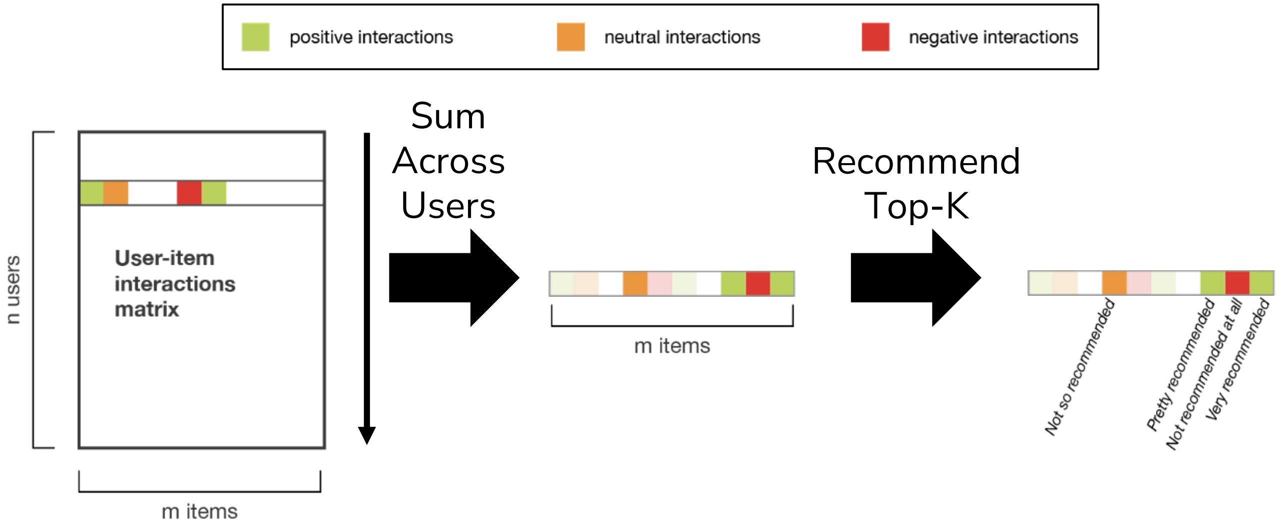
- **Task**: Given a user u_i or item v_j, predict one or more items to recommend.

Solution 0: Popularity

Solution 0: Popularity

Simplest Approach: Recommend whatever is popular

- Rank by global popularity (i.e., Squid Game)



Solution 0 (Popularity) Pros / Cons

Pros:

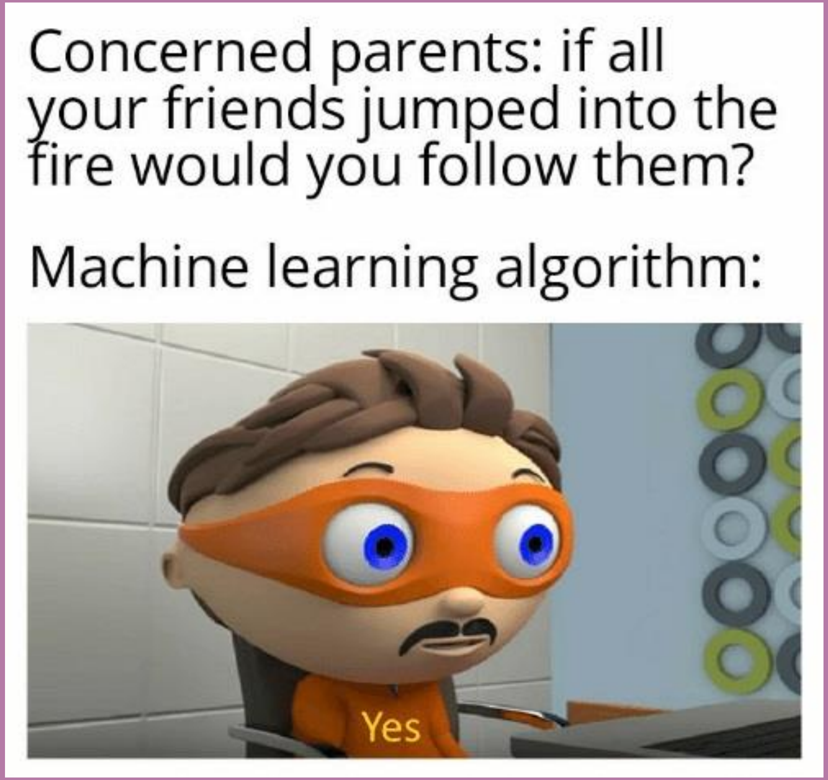
- Easy to implement

Cons:

- No Personalization
- Feedback Loops
- Top-K recommendations might be redundant
  - e.g., when a new Harry Potter movie is released, the others may also jump into top-k popularity.

