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CSE/STAT 416

Precision/Recall
k-Nearest Neighbors

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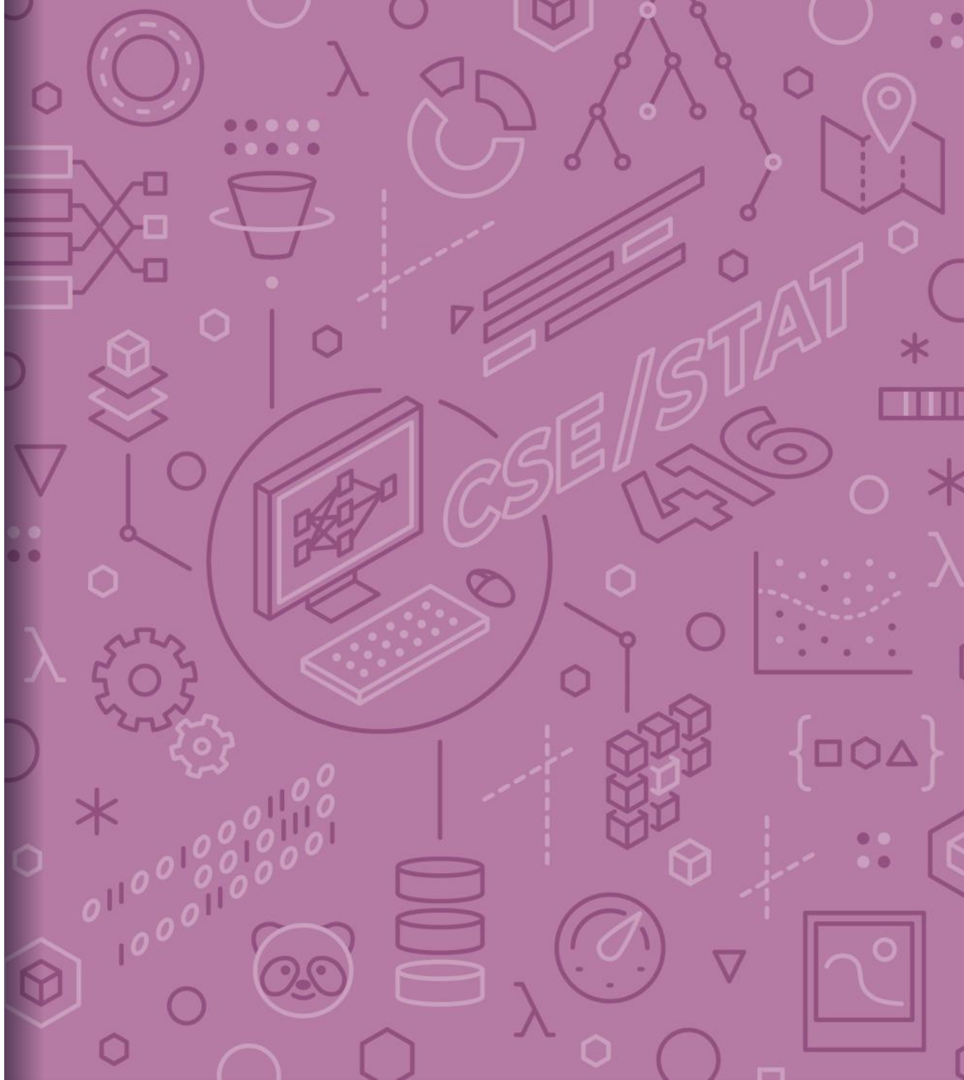
Music: Charlie XCX



Pre-Lecture Video

Detecting Spam

Imagine I made a “Dummy Classifier” for detecting spam

- The classifier ignores the input, and always predicts spam.
- This actually results in 90% accuracy! Why?
  - Most emails are spam…

This is called the **majority class classifier.**

A classifier as simple as the majority class classifier can have a high accuracy if there is a **class imbalance**.

- A class imbalance is when one class appears much more frequently than another in the dataset

This might suggest that accuracy isn’t enough to tell us if a model is a good model.

Assessing Accuracy

Always digging in and ask critical questions of your accuracy.

- Is there a **class imbalance**?
- How does it compare to a baseline approach?
  - Random guessing
  - Majority class
  - …
- Most important: **What does my application need?**
  - What's good enough for user experience?
  - What is the impact of a mistake we make?

Confusion Matrix

For binary classification, there are only two types of mistakes

- $\hat{y} = +1, \;\; y = -1$
- $\hat{y} = -1, \;\; y = +1$

Generally we make a **confusion matrix** to understand mistakes.

Predicted Label

True Label	+	−
+	True Positive (TP)	False Negative (FN)
−	False Positive (FP)	True Negative (TN)

Binary Classification Measures

Notation

- $C_{TP} = \#\mathrm{TP}, \quad \mathrm{C}_{\mathrm{FP}} = \#\mathrm{FP}, \quad \mathrm{C}_{\mathrm{TN}} = \#\mathrm{TN}, \quad \mathrm{C}_{\mathrm{FN}} = \#\mathrm{FN}$
- $N = C_{TP} + C_{FP} + C_{TN} + C_{FN}$
- $N_P = C_{TP} + C_{FP}, \quad N_N = C_{FP} + C_{TN}$

Error Rate

$$\frac{C_{FP} + C_{FN}}{N}$$

Accuracy Rate

$$\frac{C_{TP} + C_{TN}}{N}$$

False Positive rate (FPR)

$$\frac{C_{FP}}{N_N}$$

False Negative Rate (FNR)

$$\frac{C_{FN}}{N_P}$$

True Positive Rate or Recall

$$\frac{T_P}{N_P}$$

Precision

$$\frac{T_P}{C_{TP} + C_{FP}}$$

F1-Score

$$2\frac{Precision \cdot Recall}{Precison + Recall}$$

[See more!]

