Decision Tree Classification

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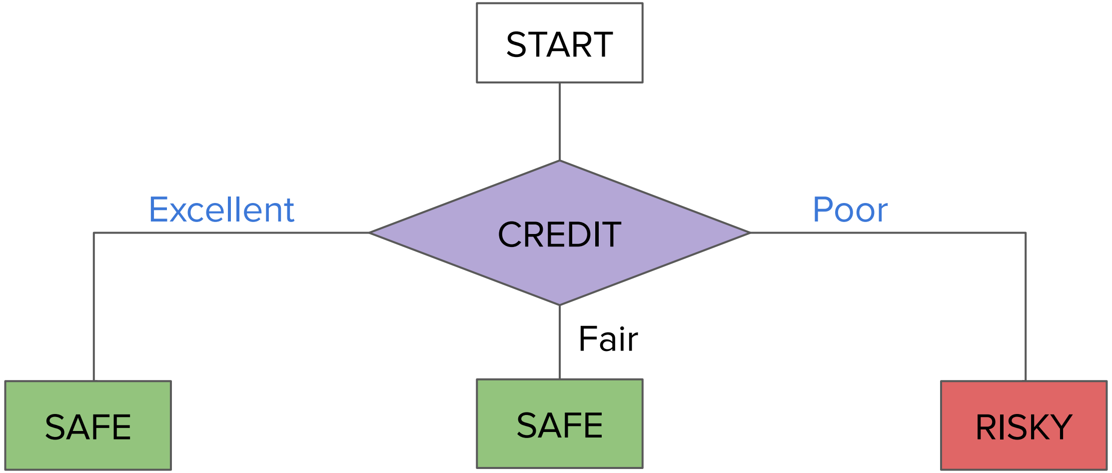
Example: Predicting potential loan defaults

- Data: discrete for now (e.g. credit rating: excellent, fair, poor)
- Goal: Given a new loan application, predict whether or not the applicant will default on their loan:

Credit	Term	Income	Y
excellent	3 years	high	safe
fair	5 years	low	risky
fair	3 years	high	risky
poor	3 years	high	risky

Decision Tree



- Internal Node: A node that tests a feature
- Branch: Splits input data based on the value of a feature
- Leaf: Assigns a class to data (i.e. SAFE, RISKY)

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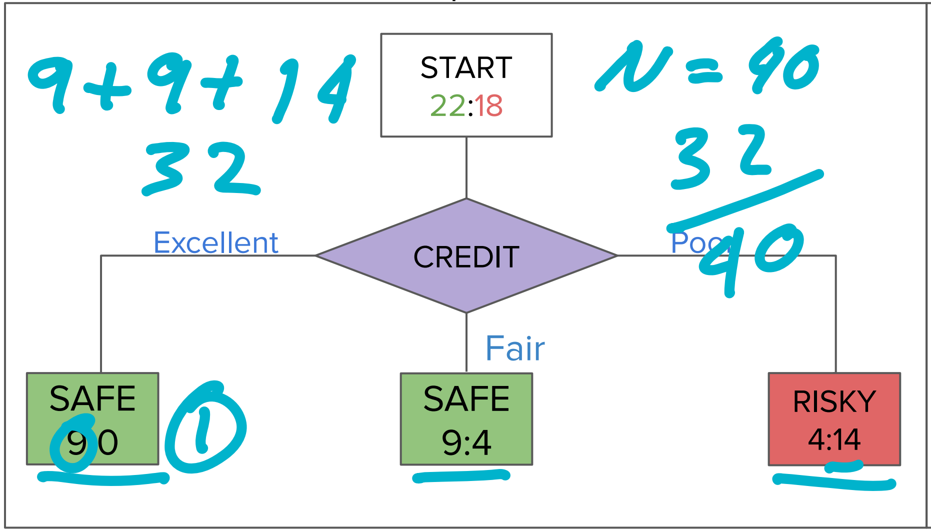
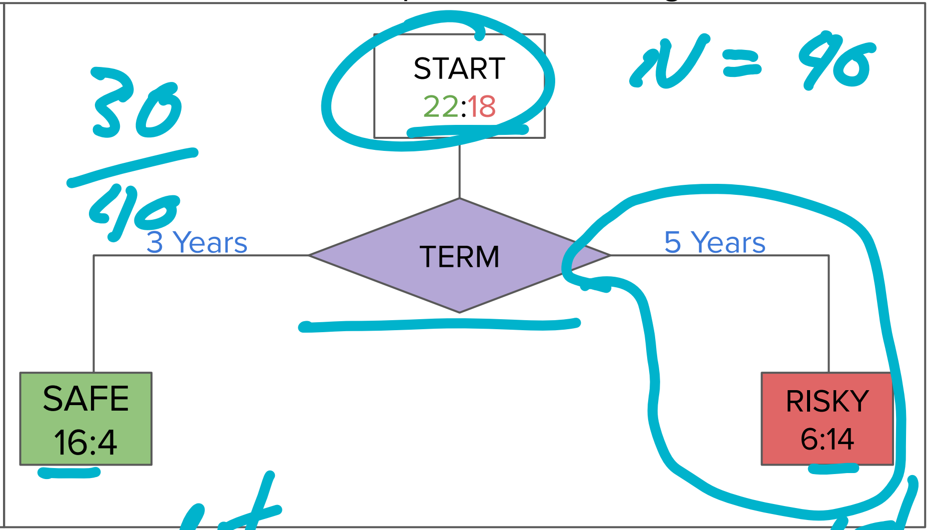
Decision Stumps

Choice 1: Split on Credit ✓



Choice 2: Split on Term Length



credit   credit

- How do we decide which split to make?
- Always pick the split which maximizes accuracy

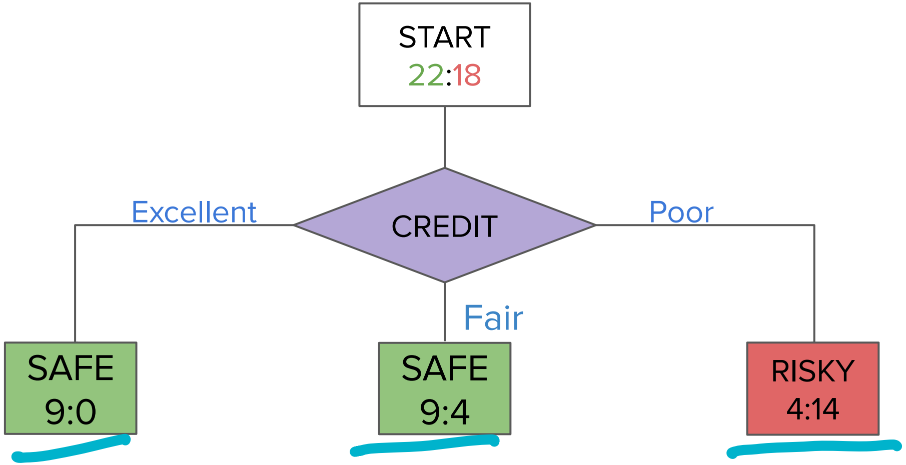
$$accuracy = \frac{\#correct\ predictions}{\#data\ points}$$

Greedy Algorithm for Growing a Decision Tree

- Start with a single root node
- Repeat while the stopping rule is not met
  - Choose a feature x[i] to split that maximizes classification accuracy
- Stopping Rule:
  - 1) Do not branch if all data has the same label (pure)
  - 2) We have already split on that feature before

