CSE 413
Programming Languages & Implementation

Hal Perkins

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Context-Free Grammars and Parsing

The Plan

- Parsing overview
- Context free grammars
- Grammar problems - ambiguity

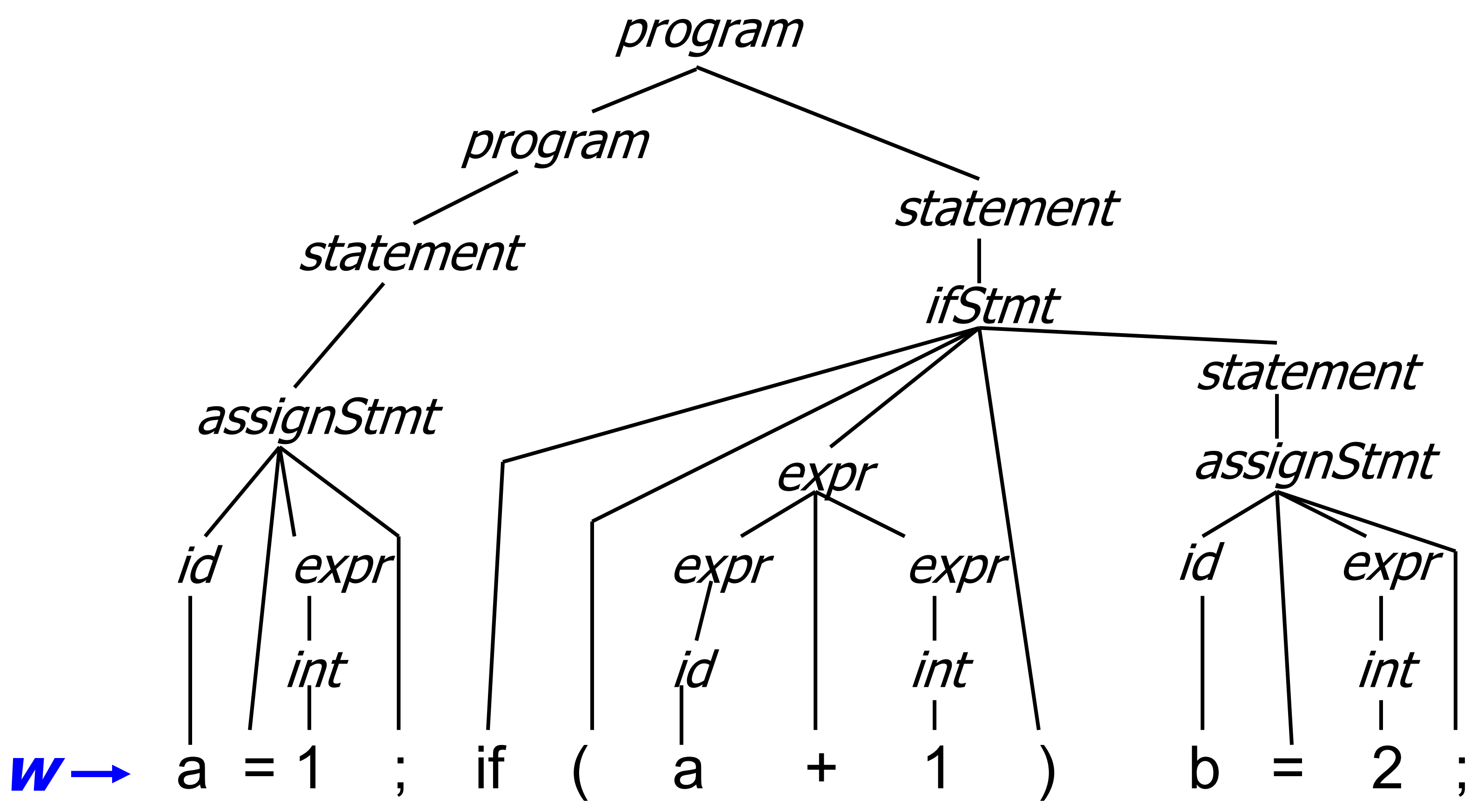
Parsing

- The syntax of most programming languages can be specified by a *context-free grammar* (CGF)
  - A grammar allowing recursive rules (*A* ::= … *A* …)
- **Parsing**: Given a grammar *G* and a sentence *w* in *L*(*G*), traverse the derivation (parse tree) for *w* in some *standard order* and do *something useful* at each node
  - The tree might not be produced explicitly, but the control flow of a parser corresponds to a traversal