Solution 1: Nearest User

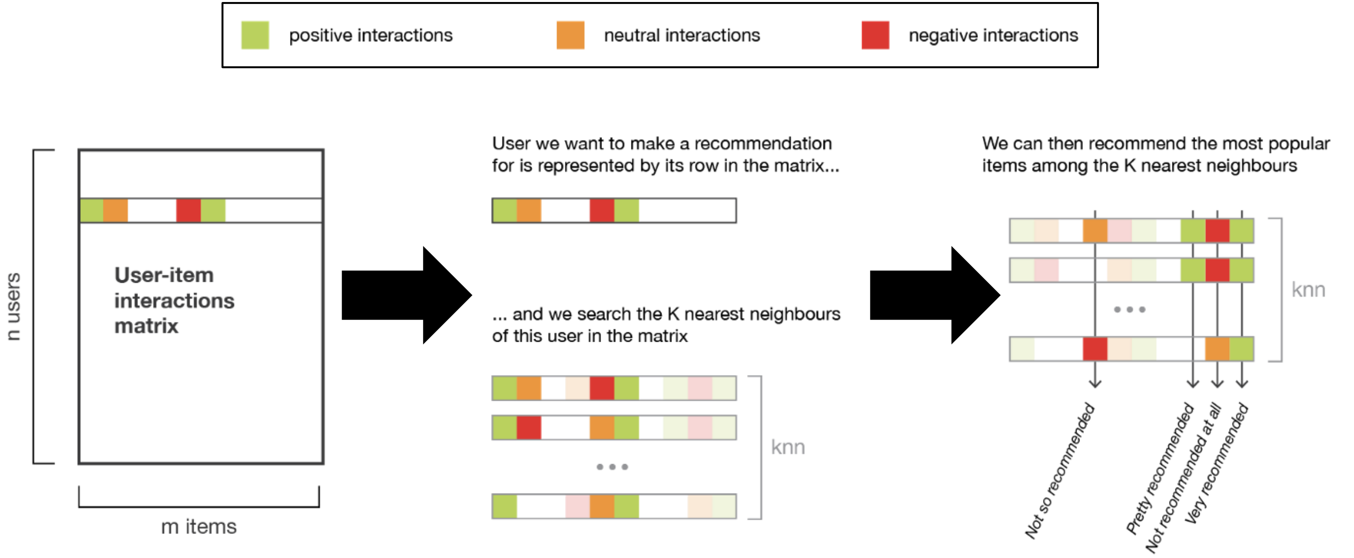
User-User



Solution 1: Nearest User (User-User)

User-User Recommendation:

- Given a user u_i, compute their k nearest neighbors.
- Recommend the items that are most popular amongst the nearest neighbors.

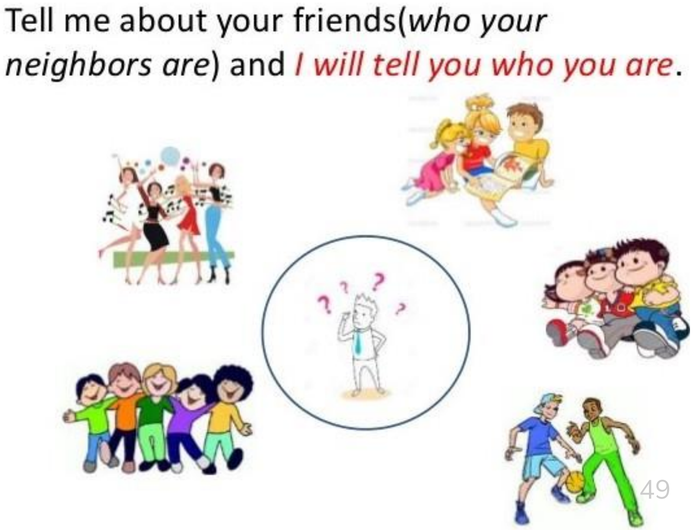


Poll Everywhere

Think

1 min

- What do you see as pros / cons of the nearest user approach to recommendations?