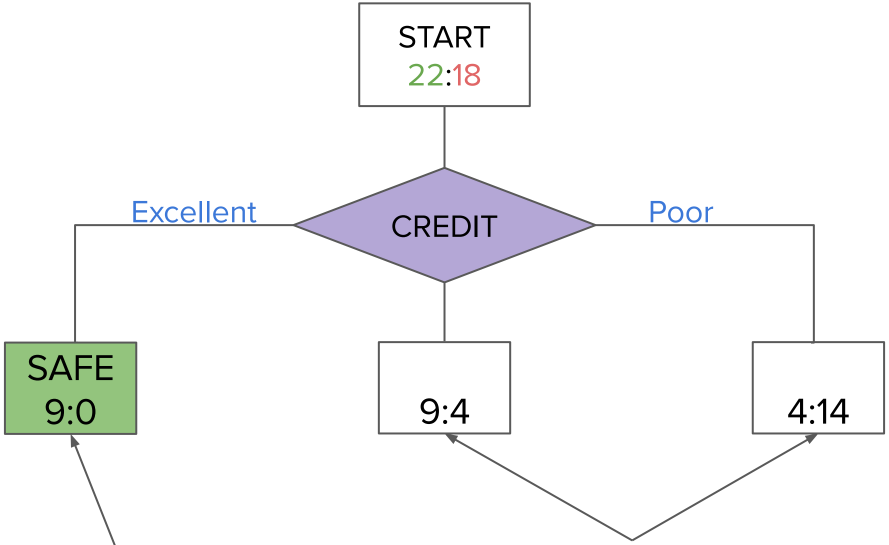
Greedy Algorithm

Classification Accuracy: 80%



Greedy Algorithm

Classification Accuracy:

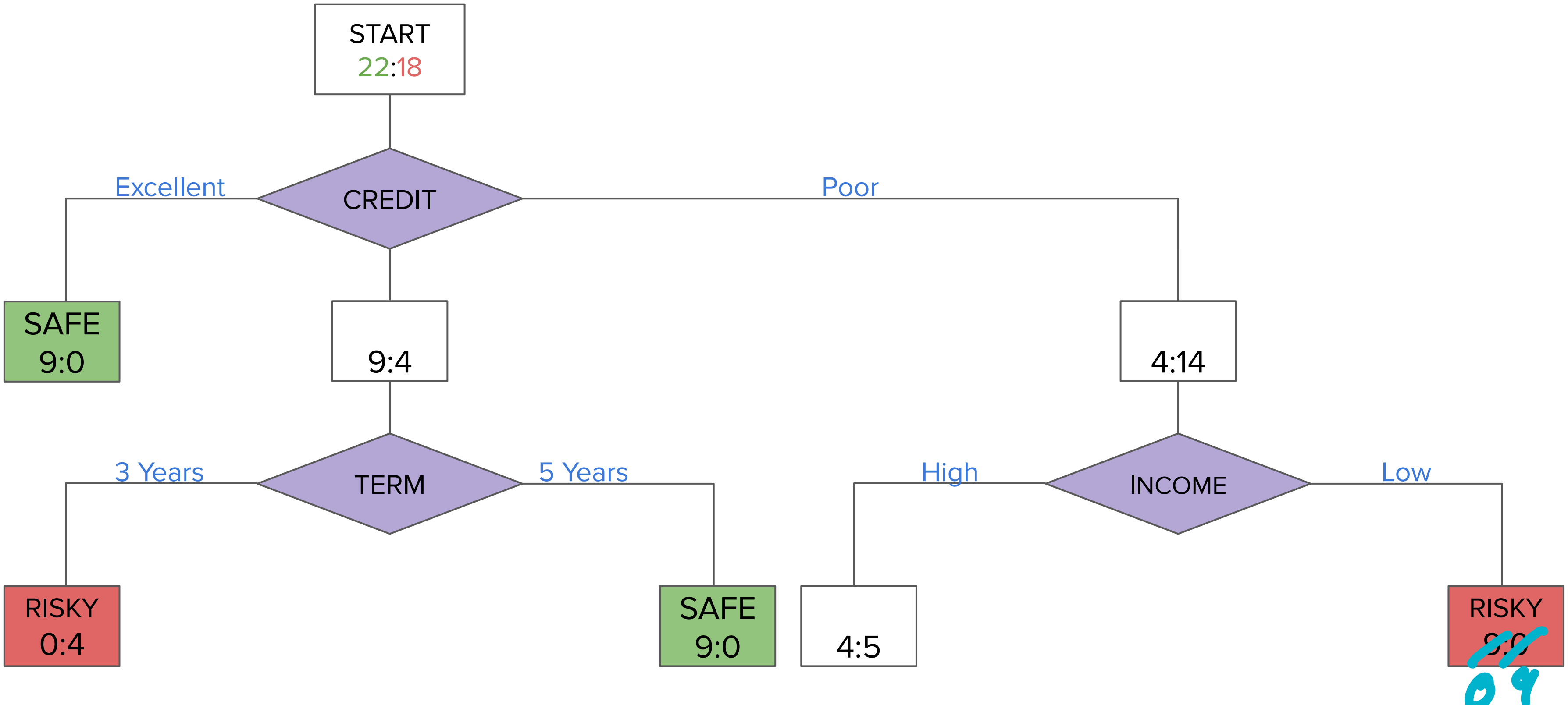


Why is it ok to stop here?

Repeat the same decision tree stump-building process with this subset of data.