Old Example

G

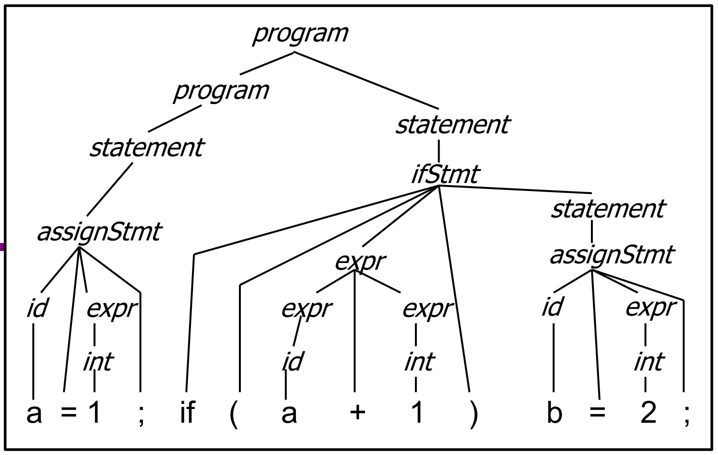
program ::= *statement* | *program statement*
statement ::= *assignStmt* | *ifStmt*
assignStmt ::= *id* = *expr* ;
ifStmt ::= if (*expr*) *statement*
expr ::= *id* | *int* | *expr* + *expr*
id ::= a | b | c | i | j | k | n | x | y | z
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9



“Standard Order”

- For practical reasons we want the parser to be *deterministic* (no backtracking), and we want to examine the source program from *left to right*.
  - (i.e., parse the program in linear time in the order it appears in the source file)

Common Orderings



- Top-down
  - Start with the root
  - Traverse the parse tree depth-first, left-to-right (leftmost derivation)
  - LL(k), recursive-descent
- Bottom-up
  - Start at leaves and build up to the root
    - Effectively a rightmost derivation in reverse(!)
  - LR(k) and subsets (LALR(k), SLR(k), etc.)

“Something Useful”

- At each point (node) in the traversal, perform some *semantic action*
  - Construct nodes of full parse tree (rare)
  - Construct abstract syntax tree (common)
  - Construct linear, lower-level representation (more common in later parts of a modern compiler)
  - Generate target code or interpret on the fly (1-pass compilers & interpreters; not common in production compilers – but works for our project)

Context-Free Grammars (review)

- Formally, a grammar G is a tuple $<N,\Sigma,P,S>$ where:
  - N a finite set of non-terminal symbols
  - Σ a finite set of terminal symbols
  - P a finite set of productions
    - A subset of $N \times (N \cup \Sigma)^*$
  - S the *start symbol,* a distinguished element of N
    - If not specified otherwise, this is usually assumed to be the non-terminal on the left of the first production

Standard Notations

- a, b, c elements of Σ
- w, x, y, z elements of Σ^*
- A, B, C elements of N
- X, Y, Z elements of $N \cup \Sigma$
- α, β, γ elements of $(N \cup \Sigma)^*$
- $A \rightarrow \alpha$ or A ::= α if <A, α> in P

Derivation Relations (1)

- α A γ => α β γ iff A ::= β in *P*
  - derives
- A =>* w if there is a *chain* of productions starting with A that generates w
  - transitive closure

Derivation Relations (2)

- $w\ A\ \gamma \Rightarrow_{lm} w\ \beta\ \gamma$ iff $A ::= \beta$ in P
  - derives leftmost
- $\alpha\ A\ w \Rightarrow_{rm} \alpha\ \beta\ w$ iff $A ::= \beta$ in P
  - derives rightmost
- Parsers normally deal with only leftmost or rightmost derivations – not random orderings

Languages

- For A in *N*, *L*(A) = { w | A =>* w }
  - i.e., set of strings (words, terminal symbols) generated by nonterminal A
- If *S* is the start symbol of grammar *G*, we define *L*(*G*) = *L*(*S*)

Reduced Grammars

- Grammar G is *reduced* iff for every production A ::= α in G there is some derivation