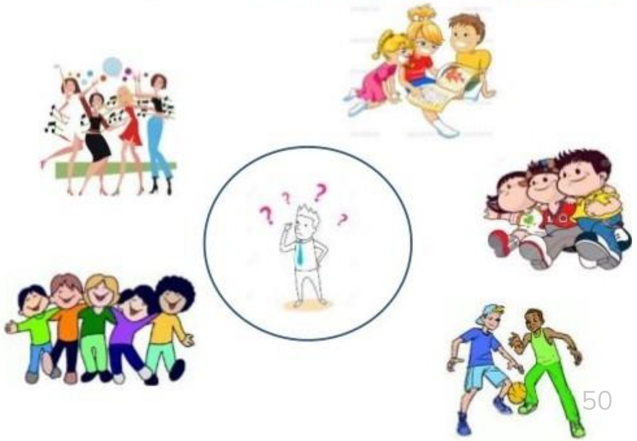
Poll Everywhere

Group

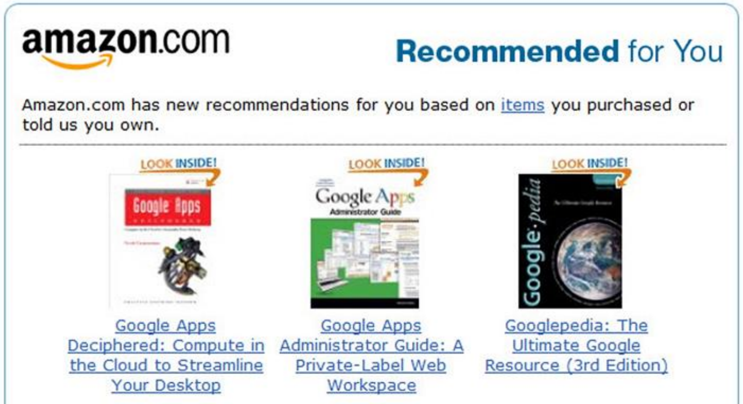
2 min

- What do you see as pros / cons of the nearest user approach to recommendations?

Tell me about your friends(*who your neighbors are*) and *I will tell you who you are*.



Solution 1 (User-User) Pros / Cons



Pros:

- Personalized to the user.

Cons:

- Nearest Neighbors might be too similar
  - This approach only works if the nearest neighbors have interacted with items that the user hasn't.
- Feedback Loop (Echo Chambers)
- Scalability
  - Must store and search through entire user-item matrix
- Cold-Start Problem
  - What do you do about new users, with no data?

Solution 2: “People Who Bought This Also Bought…”

Item-Item

Solution 2: "People Who Bought This Also Bought…" (Item-Item)

Item-Item Recommendation:

- Create a **co-occurrence matrix** $C \in \mathbb{R}^{m \times m}$ (m is the number of items). C_{ij} = # of users who bought both item i and j.
- For item i, predict the top-k items that are bought together.

	Sunglasses	Baby Bottle	…	Diapers	Swim Trunks	Baby Formula
Sunglasses	500	15	…	9	130	20
Baby Bottle	15	45	…	6	10	10
…	…	…	…	…	…	…
Diapers	9	6	…	30	9	6
Swim Trunks	130	10	…	9	200	8
Baby Formula	20	10	…	6	8	50

Normalizing Co-Occurence Matrices

Problem: popular items drown out the rest!

Solution: Normalizing using Jaccard Similarity.

$$S_{ij} = \frac{\#\text{ purchased } i \textbf{ and } j}{\#\text{ purchased } i \textbf{ or } j} = \frac{C_{ij}}{C_{ii} + C_{jj} - C_{ij}}$$

	Sunglasses	Baby Bottle	...	Diapers	Swim Trunks	Baby Formula
Sunglasses	1.00	0.03	...	0.02	0.23	0.04
Baby Bottle	0.03	1.00	...	0.09	0.04	0.12
...	...	...	...	...	...	...
Diapers	0.02	0.09	...	1.00	0.04	0.08
Swim Trunks	0.23	0.04	...	0.04	1.00	0.03
Baby Formula	0.04	0.12	...	0.08	0.03	1.00

Incorporating Purchase History

What if I know the user u has bought a baby bottle and formula?