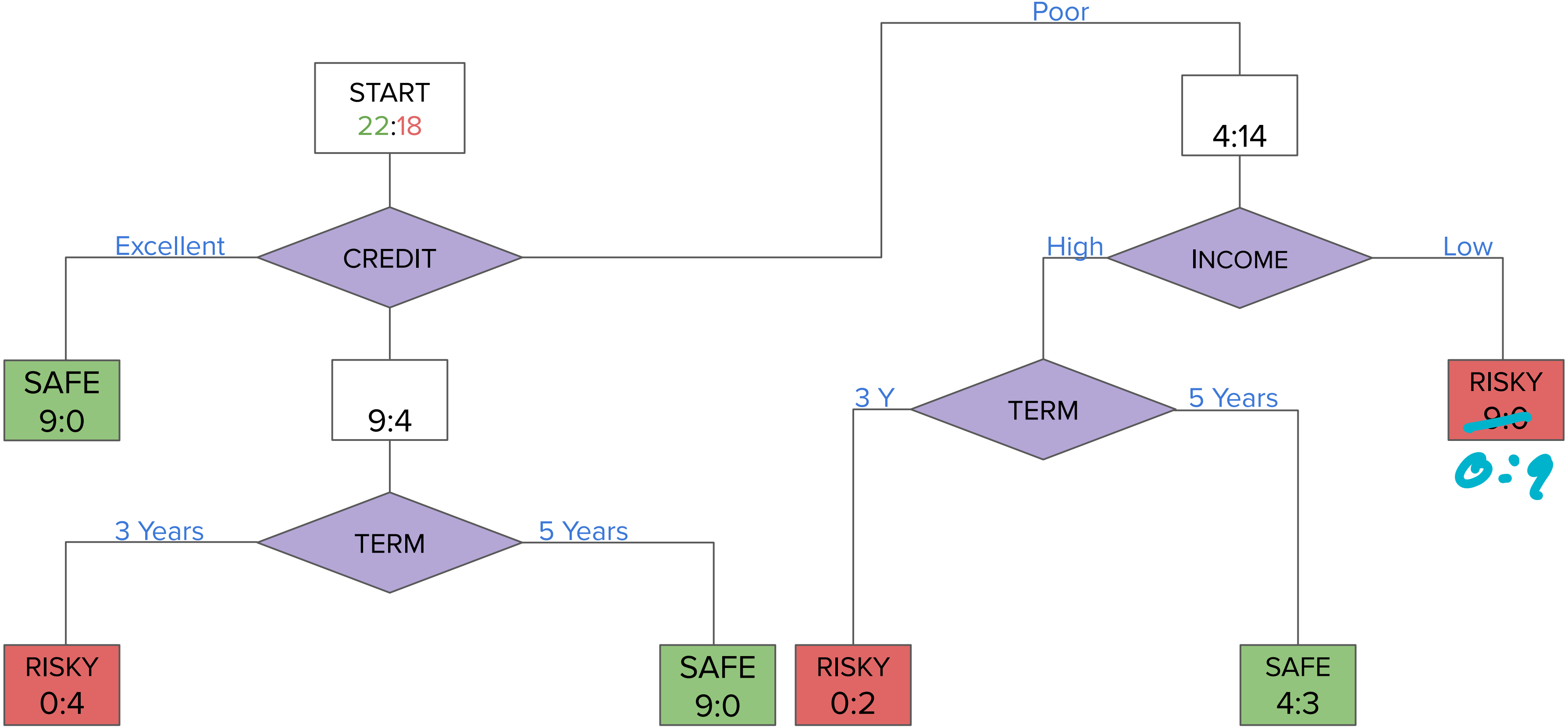
Greedy Algorithm

Classification Accuracy:



Greedy Algorithm

Classification Accuracy: 92.5%



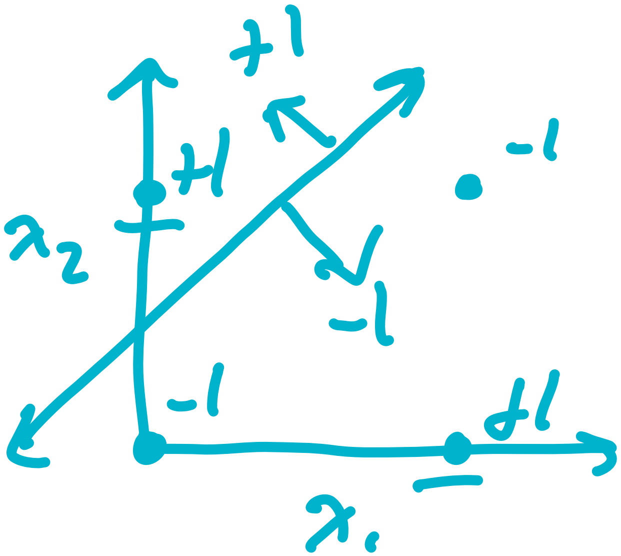
Early Stopping Rules

"exactly one of..."

- Stopping Rules:
    - 1) Do not branch if all data has the same label (pure)
    - 2) We have already split on that feature before
    - **3*) If adding a branch does not increase accuracy, should we still branch?**

XOR

x[1]	x[2]	y
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	-1



XOR: Root

KEY
+1:-1

+1
2:2

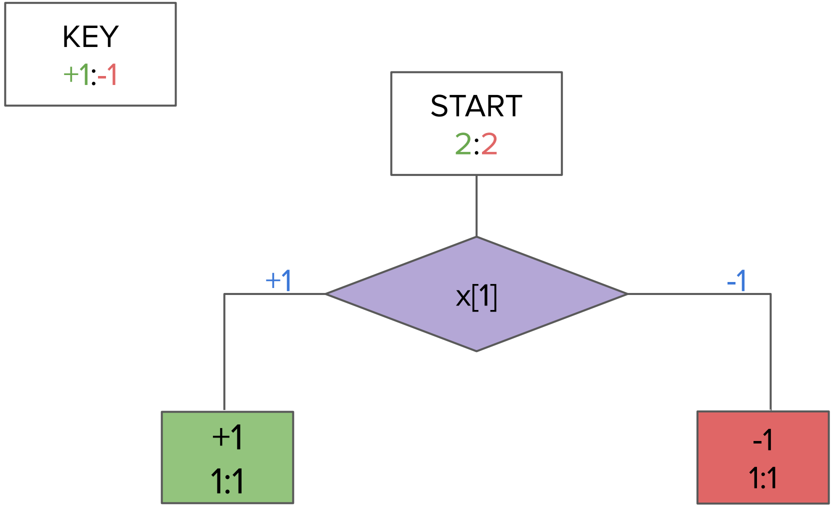
x[1]	x[2]	y
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	-1

# Levels	Accuracy
0	50%
1	?
2	?

XOR: 1 Split

x[1]	x[2]	y
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	-1

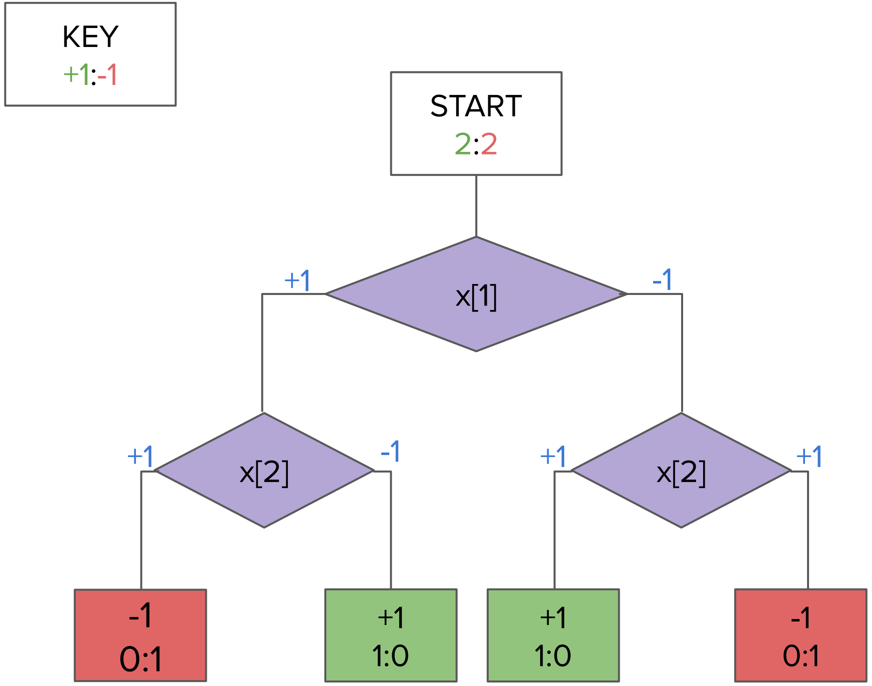
# Levels	Accuracy
0	50%
1	50%
2	?