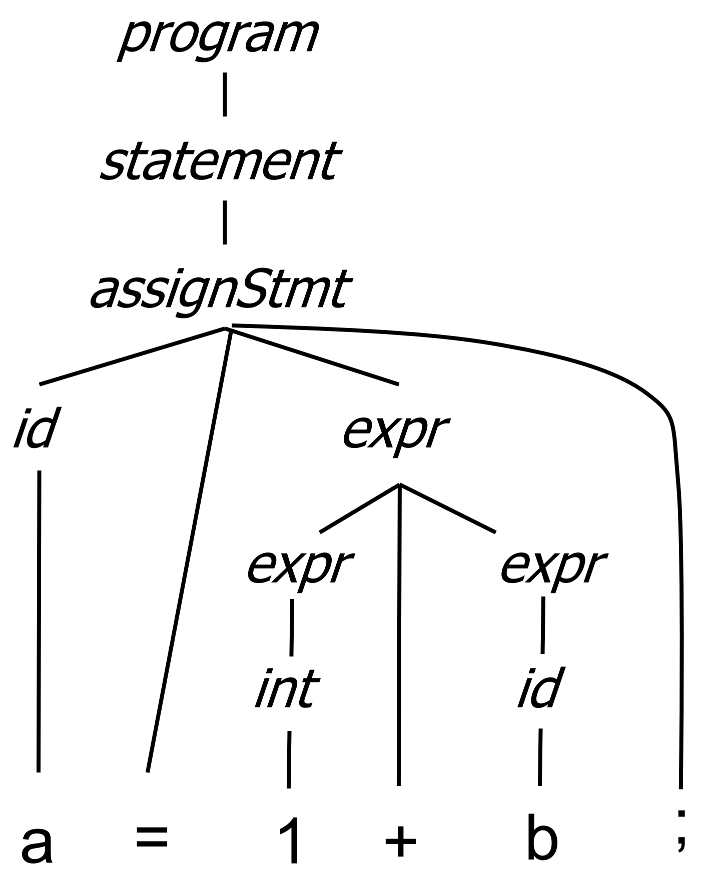
  S =>* x A z => x α z =>* xyz

  – i.e., no production is useless
- Convention: we will use only reduced grammars

Example

program ::= *statement* | *program statement*
statement ::= *assignStmt* | *ifStmt*
assignStmt ::= *id* = *expr* ;
ifStmt ::= if (*expr*) *stmt*
expr ::= *id* | *int* | *expr* + *expr*
id ::= a | b | c | i | j | k | n | x | y | z
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

- Top down, Leftmost derivation of a = 1 + b ;



Example

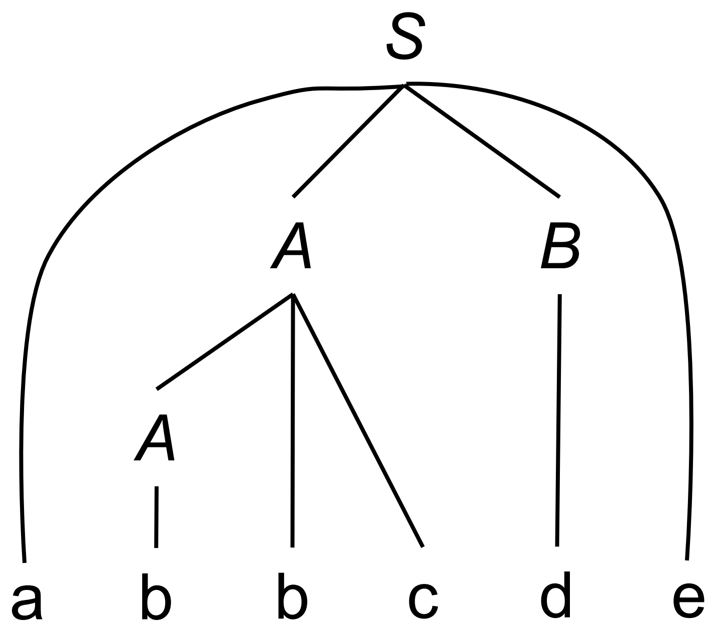
- Grammar

  S ::= a*AB*e

  A ::= *A*bc | b

  B ::= d

- Top down, leftmost derivation of: **abbcde**



Ambiguity

- Grammar G is *unambiguous* iff every w in L(G) has a unique leftmost (or rightmost) derivation
  - Fact: either unique leftmost or unique rightmost implies the other
- A grammar without this property is *ambiguous*
  - Other grammars that generate the same language might be unambiguous
- We need unambiguous grammars for parsing
  - Our compiler or interpreter shouldn’t have to choose the meaning of the input – if the grammar is unambiguous there’s only one choice