Idea: Take the average similarity between items they have bought

$$Score(u, diapers) = \frac{S_{diapers,baby\ bottle} + S_{diapers,baby\ formula}}{2}$$

Could also weight them differently based on recency of purchase!

Then all we need to do is find the item with the highest average score!

Poll Everywhere

Think

1 min

- What do you see as pros / cons of the item-item approach to recommendations?

Poll Everywhere

Group

2 min

- What do you see as pros / cons of the item-item approach to recommendations?

Solution 2 (Item-Item) Pros / Cons

Pros:

- Personalizes to item (incorporating purchase history also personalizes to the user)

Cons:

- Can still suffer from feedback loops
  - (As can all recommender systems – but in some cases it's worse than others)
- Scalability (must store entire item-item matrix)
- Cold-Start Problem
  - What do you do about new ***items***, with no data?

Solution 3: Feature-Based

Solution 3: Feature-Based

What if we know what factors lead users to like an item?

Idea: Create a feature vector for each item. Learn a regression model!

Genre	Year	Director	...
Action	1994	Quentin Tarantino	...
Sci-Fi	1977	George Lucas	...

Define weights on these features for **all users** (global)

$$w_G \in \mathbb{R}^d$$

Fit linear model

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Sci-Fi	1977	George Lucas	...

Define weights on these features for **all users** (global)

$$w_G \in \mathbb{R}^d$$

Fit linear model

$$\hat{r}_{uv} = w_G^T h(v) = \sum_i w_{G,i} \, h_i(v)$$

$$\hat{w}_G = argmin_w \frac{1}{\# \, ratings} \sum_{u,v: r_{uv} \neq ?} \left(w_G^T h(v) - r_{uv}\right)^2 + \lambda \|w_G\|$$

Personalization: Option A

Add user-specific features to the feature vector!

Genre	**Year**	**Director**	**…**	**Gender**	**Age**	**…**
Action	1994	Quentin Tarantino	…	F	25	…
Sci-Fi	1977	George Lucas	…	M	42	…

Personalization: Option B

Include a user-specified deviation from the global model.

$$\hat{r}_{uv} = (\hat{w}_G + \hat{w}_u)^T h(v)$$

Start a new user at $\hat{w}_u = 0$, update over time.

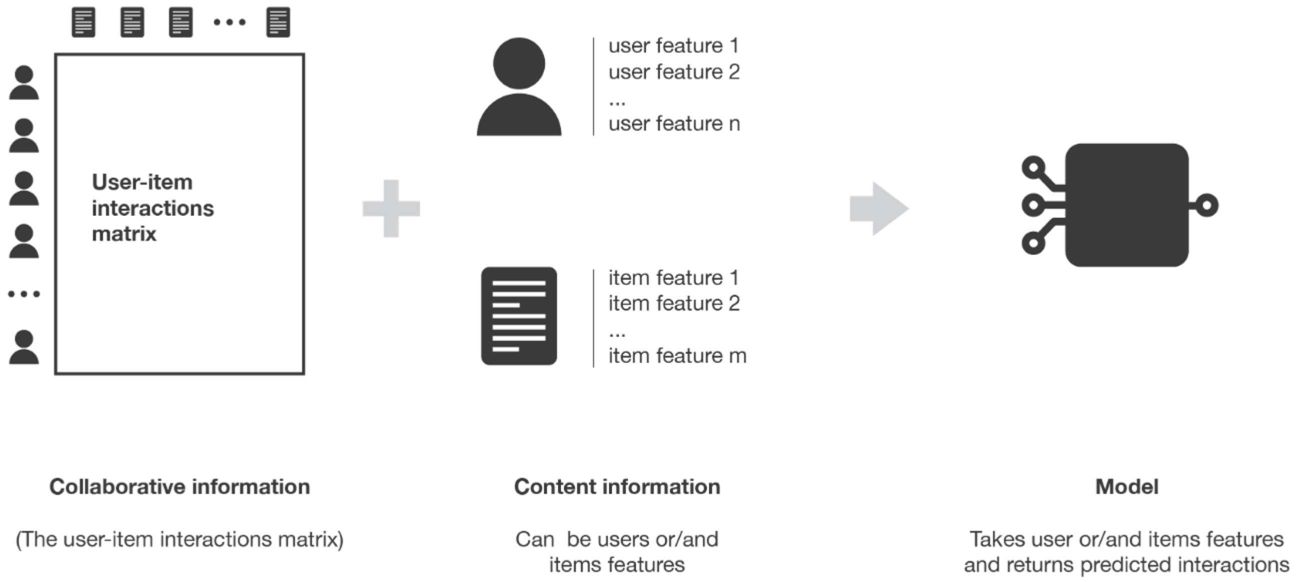
- OLS on the residuals of the global model
- Bayesian Update (start with a probability distribution over user-specific deviations, update as you get more data)