Change Threshold

What if I never want to make a false positive prediction?

Always predict negative $(\alpha = \infty)$

What if I never want to make a false negative prediction?

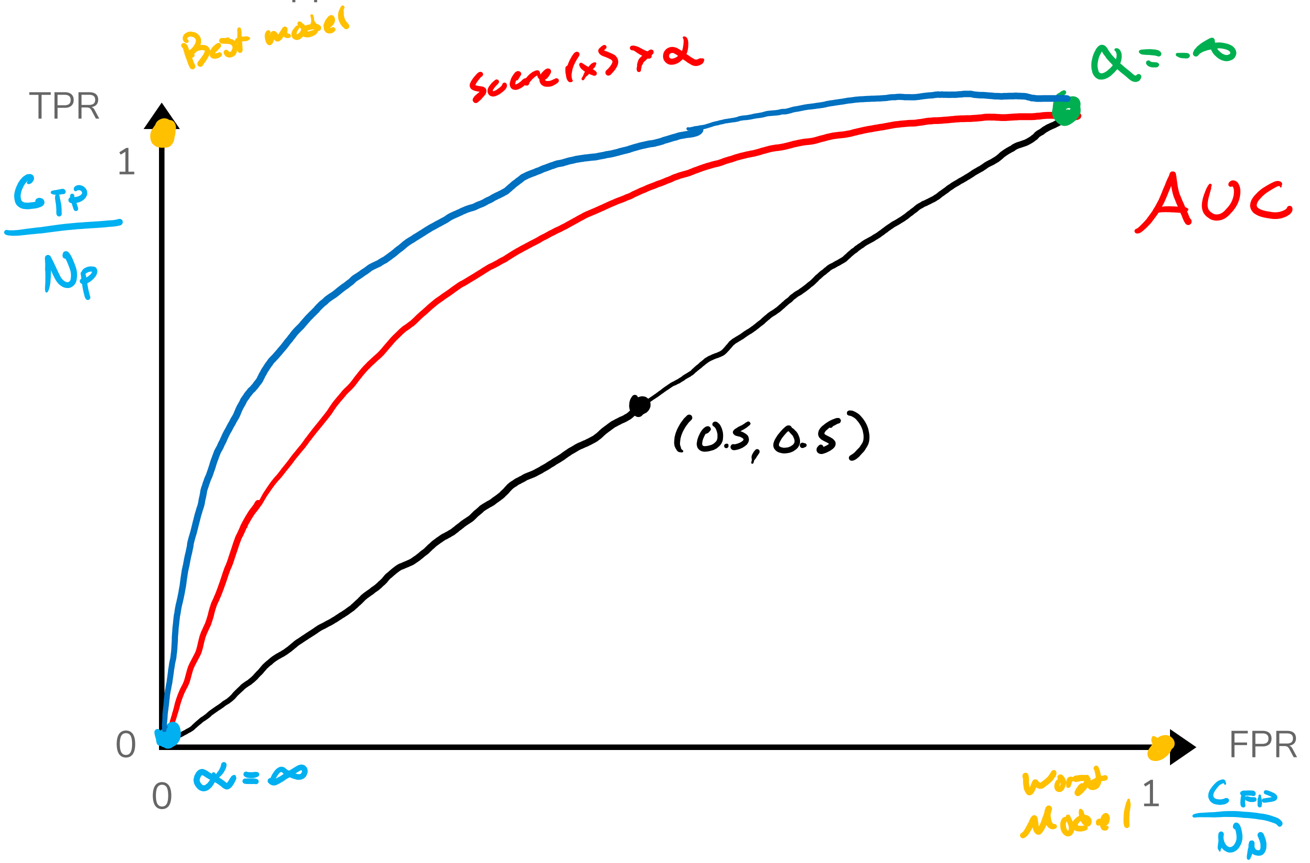
Always predicting positive $(\alpha = -\infty)$

One way to control for our application is to change the scoring threshold. (Could also change intercept!)

- If $\underline{Score(x) > \alpha}$:
  - Predict $\hat{y} = +1$
- Else:
  - Predict $\hat{y} = -1$

ROC Curve

What happens to our TPR and FPR as we increase the threshold?



Assessing Accuracy

Often with binary classification, we treat the positive label as being the more important of the two. We then often then focus on these metrics:

Precision: Of the ones I predicted positive, how many of them were actually positive?

Recall: Of all the things that are truly positive, how many of them did I correctly predict as positive?

Precision

What fraction of the examples I predicted positive were correct?

Sentences predicted to be positive:

$\hat{y}_i = +1$

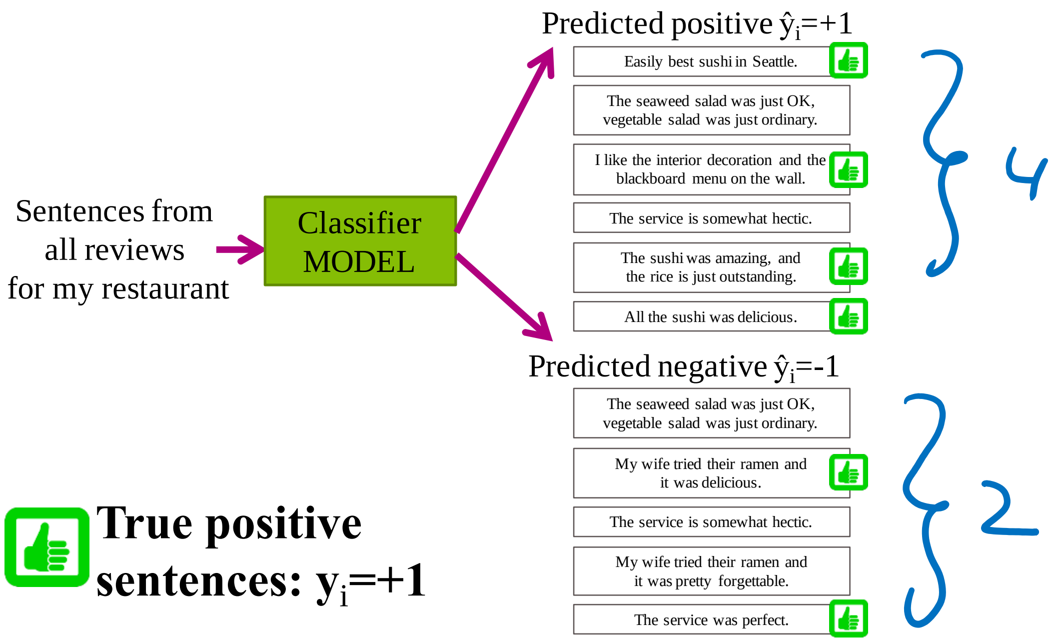
Easily best sushi in Seattle.	✓
The seaweed salad was just OK, vegetable salad was just ordinary.	✗
I like the interior decoration and the blackboard menu on the wall.	✓
The service is somewhat hectic.	✗
The sushi was amazing, and the rice is just outstanding.	✓
All the sushi was delicious.	✓

Only 4 out of 6 sentences predicted to be positive are actually positive

$$precision = \frac{C_{TP}}{C_{TP} + C_{FP}} = \frac{4}{4+2} = \frac{2}{3}$$

Recall

Of the truly positive examples, how many were predicted positive?



$$recall = \frac{C_{TP}}{N_P} = \frac{C_{TP}}{C_{TP} + C_{FN}} = \frac{4}{4+2} = \frac{2}{3}$$

Precision & Recall

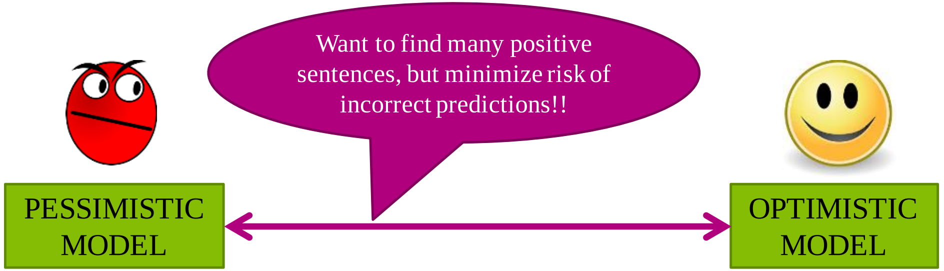
An optimistic model will predict almost everything as positive 

- High recall, low precision

$\frac{0}{0} = 1$

A pessimistic model will predict almost everything as negative 

- High precision, low recall



Finds few positive sentences, but includes no false positives

Finds all positive sentences, but includes many false positives

$\alpha = \infty$ $\longleftarrow \alpha \longrightarrow$ $\alpha = -\infty$

Controlling Precision/Recall

Depending on your application, precision or recall might be more important

- Ideally you will have high values for both, but generally increasing recall will decrease precision and vice versa.

For logistic regression, we can control for how optimistic the model is by changing the threshold for positive classification

Before

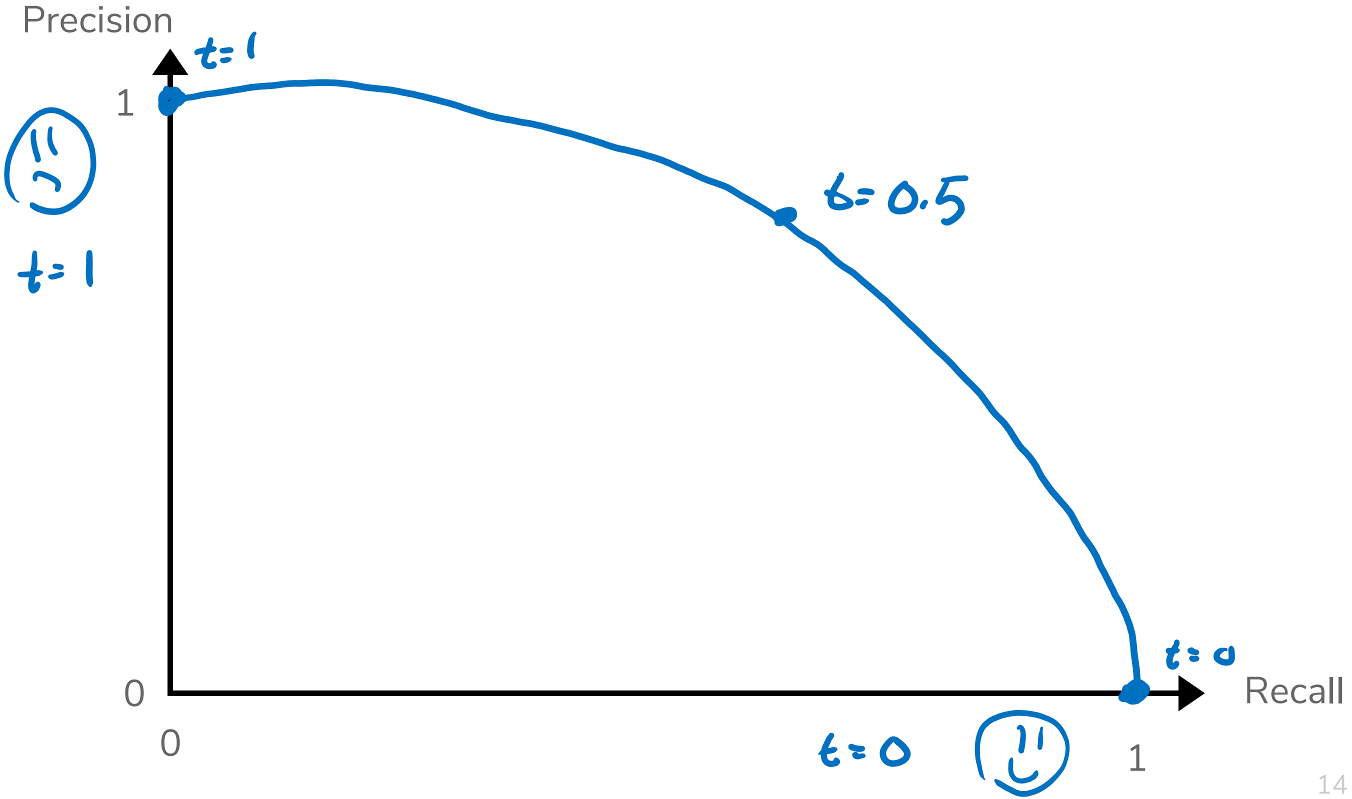
$\hat{y}_i = +1$ if $\hat{P}(y = +1|x_i) > 0.5$ else $\hat{y}_i = -1$