Example: Ambiguous Grammar for Arithmetic Expressions

expr ::= *expr* + *expr* | *expr* - *expr*
  | *expr* * *expr* | *expr* / *expr* | *int*

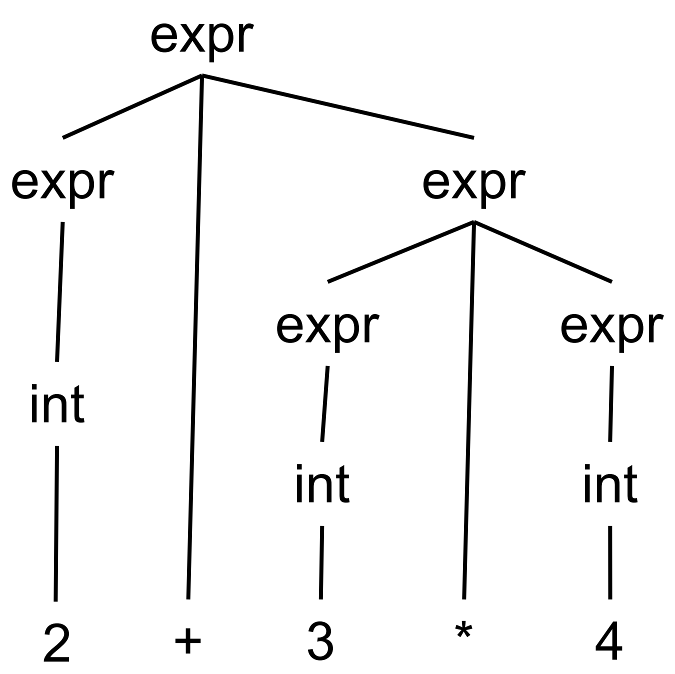
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

- Exercise: show that this is ambiguous
  - How? Show two different leftmost or rightmost derivations for the same string
  - Equivalently: show two different parse trees for the same string

Example (cont)

expr ::= *expr* + *expr* | *expr* - *expr*
| *expr* * *expr* | *expr* / *expr* | *int*
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

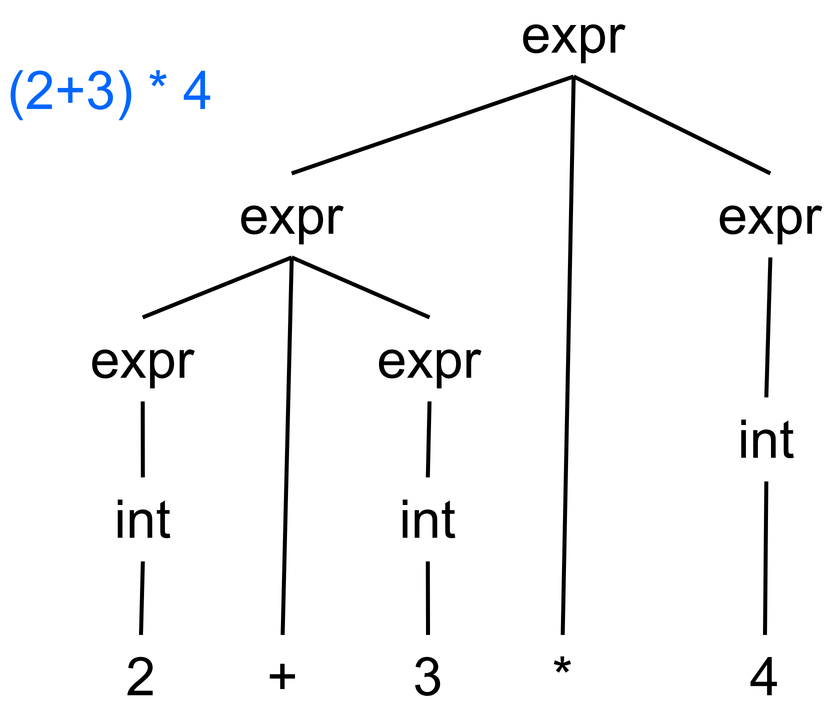
- Give a leftmost derivation of 2+3*4 and show the parse tree