Poll Everywhere

Think

1 min

- Will feature-based recommender systems suffer from the cold start problem? Why or why not?
- What about other pros/cons of feature-based?

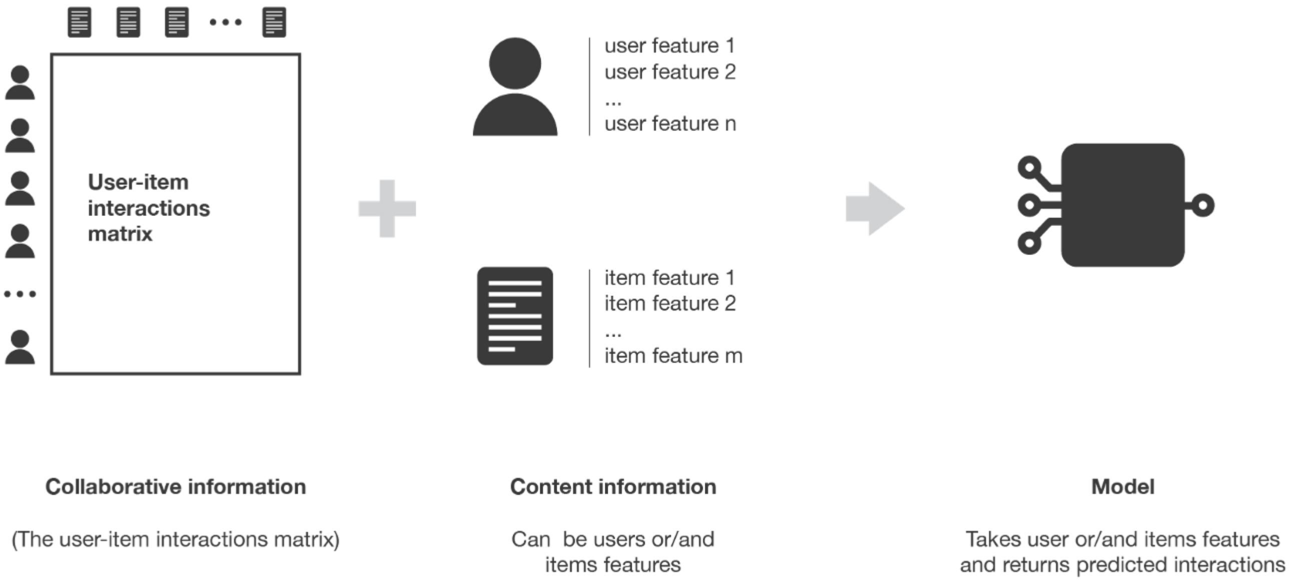


Poll Everywhere

Group

2 min

- Will feature-based recommender systems suffer from the cold start problem? Why or why not?
- What about other pros/cons of feature-based?



Solution 3 (Feature-Based) Pros / Cons

Pros:

- No cold-start issue!
  - Even if a new user/item has no purchase history, you know features about them.
- Personalizes to the user and item.
- Scalable (only need to store weights per feature)
- Can add arbitrary features (e.g., time of day)

Cons:

- Hand-crafting features is very tedious and unscalable ☹

Recap

Dimensionality Reduction & PCA:

- Why and when it's important
- High level intuition for PCA
- Linear Projections & Reconstruction
- Eigenvectors / Eigenvalues

Recommender Systems:

- Sol 0: Popularity
- Sol 1: Nearest User (User-User)
- Sol 2: "People who bought this also bought" (item-item)
- Sol 3: Feature-Base

Next Time (Rec System Continued):

- Sol 4: Matrix Factorization
- Sol 5: Hybrid Model
- Addressing common issues with Recommender Systems
- Evaluating Recommender Systems