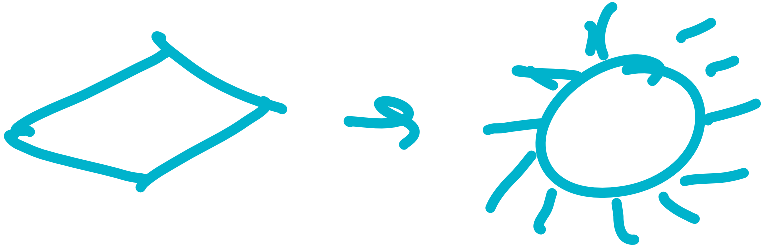
XOR: 2 Splits

x[1]	x[2]	y
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	-1

# Levels	Accuracy
0	50%
1	50%
2	100%



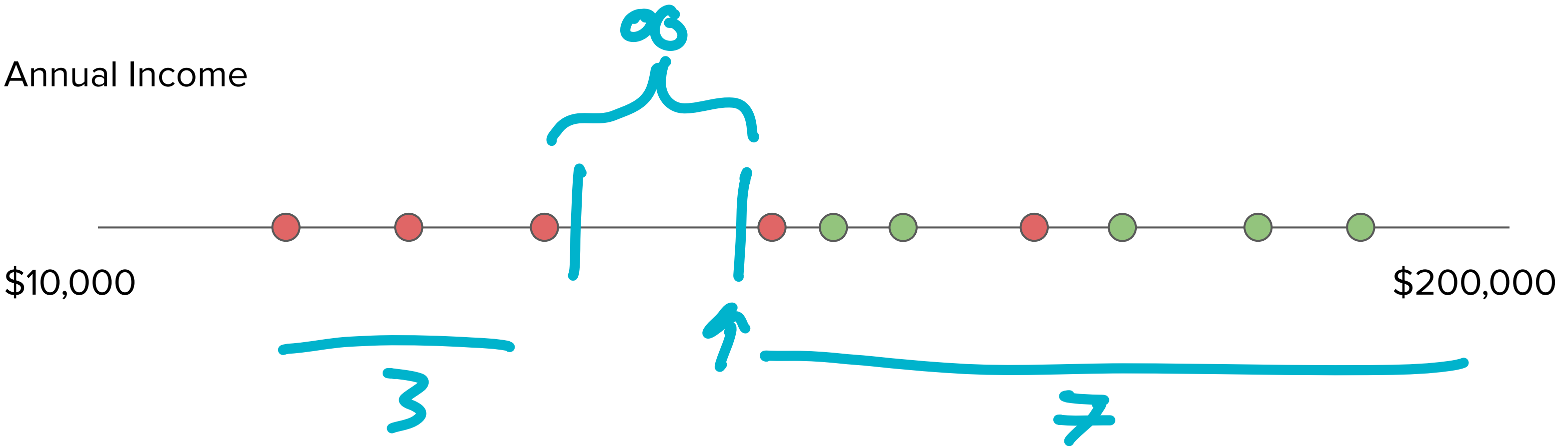
Real Valued Data



- We've been making an assumption so far that our data takes on discrete values.
- How do we know here to split our data? There are an infinite number of possible splits we can make.

Credit	Term	Income	Y
excellent	3 years	$105,000.00	safe
fair	5 years	$63,000.00	risky
fair	3 years	$85.000.00	risky
poor	3 years	$99,000.00	risky

Real Valued Data

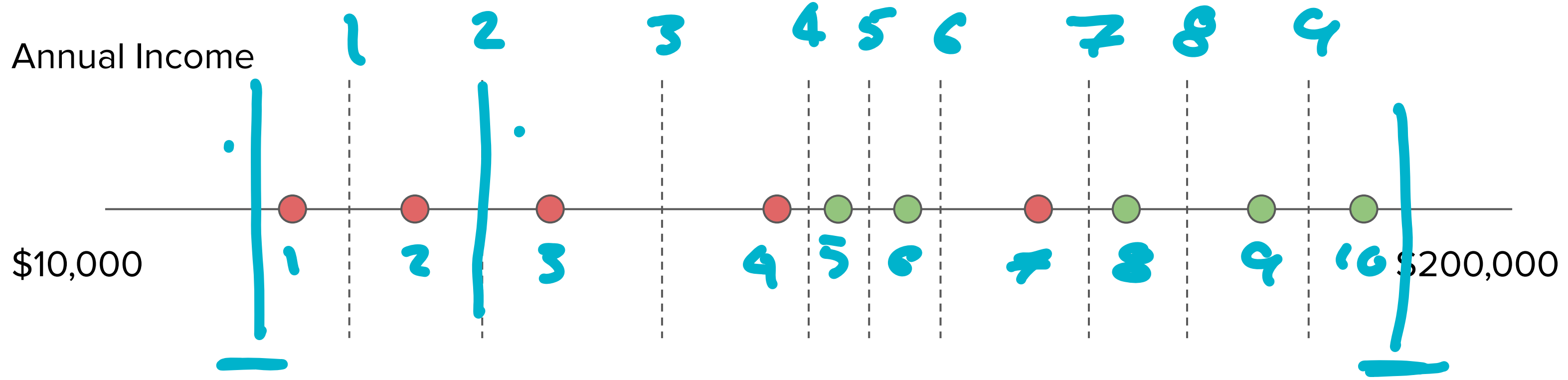


How do we know where to split our data?

Real Valued Data

N data points

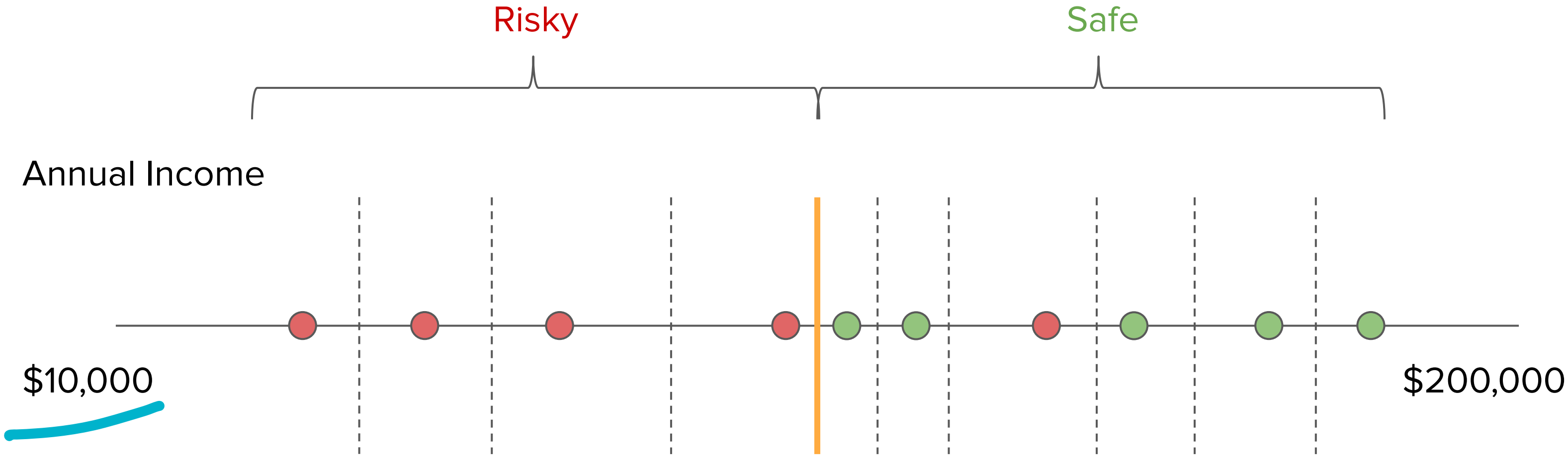
$< N-1$



Key Insight: Sort the data and split halfway between each pair of adjacent points. There will always be a finite number of splits. How many splits are there?