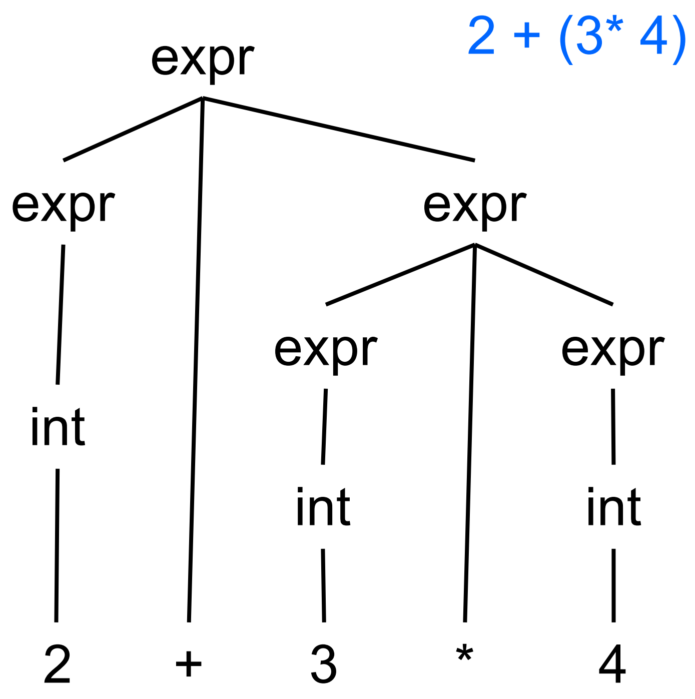


Example (cont)

expr ::= *expr* + *expr* | *expr* - *expr*
| *expr* * *expr* | *expr* / *expr* | *int*
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