Now

$\hat{y}_i = +1$ if $\hat{P}(y = +1|x_i) > t$ else $\hat{y}_i = -1$

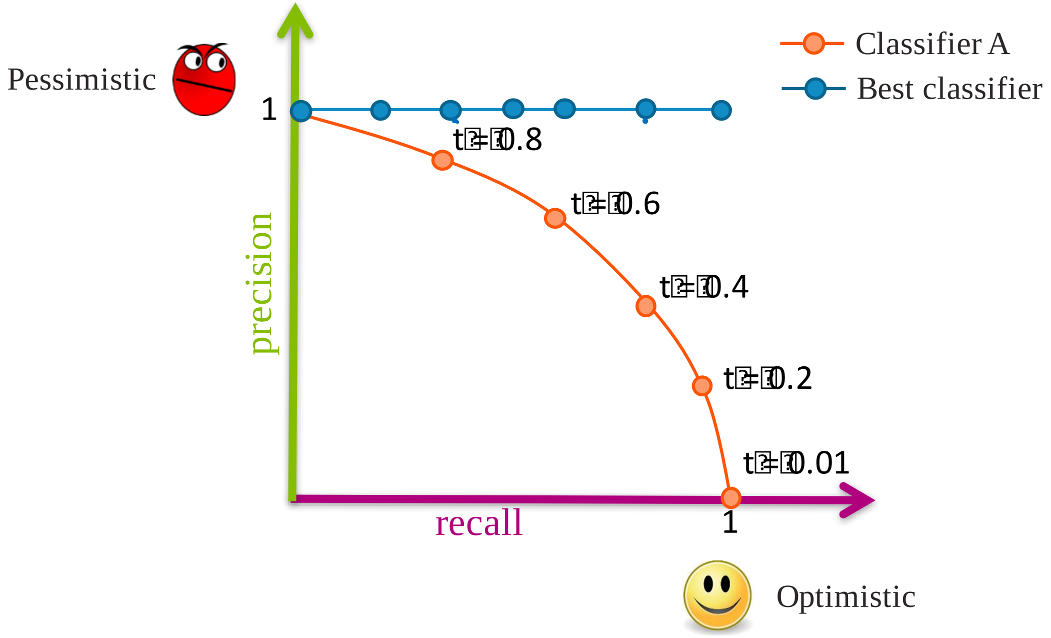
Precision-Recall Curve

$$\frac{C_{TP}}{C_{TP}+C_{FP}} = \frac{0}{0} = 1$$



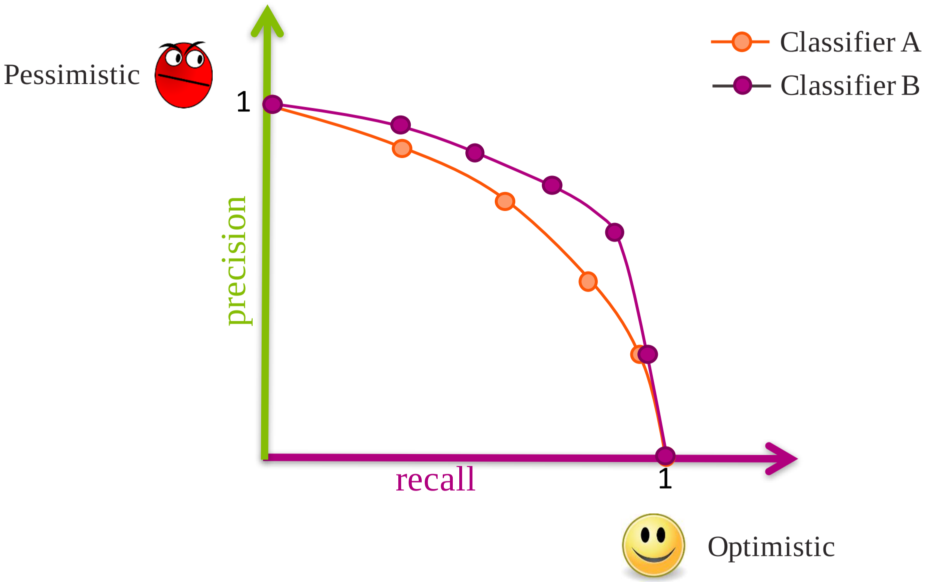
Precision-Recall Curve

Can try every threshold to get a curve like below



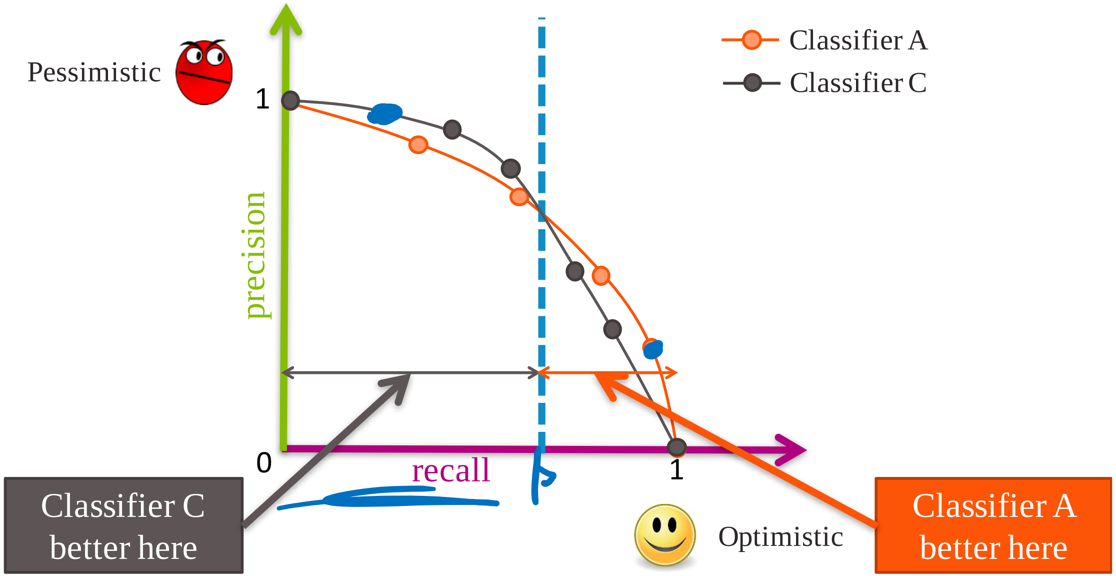
Precision-Recall Curve

Sometimes, Classifier B is strictly better than Classifier A



Precision-Recall Curve

Most times, the classifiers are incomparable



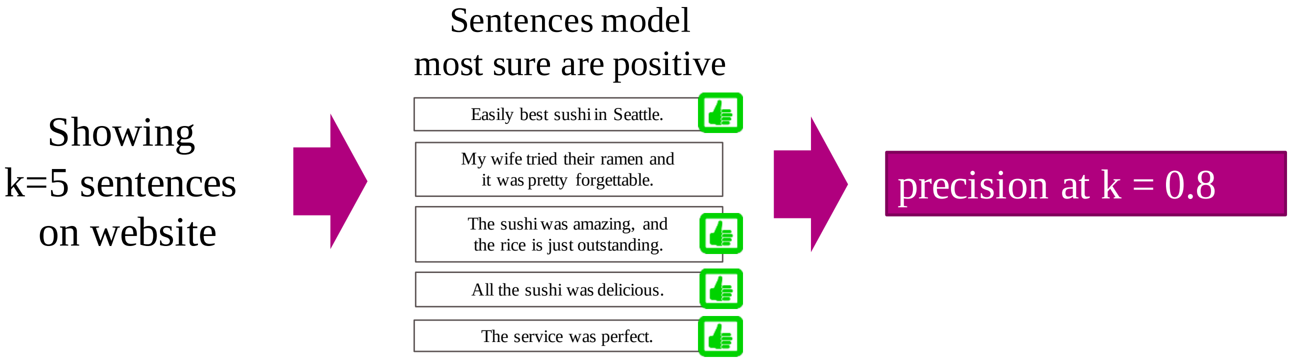
Compare Classifiers

Often come up with a single number to describe it

- F1-score, AUC, etc.
- Remember, what your application needs is most important

Also common to use **precision at k**

- If you show the top **k** most likely positive examples, how many of them are true positives



