Consider a graph of colored vertices and undirected edges where the vertices can be red, green, blue. In particular, you have the relations

```
Vertex(x, color)

Edge(x, y)
```

The Edge relation is symmetric in the if (x, y) is in Edge, then (y, x) is in Edge. Your goal is to write a datalog program to answer each of the following questions.

1. Find all green vertices.

```
GreenV(x) :- Vertex(x, 'green')
```

2. Find all pairs of blue vertices connected by one edge.

```
BluePairs(x) :- Vertex(x, 'blue'), Vertex(y, 'blue'), Edge(x, y)
```

3. Find all triangles where all the vertices are the same color. Output the three vertices and their shared color.

```
\begin{aligned} \text{Triangle}\left(x,\ y,\ z\,,\ a\right) := \ & \text{Vertex}\left(x,\ a\right),\ \text{Vertex}\left(y,\ a\right),\ \text{Vertex}\left(z\,,\ a\right),\\ & \text{Edge}\left(x,\ y\right),\ \text{Edge}\left(y,\ z\right),\ \text{Edge}\left(z\,,\ x\right) \end{aligned}
```

4. Find all vertices that don't have any neighbors.

```
WRONG ANSWER (UNSAFE)
```

```
LonleyV(x) := not Edge(x, )
```

WRONG ANSWER (UNSAFE)

```
LonleyV(x) := Vertex(x, ), not Edge(x, )
```

RIGHT ANSWER (SAFE)

```
\operatorname{OnlyX}(x) := \operatorname{Edge}(x, _{-})

\operatorname{LonleyV}(x) := \operatorname{Vertex}(x, _{-}), \text{ not } \operatorname{OnlyX}(x)
```

5. Find all vertices such that they only have red neighbors.

```
BlueV(x) := Vertex(x, ), Edge(x, y), Vertex(y, 'blue')

GreenV(x) := Vertex(x, ), Edge(x, y), Vertex(y, 'green')

RedV(x) := Vertex(x, ), not BlueV(x), not GreenV(x)
```

6. Find all vertices such that they only have neighbors with the same color. Return the vertex and color.

```
\begin{split} & SameColor(x,\ y,\ a):-\ Vertex(x,\ a),\ Vertex(y,\ a) \\ & NotSameNeigh(x):-\ Vertex(x,\_),\ Edge(x,\ y),\ Edge(x,\ z),\ not\ SameColor(y,\ z) \\ & OnlySameNeigh(x,\ a):-\ Vertex(x,\ a),\ not\ NotSameNeigh(x) \end{split}
```

OR

```
\begin{array}{lll} Neigh(x,\ y,\ a) := Edge(x,\ y)\,,\ Vertex(y,\ a) \\ DifferentNeigh(x) := Neigh(x,\ y,\ a)\,,\ Neigh(x,\ z,\ b)\,,\ a \mathrel{!=} b \\ OnlySameNeigh(x,\ a) := Vertex(x,\ a)\,,\ not\ DifferentNeigh(x) \end{array}
```

7. (Optional) For some vertex v, find all vertexes connected to v by blue vertexes (this one require recursion).

```
\begin{array}{lll} ConnectedTo(x) :- & Vertex(x, `blue'), & Edge(x, v) \\ ConnectedTo(x) :- & Vertex(x, `blue'), & Edge(x, y), & ConnectedTo(y) \end{array}
```