- Give a different leftmost derivation of 2+3*4 and show the parse tree

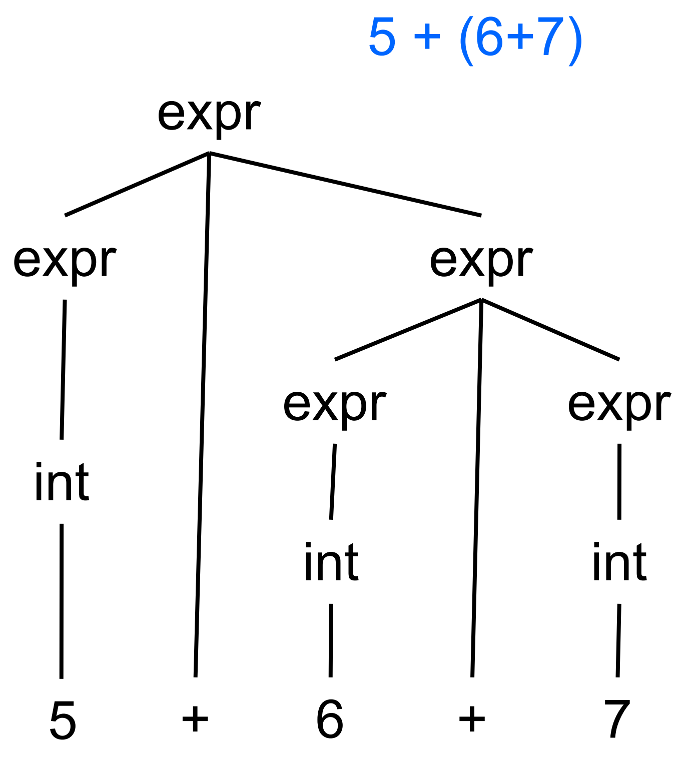


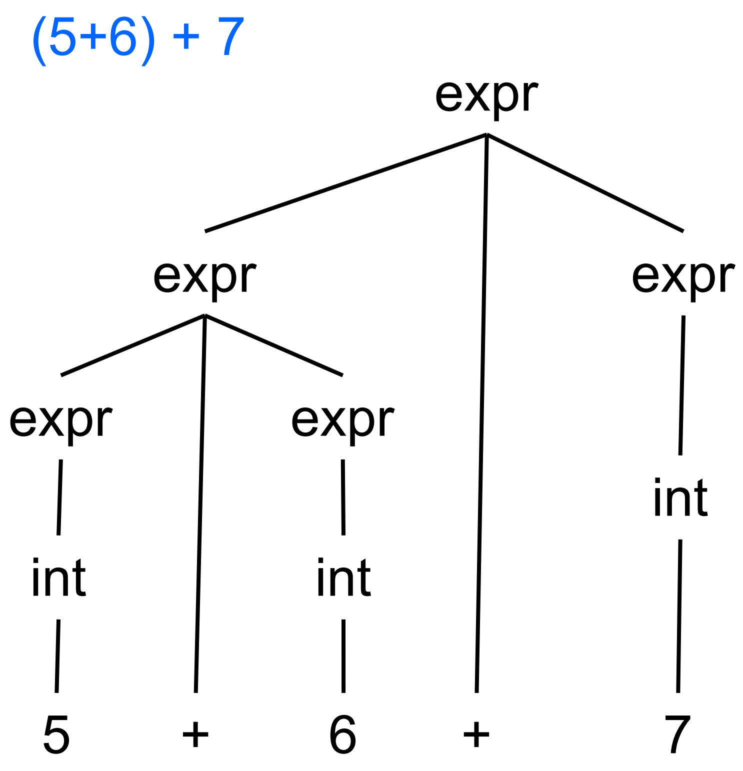


Another example

expr ::= *expr* + *expr* | *expr* - *expr*
| *expr* * *expr* | *expr* / *expr* | *int*
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

- Give two different leftmost derivations of 5+6+7





What’s going on here?

- This grammar has no notion of precedence or associatively
- Standard solution
  - Create a non-terminal for each level of precedence
  - Isolate the corresponding part of the grammar
  - Force the parser to recognize higher precedence subexpressions first

Classic Expression Grammar

expr ::= *expr* + *term* | *expr* – *term* | *term*

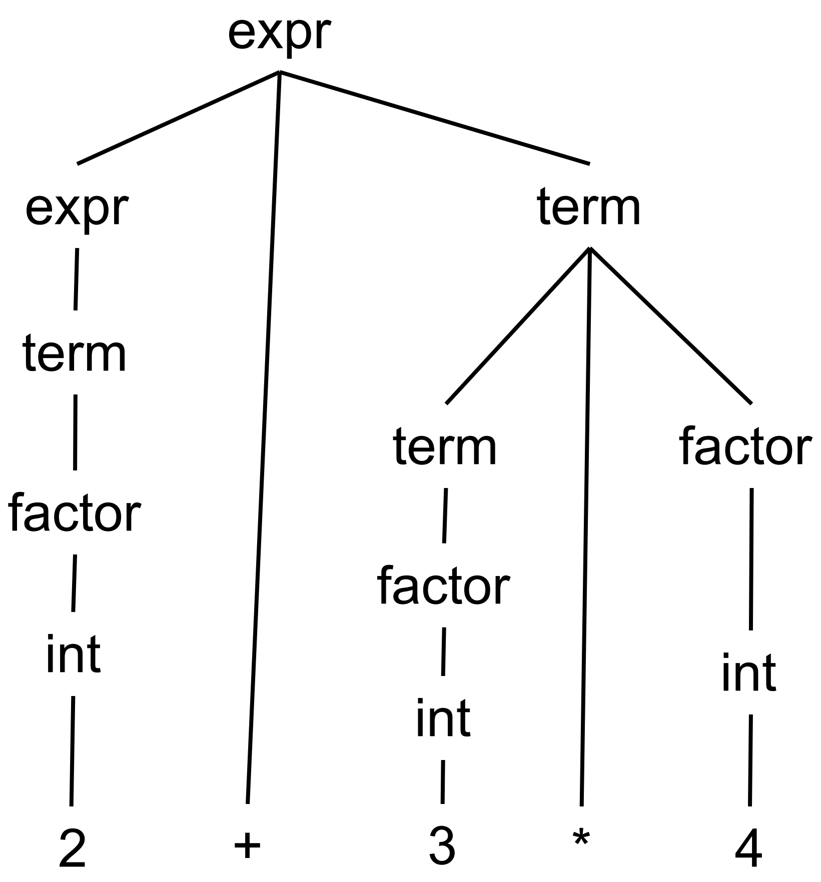
term ::= *term* * *factor* | *term* / *factor* | *factor*