Class Session

Roadmap

1. Housing Prices - Regression
   - Regression Model
   - Assessing Performance
   - Ridge Regression
   - LASSO
2. Sentiment Analysis – Classification
   - Classification Overview
   - Logistic Regression
   - Bias / Fairness
   - Decision Trees
   - Ensemble Methods
3. Document Retrieval – Clustering and Similarity
   - Precision / Recall
   - k-Nearest Neighbor
   - Kernel Methods
   - Locality Sensitive Hashing
   - Clustering
   - Hierarchical Clustering

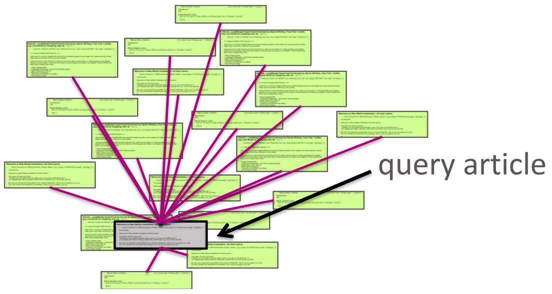
Supervised Learning

Unsupervised Learning

Document Retrieval

- Consider you had some time to read a book and wanted to find other books similar to that one.
- If we wanted to write an system to recommend books
  - How do we measure similarity?
  - How do we search over books?
  - How do we measure accuracy?

