

## PCA

1. Assume we are given three data points in  $\mathbb{R}^3$  :  $(1, 1), (2, 2), (3, 3)$ .

(a) Compute the covariance matrix. Recall the following formula:

$$\Sigma[t, s] = \frac{1}{n-1} \sum_{i=1}^n x_{c,i}[t] x_{c,i}[s]$$

(b) The associated eigenvalues and eigenvectors of the previous matrix are

$$\lambda_1 = 2, \lambda_2 = 0, v_1 = (1, 1), v_2 = (-1, 1)$$

If our goal is to minimize reconstruction error, which eigenvalue/eigenvector pair would we choose?

(c) What is the reconstruction error for the first principal component?

(d) Create an  $x/y$  axis and draw the data points and the principal component from part (c).

## Recommender Systems

1. (a) When doing matrix factorization, how do we find the best  $k$  latent factors?
  - (b) Consider the case of learning a featurized matrix factorization model. Suppose instead of using a linear model we use a neural network. Is it possible to use a neural network like this? If not, explain why. If it is possible, give one benefit and drawback of using a neural network here.
  
2. (a) Do recommendation systems fit into supervised or unsupervised learning?
  - (b) Recall the matrix of users and ratings with missing data from class. Denote the  $(u, v)$ -th entry of this matrix as  $r_{u,v}$ , the user specific weights as  $L_u$  and rating specific weights as  $R_v$ .
    - i. In matrix factorization, how is  $r_{u,v}$  modeled in terms of  $L_u, R_v$ ?
  
    - ii. What is the quality metric that is used to estimate  $L_u, R_v$ ?
  
    - iii. If your movie ratings were between 1 and 5, is it possible for the approximate ratings found by minimizing the above quality metric to be above 5?