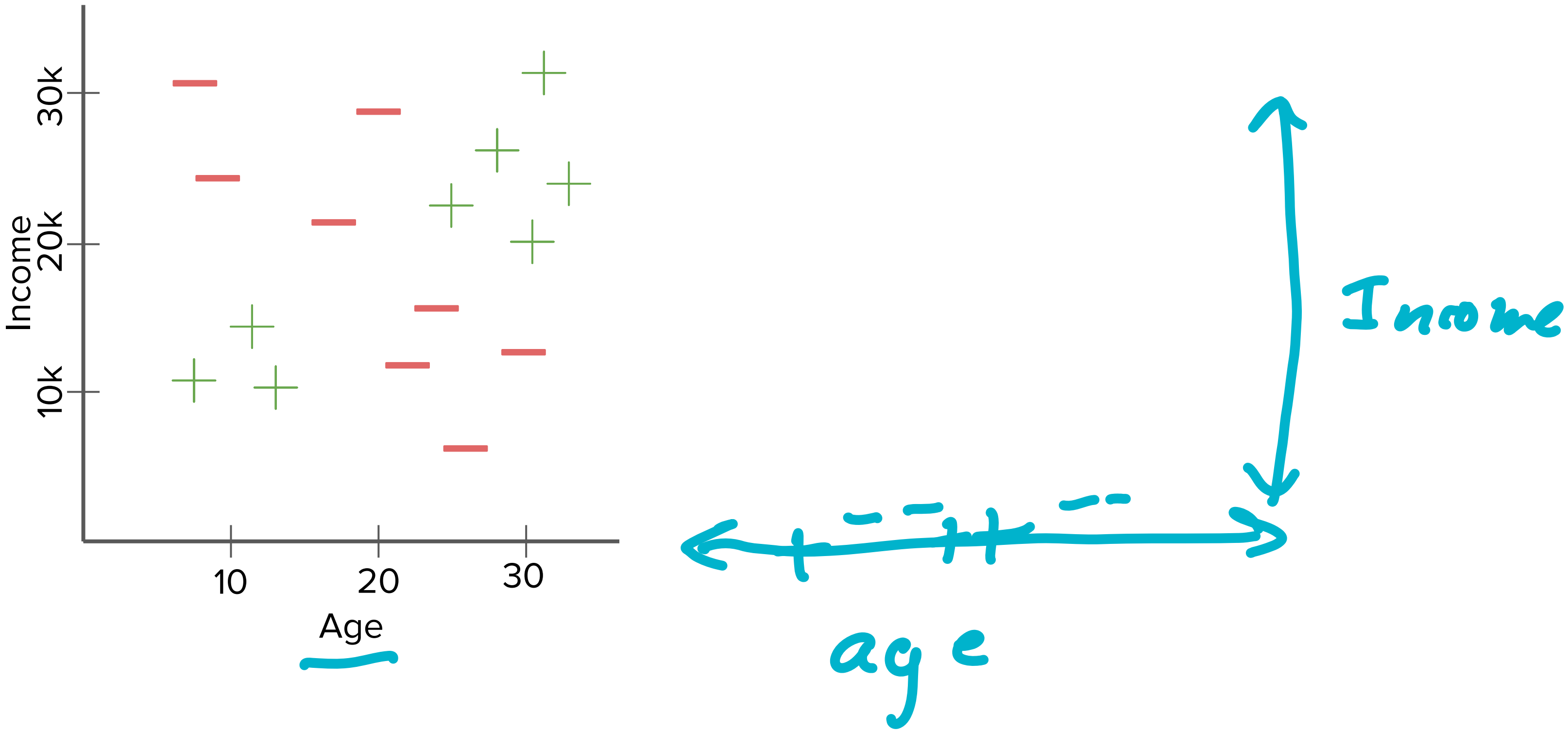
$O(n \log n)$

Real Valued Data

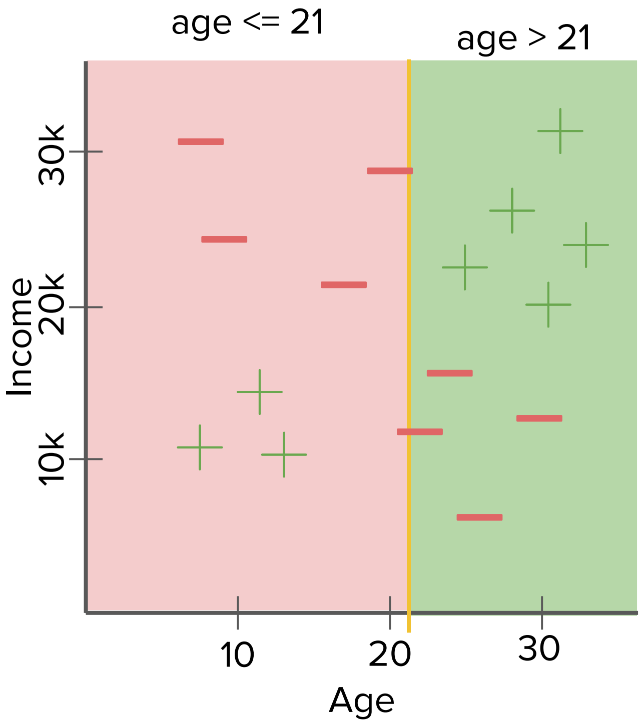


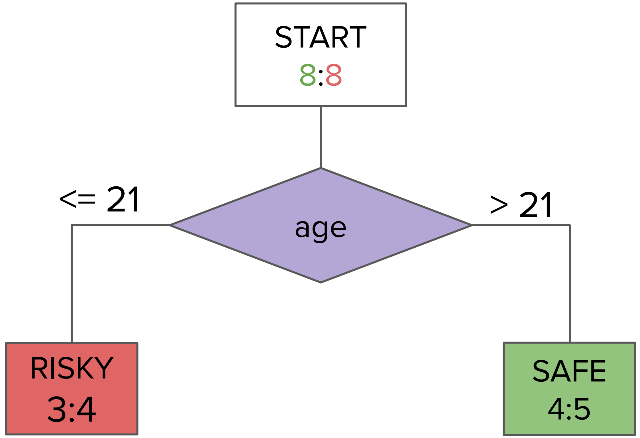
Which split is best? Pick the one that maximizes accuracy.