Big Idea: Define an **embedding** and a **similarity metric** for the books, and find the **“nearest neighbor”** to some query book.



Nearest Neighbors

1-Nearest Neighbor

Input

- x_q: Query example (e.g. my book)
- $x_1, \dots, x_n$: Corpus of documents (e.g. Amazon books)

Output

- The document in corpus that is most similar to x_q

$$x^{NN} = \underset{x_i \in [x_1, \dots, x_n]}{\arg\min} \; distance(x_q, x_i)$$

It's very critical to properly define how we represent each document x_i and the similarity metric $distance$! Different definitions will lead to very different results.

1-Nearest Neighbor

How long does it take to find the 1-NN? About n operations

$n = 100{,}000{,}000$

Input: x_q

$x^{NN} = \emptyset$

$nn\_dist = \infty$

$for\ x_i \in [x_1, \dots, x_n]$:

  $dist = distance(x_q, x_i)$

  $if\ dist < nn\_dist$:

    $x^{NN} = x_i$

    $nn\_dist = dist$

Output: x^{NN}

$O(n)$

k-Nearest Neighbors

Input

- x_q: Query example (e.g. my book)
- $x_1, \ldots, x_n$: Corpus of documents (e.g. Amazon books)

Output

- List of k documents most similar to x_q

Formally

k-Nearest Neighbors

Current

$X^{k\text{-}NN} = [x_1, x_2, x_3]$

nn_dists = [4, 7, 5]

See

x_4

3

Next

$X^{k\text{-}NN} = [x_1, x_3, x_4]$

nn_dists = [4, 5, 3]

Same idea as 1-NN algorithm, but maintain list of k-NN

current k-NNs

Input: x_q

$X^{k-NN} = [x_1, \ldots, x_k]$

$nn\_dists = [dist(x_1, x_q), dist(x_2, x_q), \ldots, dist(x_k, x_q)]$

$for\ x_i \in [x_{k+1}, \ldots, x_n]:$

$\quad dist = distance(x_q, x_i)$

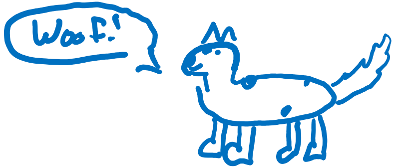
$\quad if\ dist < \max(nn\_dists):$

$\quad\quad remove\ largest\ dist\ from\ X^{k-NN}\ and\ nn\_dists$

$\quad\quad add\ x_i\ to\ X^{k-NN} and\ distance(x_q, x_i)\ to\ nn\_dists$

Output: X^{k-NN}

Brain Break: 3:06



k-Nearest Neighbors

Can be used in many circumstances!

Retrieval

Return X^{k-NN}

Regression

$$\hat{y}_i = \frac{1}{k}\sum_{j=1}^{k} x^{NN_j}$$

Classification

$$\hat{y}_i = majority\_class(X^{k-NN})$$

will discuss Wed.

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 4 \\ 0 \\ 5 \end{bmatrix}$$

Embeddings

Important Points

While the formalization of these algorithms are fairly tedious, the intuition is fairly simple. Find the 1 or k nearest neighbors to a given document and return those as the answer.

This intuition relies on answering two important questions

- How do we represent the documents x_i?
- How do we measure the distance $distance(x_q, x_i)$?

D = # unique words in our corpus

x_i = [#I, # like, # dogs, # cats, ...]

Document Representation

Like our previous ML algorithms, we will want to make a vector out of the document to represent it as a point in space.

Simplest representation is the **bag-of-words** representation.

- Each document will become a W dimension vector where W is the number of words in the entire corpus of documents
- The value of $x_i[j]$ will be the number of times word j appears in document i.
- This ignores order of words in the document, just the counts.

"I like dogs" -> [1, 1, 1, 0]

"I like cats" -> [1, 1, 0, 1]

"I like dogs dogs" -> [1, 1, 2, 0]

Bag of Words

Pros

- Very simple to describe
- Very simple to compute

Cons

- Common words like “the” and “a” dominate counts of uncommon words
- Often it’s the uncommon words that uniquely define a doc.

TF-IDF = Term Frequency × Inverse Document Frequency

TF-IDF

Goal: Emphasize important words

- Appear frequently in the document (common locally)

Term frequency = word counts

"the" appears in Doc1 200 times, TF("the", doc1) = 200

- Appears rarely in the corpus (rare globally)

Inverse doc freq. = $\log \frac{\#\ \text{docs}}{1 + \#\ \text{docs using word}}$

"the" appears in all 1000 docs

$$IDF(\text{"the"}) = \log\left(\frac{1000}{1+1000}\right)$$
$$\approx \log(1) = 0$$

tf * idf

Do a pair-wise multiplication to compute the TF-IDF for each word

- Words that appear in every document will have a small IDF making the TF-IDF small!

num features → D = W
← num unique words

Distance

Euclidian Distance

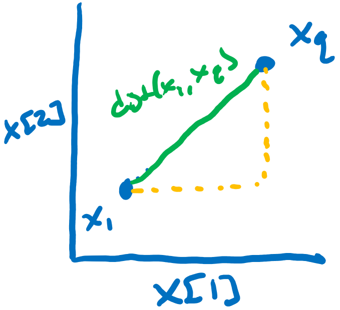
Now we will define what similarity/distance means

Want to define how "close" two vectors are. A smaller value for distance means they are closer, a large value for distance means they are farther away.