factor ::= *int* | (*expr*)

int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7

Check: Derive 2+3*4

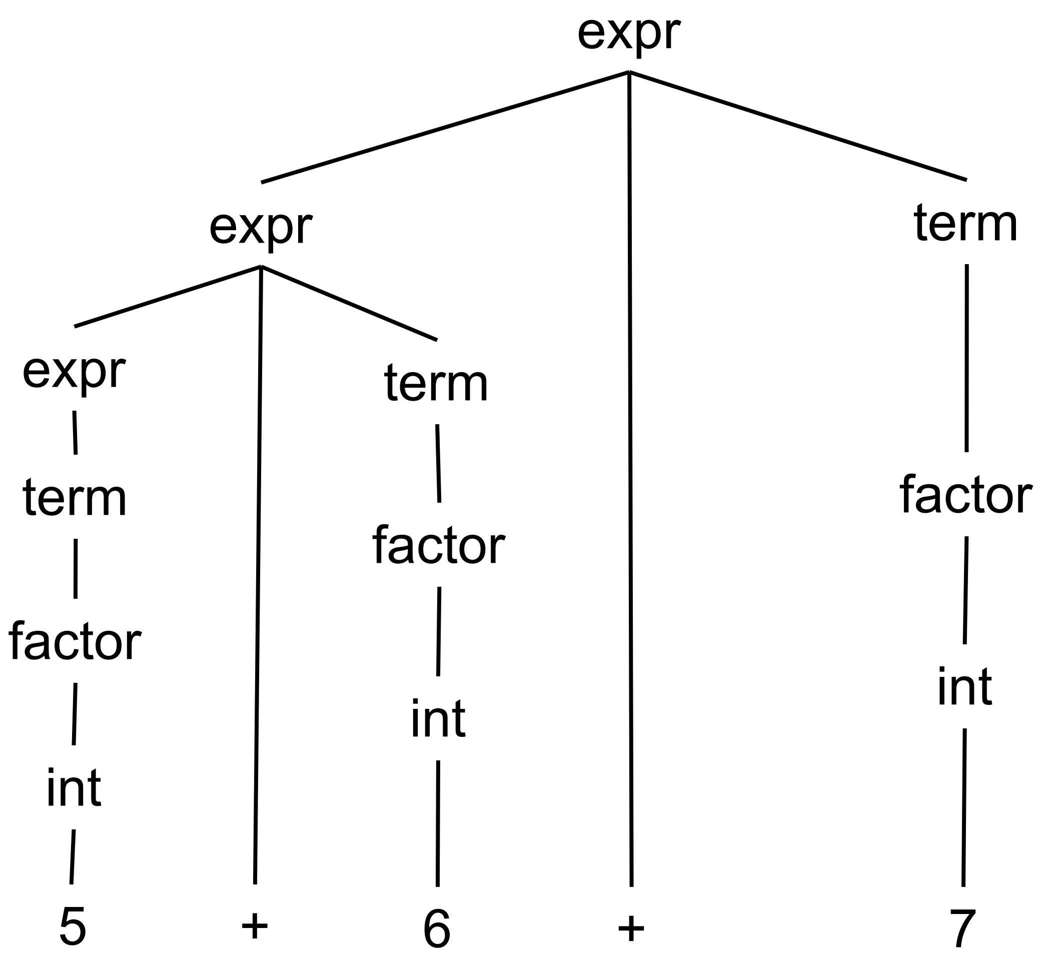
expr ::= *expr* + *term* | *expr* – *term* | *term*
term ::= *term* * *factor* | *term* / *factor* | *factor*
factor ::= *int* | (*expr*)
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7



Separation of non-terminals enforces precedence

Check: Derive 5+6+7

expr ::= *expr* + *term* | *expr* – *term* | *term*
term ::= *term* * *factor* | *term* / *factor* | *factor*
factor ::= *int* | (*expr*)
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7



Note interaction between left- vs right-recursive rules and resulting associativity

