HW 3 Sample Solutions

February 25, 2003

1. (6.1) (Note that you could justify your answers in different ways.)

   (a) $P \rightarrow \neg P$ is a satisfiable non-tautology. Proof:
   - $P \rightarrow \neg P$
   - $\neg P \vee \neg P$ (def. of $\rightarrow$)
   - $\neg P$ ($\vee$-elim)

   (b) $(P \vee Q) \wedge (\neg P \vee \neg Q)$ is a satisfiable non-tautology. Proof:
   - if P, Q false: $(P \vee Q)$ is false, formula is false
   - if P true, Q false: formula is true

   (c) $(P \rightarrow Q) \wedge (Q \rightarrow \neg P)$ is satisfiable non-tautology. Proof:
   - $(P \rightarrow Q) \wedge (Q \rightarrow \neg P)$
   - $(\neg P \vee Q) \wedge (\neg Q \vee \neg P)$ (def. of $\rightarrow$)
   - $\neg P \vee \neg P$ (resolution)
   - $\neg P$ ($\vee$-elim.)

   (d) $P \rightarrow (\neg P \rightarrow P)$ is a tautology. Proof:
   - $P \rightarrow (\neg P \rightarrow P)$
   - $\neg P \vee (P \vee P)$ (def. of $\rightarrow$, twice)
   - $\neg P \vee P$ ($\vee$-elim, this is a tautology)

2. (6.9)

P	Q	A=(P ∨ Q)	R	B=(¬ P ∨ R)	A ∧ B	C=(Q ∨ R)	A ∧ B → C
T	T	T	T	T	T	T	T
T	T	T	F	F	F	T	T
T	F	T	T	T	T	T	T
T	F	T	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	T	F	T	T	T	T
F	F	F	T	T	F	T	T
F	F	F	F	T	F	F	T

   Thus $(P \vee Q)(\neg P \vee R) \rightarrow (Q \vee R)$ is a tautology.

3. (6.10) We will use the following variables:
   - $stoleJewelry_x$: x stole jewelry
   - $guilty_x$: x is guilty

- $milkedCows_x$: x milked the cows
- $gotCream_x$: x got the cream.

Given:

- $stoleJewelry_{Maid} \rightarrow \neg\ guilty_{Butler}$
- $\neg\ stoleJewelry_{Maid} \rightarrow milkedCows_{Maid}$
- $milkedCows_{Maid} \rightarrow gotCream_{Butler}$
- $gotCream_{Maid} \rightarrow gotCream_{Butler}$

Negation of conclusion: $guilty_{Butler} \wedge \neg\ gotCream_{Butler}$

Resolution proof:

(a) $\neg stoleJewelry_{Maid} \vee \neg\ guilty_{Butler}$ (given)
(b) $stoleJewelry_{Maid} \vee milkedCows_{Maid}$ (given)
(c) $\neg milkedCows_{Maid} \wedge gotCream_{Butler}$ (given)
(d) $guilty_{Butler}$ (neg. concl.)
(e) $\neg gotCream_{Butler}$ (neg. concl.)
(f) $\neg stoleJewelry_{Maid}$ (resolve (a) and (d))
(g) $\neg milkedCows_{Maid}$ (resolve (c) and (e))
(h) $milkedCows_{Maid}$ (resolve (b) and (f))
(i) (null clause) (resolve (g) and (h))

4. (6.12)

(a) $\forall x \forall y((P(x) \wedge Q(y)) \rightarrow \exists z R(x, y, z))$
- $(P(x) \wedge Q(y)) \rightarrow R(x, y, f(x, y))$
- $\neg(P(x) \wedge Q(y)) \vee R(x, y, f(x, y))$
- $\neg P(x) \vee \neg Q(y) \vee R(x, y, f(x, y))$

(b) $\exists x \forall y \exists z P(x) \rightarrow (Q(y) \rightarrow R(z))$
- $P(a) \rightarrow (Q(y) \rightarrow R(f(y)))$
- $\neg P(a) \vee \neg Q(y) \vee R(f(y))$

5. (6.13)

(a) $\{b|x, a|y\}$
(b) not unifiable
(c) $\{f(g(z))|x, g(z)|y\}$
(d) not unifiable
(e) not unifiable

6. (6.14)

(a) $\forall x(S(x) \rightarrow L(x))$ (given)
(b) $\neg S(x) \vee L(x)$ ($\forall$ elim, def. of $\rightarrow$)

(c) $\neg\exists x S(x)$ (given)

(d) $S(x)$ (neg.of $\exists$, $\forall$ elim for (c))

(e) $\exists x \neg L(x))$ (negated conclusion)

(f) $\neg L(a)$ (skolemized)

(g) $L(x)$ (resolution of (b) and (d)

(h) $L(a)$ (substitute $\{a|x$, unify (f) and (g))

(i) (null clause) (resolution of (f) and (h))

7. (6.15)

   Predicates:

   - hasHair(x) : x has hair
   - givesMilk(x) : x gives milk
   - isMammal(x) : x is mammal
   - isCoconut(x) : x is coconut

   Given:

   - $\forall$ x hasHair(x) $\wedge$ givesMilk(x) $\rightarrow$ isMammal(x)
     $\neg$ hasHair(x) $\vee$ givesMilk(x) $\vee$ isMammal(x)
   - isCoconut(x) $\rightarrow$ hasHair(x)
     $\neg$ isCoconut(x) $\vee$ hasHair(x)
   - isCoconut(x) $\rightarrow$ givesMilk(x)
     $\neg$ isCoconut(x) $\vee$ givesMilk(x)
   - isCoconut(x) $\rightarrow$ isMammal(x)
     $\neg$ isCoconut(x) $\vee$ isMammal(x)

   Negated conclusion: isCoconut(x) $\wedge\neg$ isMammal(x)

   Proof:

   (a) $\neg$ hasHair(x) $\vee$ givesMilk(x) $\vee$ isMammal(x)

   (b) $\neg$ isCoconut(x) $\vee$ hasHair(x)

   (c) $\neg$ isCoconut(x) $\vee$ givesMilk(x)

   (d) isCoconut(x) (neg. concl.)

   (e) $\neg$ isMammal(x) (neg. concl.)

   (f) $\neg$ hasHair(x) $\vee\neg$ givesMilk(x) (res. (a) and (e))

   (g) $\neg$ isCoconut(x) $\vee\neg$ givesMilk(x) (res. (b) and (f))

   (h) $\neg$ isCoconut(x) (res. (c) and (g), $\vee$ elim.)

   (i) (null clause) (res. (d) and (h))