The simplest way to define distance between vectors is the **Euclidean distance**

$$distance(x_i, x_q) = \left\|x_i - x_q\right\|_2$$

$$= \sqrt{\sum_{j=1}^{D} \left(x_i[j] - x_q[j]\right)^2}$$

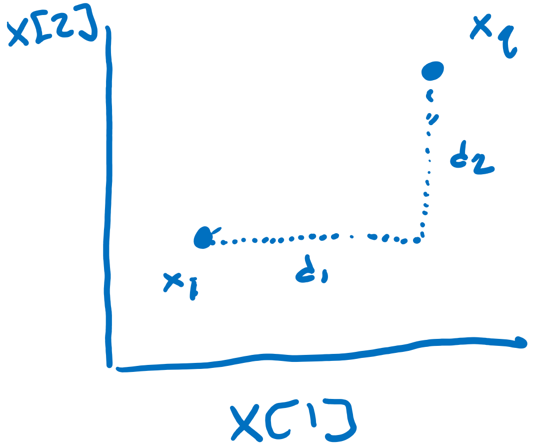


Manhattan Distance

Another common choice of distance is the **Manhattan Distance**

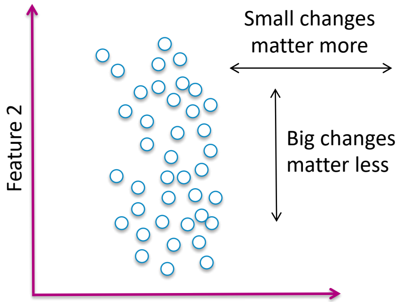
$$distance(x_i, x_q) = \left|\left|x_i - x_q\right|\right|_1$$

$$= \sum_{j=1}^{D} \left|x_i[j] - x_q[j]\right|$$



Weighted Distances

Some features vary more than others or are measured in different units. We can weight different dimensions differently to make the distance metric more reasonable.



Specify weights as a function of feature spread

For feature j: $\frac{1}{\max_i(x_i[j]) - \min_i(x_i[j])} = a_j$

Weighted Educlidean distance

$$distance(x_i, x_q) = \sqrt{\sum_{j=1}^{D} a_j^2 \left(x_i[j] - x_q[j]\right)^2}$$

Similarity

Another natural similarity measure would use

$$x_i^T x_q = \sum_{j=1}^{D} x_i[j] x_q[j]$$

Notice this is a measure of similarity, not distance

- This means a bigger number is better

1	0	0	0	5	3	0	0	1	0	0	0	0
3	0	0	0	2	0	0	1	0	1	0	0	0

Similarity

$= x_i^T x_q$

$= \sum_{j=1}^{d} x_i[j]\, x_q[j]$

$= 13$

1	0	0	0	5	3	0	0	1	0	0	0	0
0	0	1	0	0	0	9	0	0	6	0	4	0

Similarity $= 0$

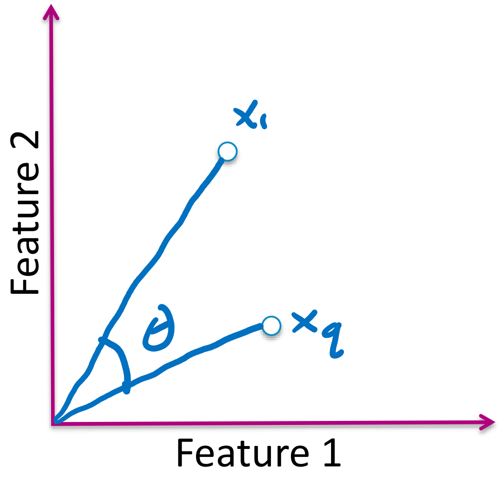
Cosine Similarity

Should we normalize the vectors before finding the similarity?

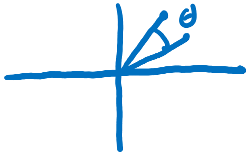
$$similarity = \frac{x_i^T x_q}{||x_i||_2 \, ||x_q||_2} = \cos(\theta)$$

Note:

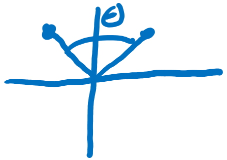
- Not a true distance metric
- Efficient for sparse vectors!



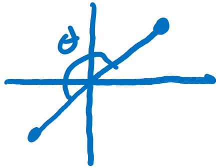
Cosine Similarity



$\cos(\theta) \approx 1$
dist ≈ 0



$\cos(\theta) \approx 0$
dist ≈ 1



$\cos(\theta) \approx -1$
dist ≈ 2

In general

$$-1 \leq cosine\ similarity \leq 1$$

For positive features (like TF-IDF)

$$0 \leq cosine\ similarity \leq 1$$

Define

$$distance = 1\ - similarity$$

To Normalize or Not To Normalize?