Real Valued Data

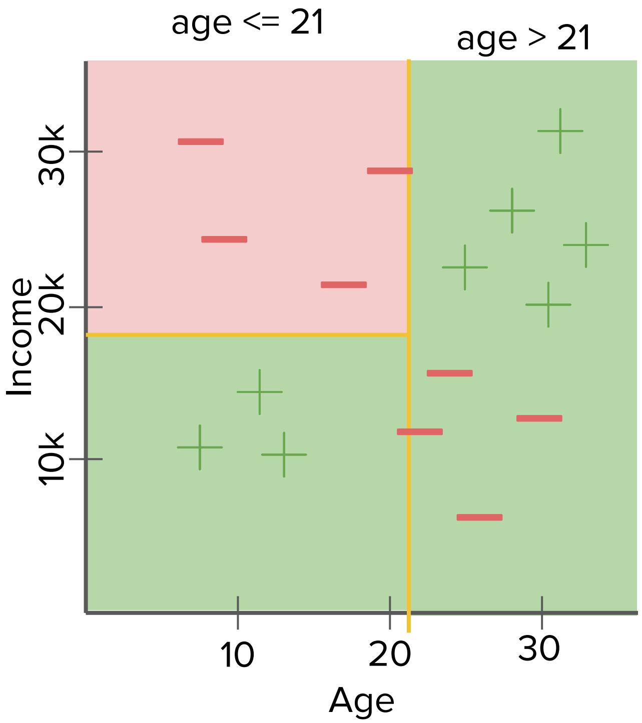


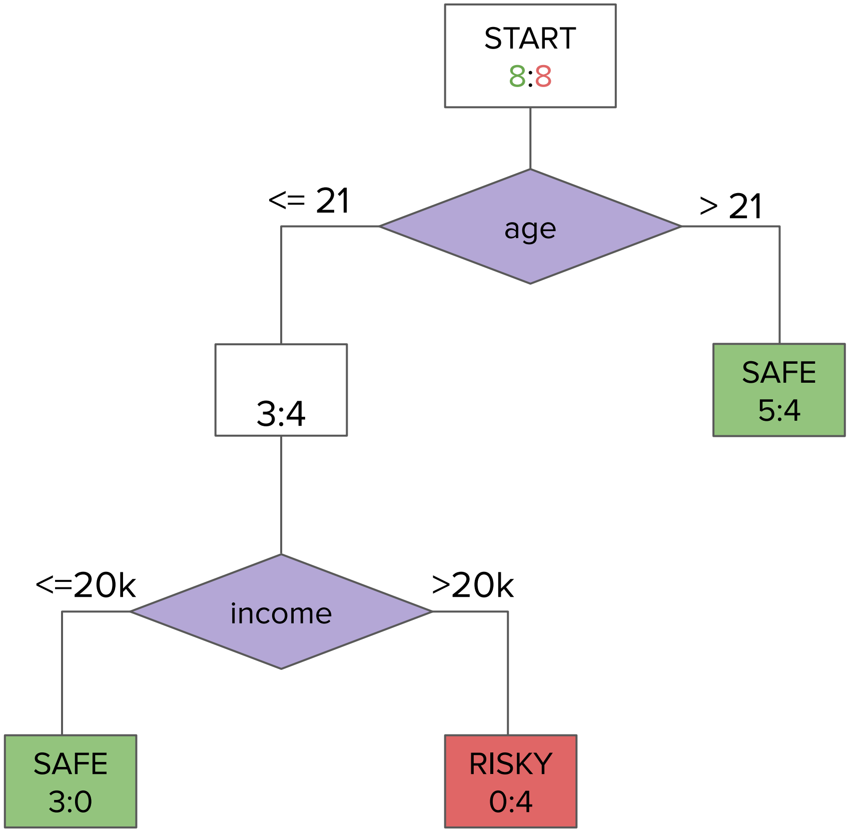
Real Valued Data



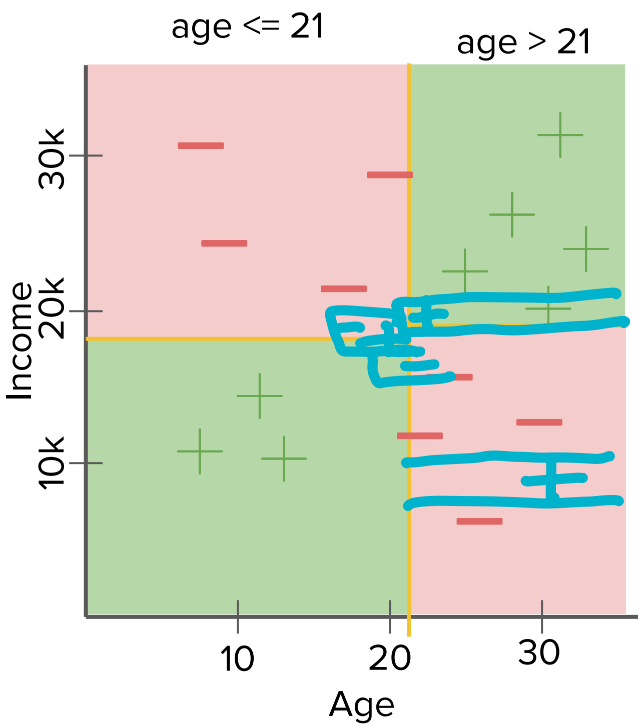


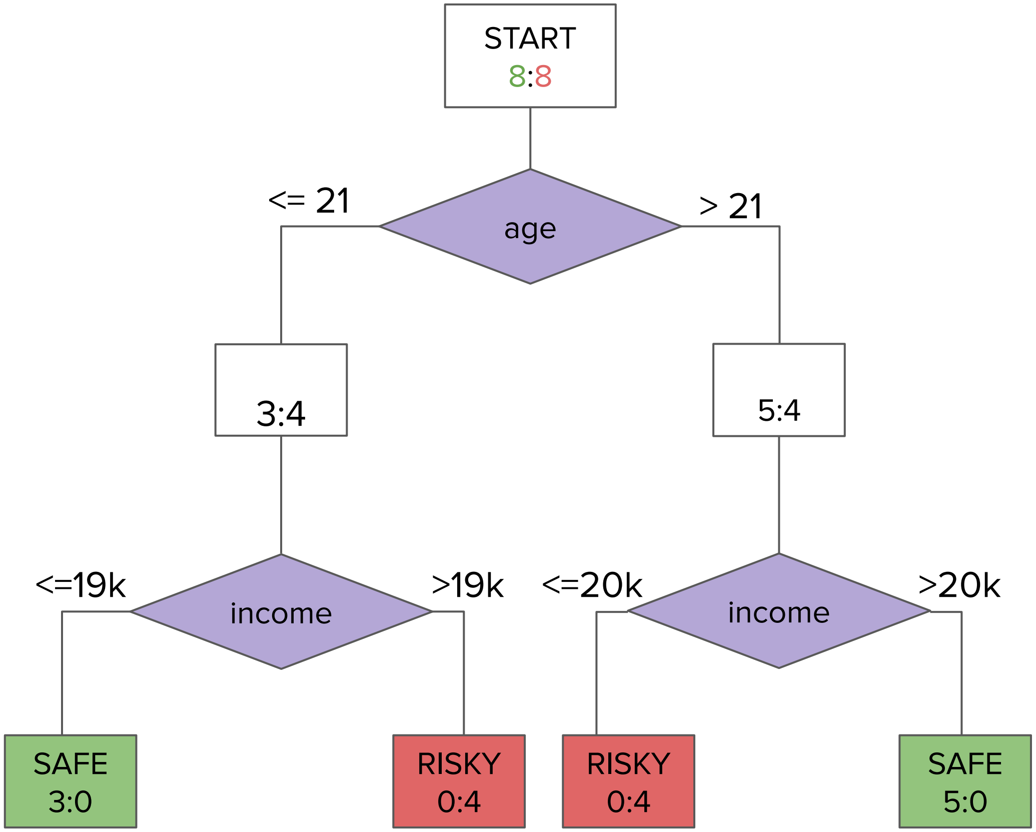
Real Valued Data



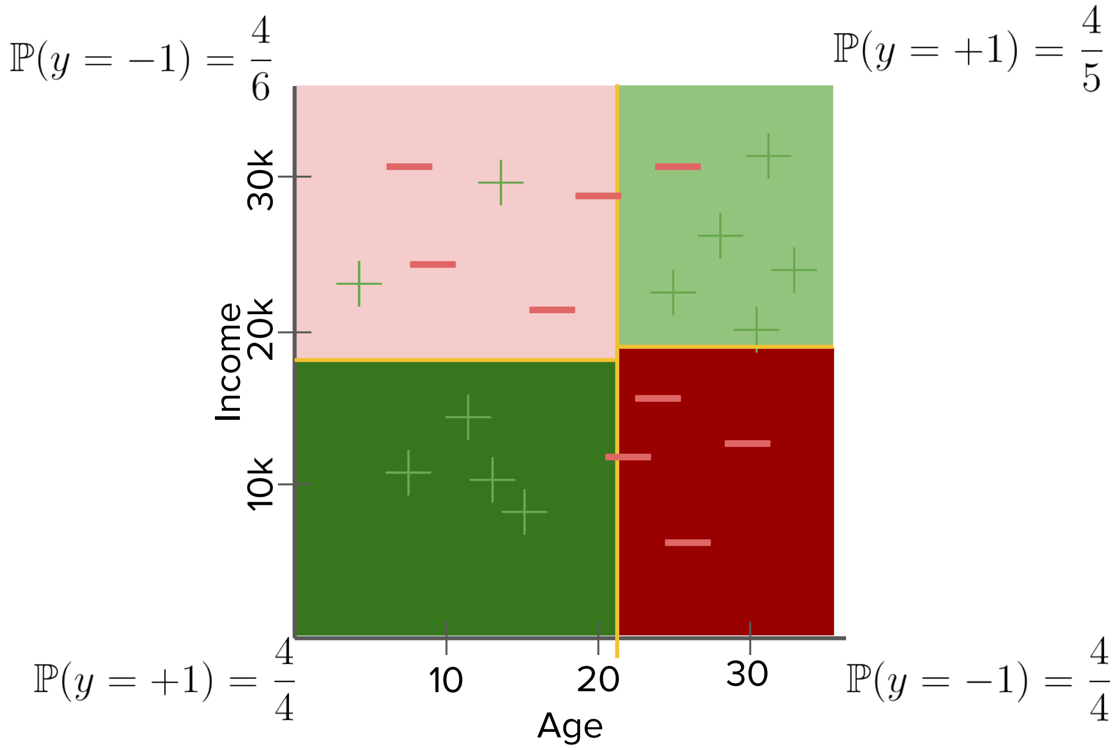


Real Valued Data



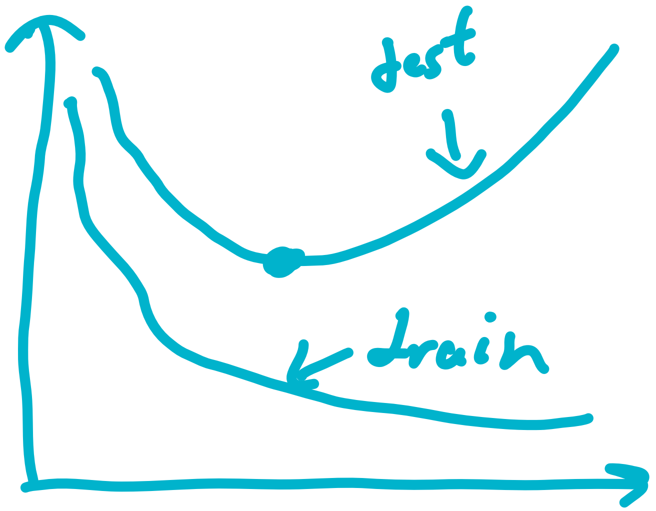


Probabilistic Prediction

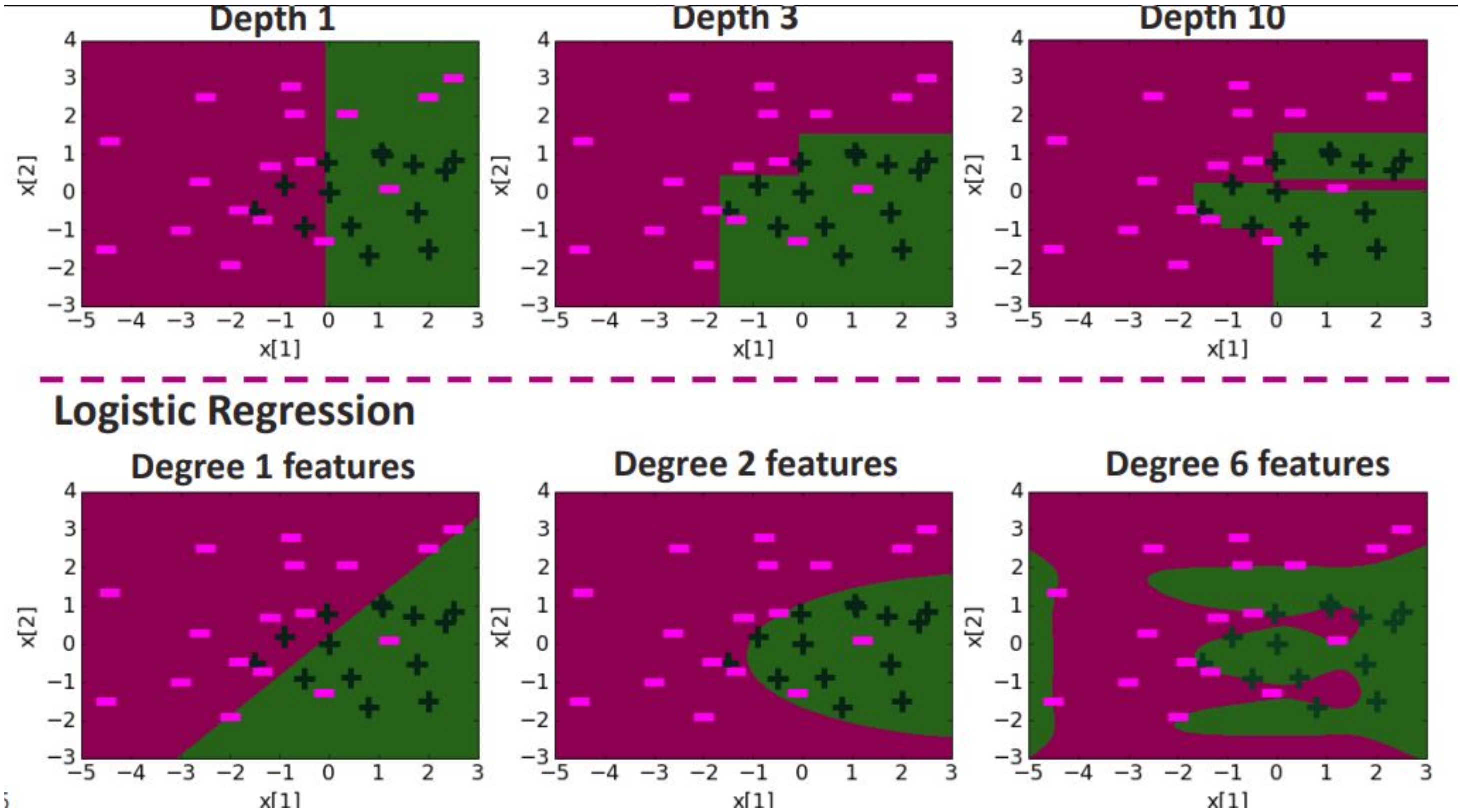


Overfitting

- Similar to regression, training error monotonically non-increases with model complexity.
- Model complexity with decision trees is commonly measured in the depth of the tree.
- Two methods for preventing overfitting:
    - 1) Early stopping
        - Stop the tree before it can get too complex
    - 2) Pruning
        - Create a complex tre and make it more simple



Overfitting



Overfitting: Early Stopping

- Stopping Rules:
  - 1) All data in the subset have the same label
  - 2) No more features left to split
- Early Stopping Rule
  - Only grow up to a max depth hyperparameter (choose via validation)
    - Can be difficult to know the depth.
    - Oftentimes the correct tree is one that is imbalanced
  - Don’t split if there is not a sufficient decrease in error
    - Problem: difficult to classify XOR problems

Exercise: Overfitting and cross validation

/cse416

80% Train Data | valid. | 20% Test

Fold-1	Fold-2	Fold-3	Test

```
cross-validation(data d, folds k):
  fold_1, fold_k = split_data(d, k)

    for each model m:
      for i from 1 to k:
        model = train_model(m, fold -i)
        err = error(model, fold_i)
      avg_err = average err over folds
      keep track of m with smallest avg_err

  return m with smallest avg_err
```

Max Height	Fold-1 Error	Fold-2 Error	Fold-3 Error	Test Error
5	10.3	14.2	12.5	14.5
10	5.6	4.3	7.3	8.7
15	3.1	10.4	8.8	6.9

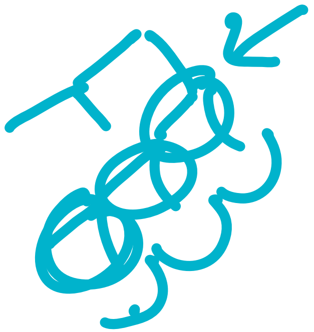
≈12 err validation

≈5.7 validation

≈7.9 validation

Overfitting: Pruning

- Basic Idea: Train a tall, overfit model and then simplify it.
- Pruning is defined by a quality metric that balances classification error and model complexity.



$$Loss(T) = Error(T) + \lambda r(T)$$

leaves in model

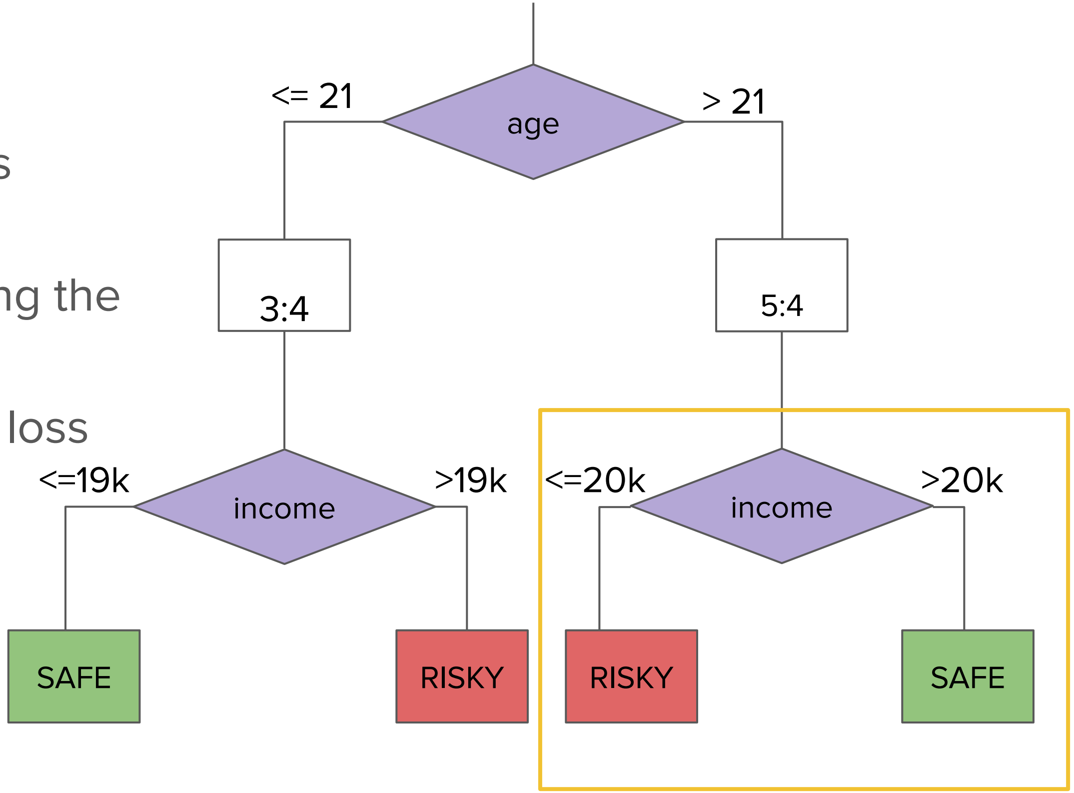
Total(T) = Error(T) + λ # leaves(T)

Pruning Algorithm

1. Consider some arbitrary split
2. Compute the error if the split is taken away
3. Compute the penalty of keeping the split
4. Pick whichever one minimizes loss
5. Repeat 1-4 for all splits

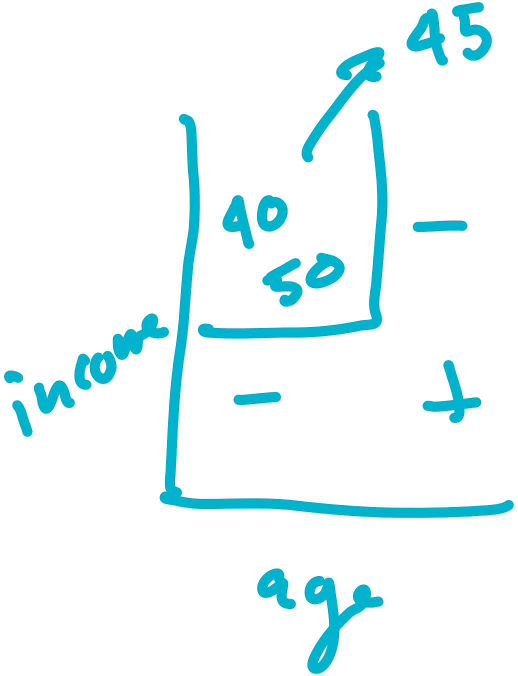
Tree	Error	# Leaves	Total
T	0.25	4	0.43
T’	0.26	3	0.41

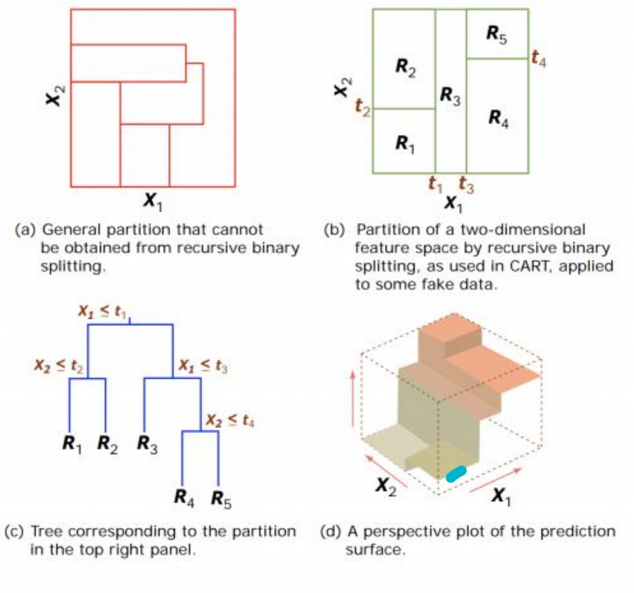
λ = 0.03



Decision Trees for Regression

- Error measured by mean squared error
- Prediction is the mean value of all partitions in the sample





(a) General partition that cannot be obtained from recursive binary splitting.

(b) Partition of a two-dimensional feature space by recursive binary splitting, as used in CART, applied to some fake data.

(c) Tree corresponding to the partition in the top right panel.

(d) A perspective plot of the prediction surface.