Not normalized

1	0	0	0	5	3	0	0	1	0	0	0	0

Similarity = 13

3	1	0	0	2	0	0	1	0	1	0	0	0

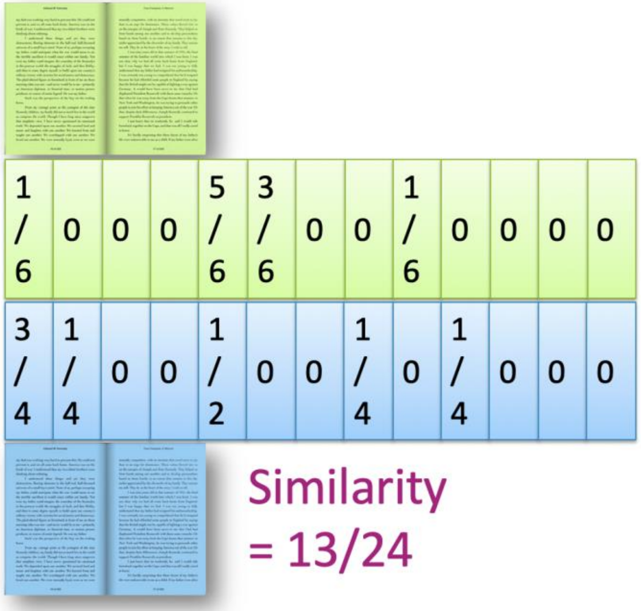
2	0	0	0	10	6	0	0	2	0	0	0	0

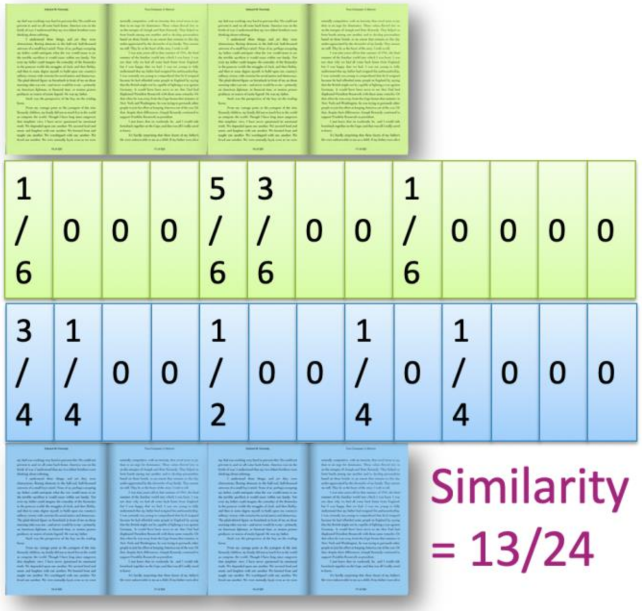
Similarity = 52

6	2	0	0	4	0	0	2	0	2	0	0	0

To Normalize or Not To Normalize?

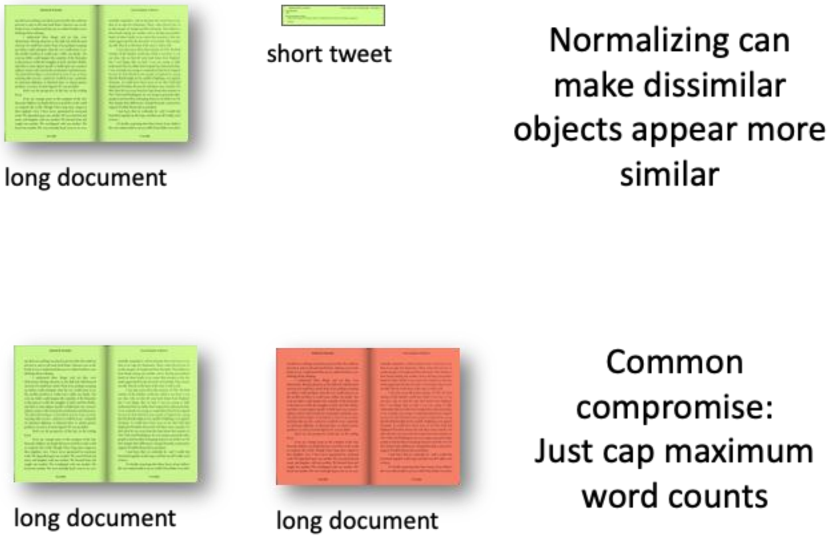
Normalized





To Normalize or Not To Normalize?

Normalization is not desired when comparing documents of different sizes since it ignores length.



In practice, can use multiple distance metrics and combine them using some defined weights

Poll Everywhere

Think

3 min

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Not a real Poll Everywhere question, just time to work!

For the given documents, what are their Euclidean Distance and Cosine Similarity?

Assume we are using a bag of words representation

Document 1: “I really like dogs”

Document 2: “dogs are really really awesome”

Steps:

- Write out bag of words vectors
- Compute Euclidean distance
- Compute Cosine similarity

Poll Everywhere

Think

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Doc1 = "I really like dogs" Doc2 = "dogs are really really awesome"

BoW: [I, really, like, dogs, are, awesome]

$x_1 = [1, 1, 1, 1, 0, 0]$ $x_2 = [0, 2, 0, 1, 1, 1]$

Euclidean Distance = $\|x_1 - x_2\|_2$

$$dist(x_1, x_2) = \sqrt{(1-0)^2 + (1-2)^2 + (1-0)^2 + (1-1)^2 + (0-1)^2 + (0-1)^2}$$

$$= \sqrt{5}$$

Cosine Distance = $1 - \dfrac{x_1^T x_2}{\|x_1\|_2 \|x_2\|_2}$

$$dist(x_1, x_2) = 1 - \frac{1\cdot 0 + 1\cdot 2 + 1\cdot 0 + 1\cdot 1 + 0\cdot 1 + 0\cdot 1}{\sqrt{1^2+1^2+1^2+1^2+0^2+0^2}\sqrt{0^2+2^2+0^2+1^2+1^2+1^2}}$$

$$= 1 - \frac{3}{\sqrt{4}\sqrt{7}} \approx 0.433$$

Recap

Theme: Use nearest neighbors to recommend documents.

Ideas:

- Precision and Recall Curves
- Implement a nearest neighbor algorithm
- Compare and contrast different document representations
  - Emphasize important words with TF-IDF
- Compare and contrast different measurements of similarity
  - Euclidean and weighted Euclidean
  - Cosine similarity and inner-product similarity