Check: Derive 5+(6+7)

expr ::= *expr* + *term* | *expr* – *term* | *term*
term ::= *term* * *factor* | *term* / *factor* | *factor*
factor ::= *int* | (*expr*)
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7

Another Classic Example

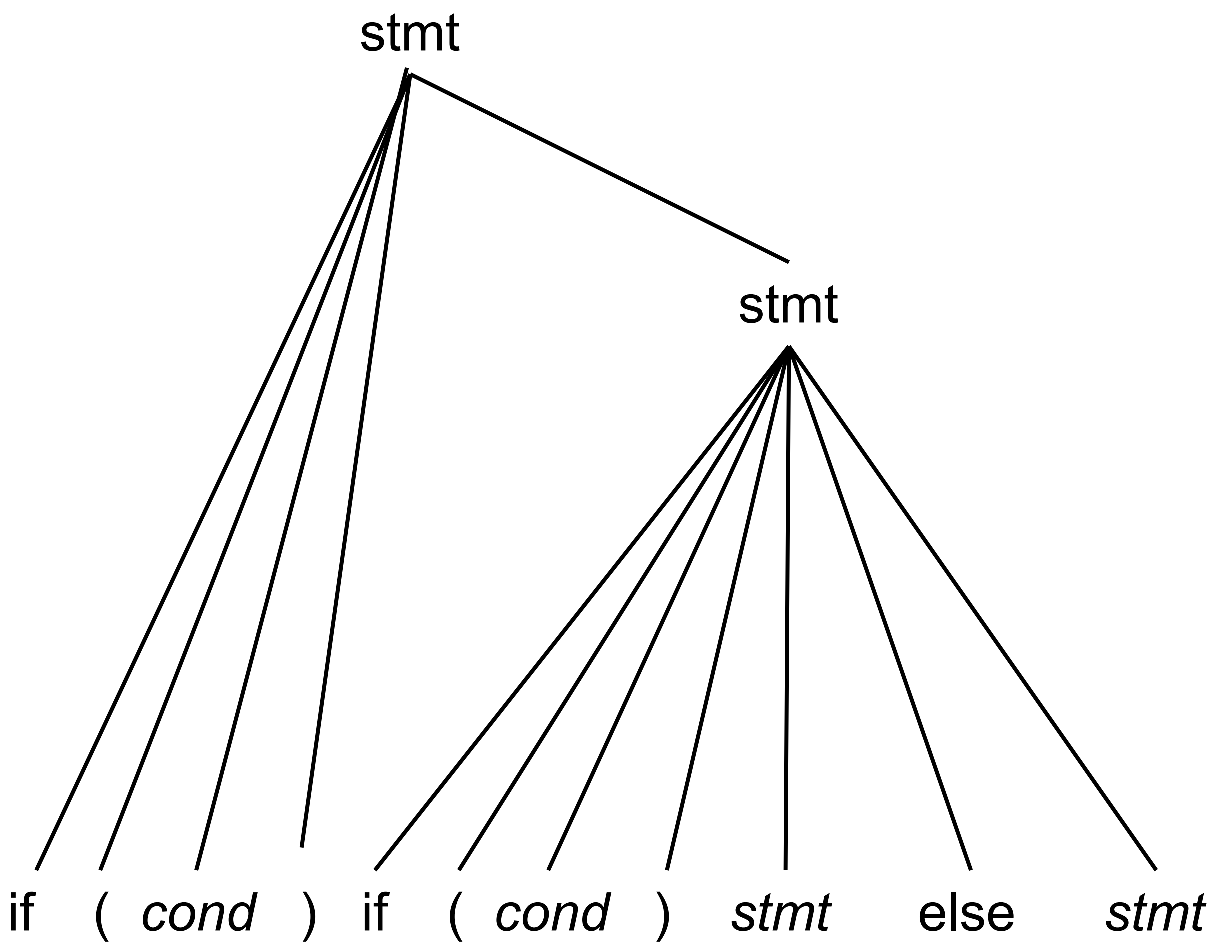
- Grammar for conditional statements

  stmt ::= if (*cond*) *stmt*
  | if (*cond*) *stmt* else *stmt*
  | *assign*

- Exercise: show that this is ambiguous
  - How?

stmt ::= if (*cond*) *stmt*
| if (*cond*) *stmt* else *stmt*
| *assign*

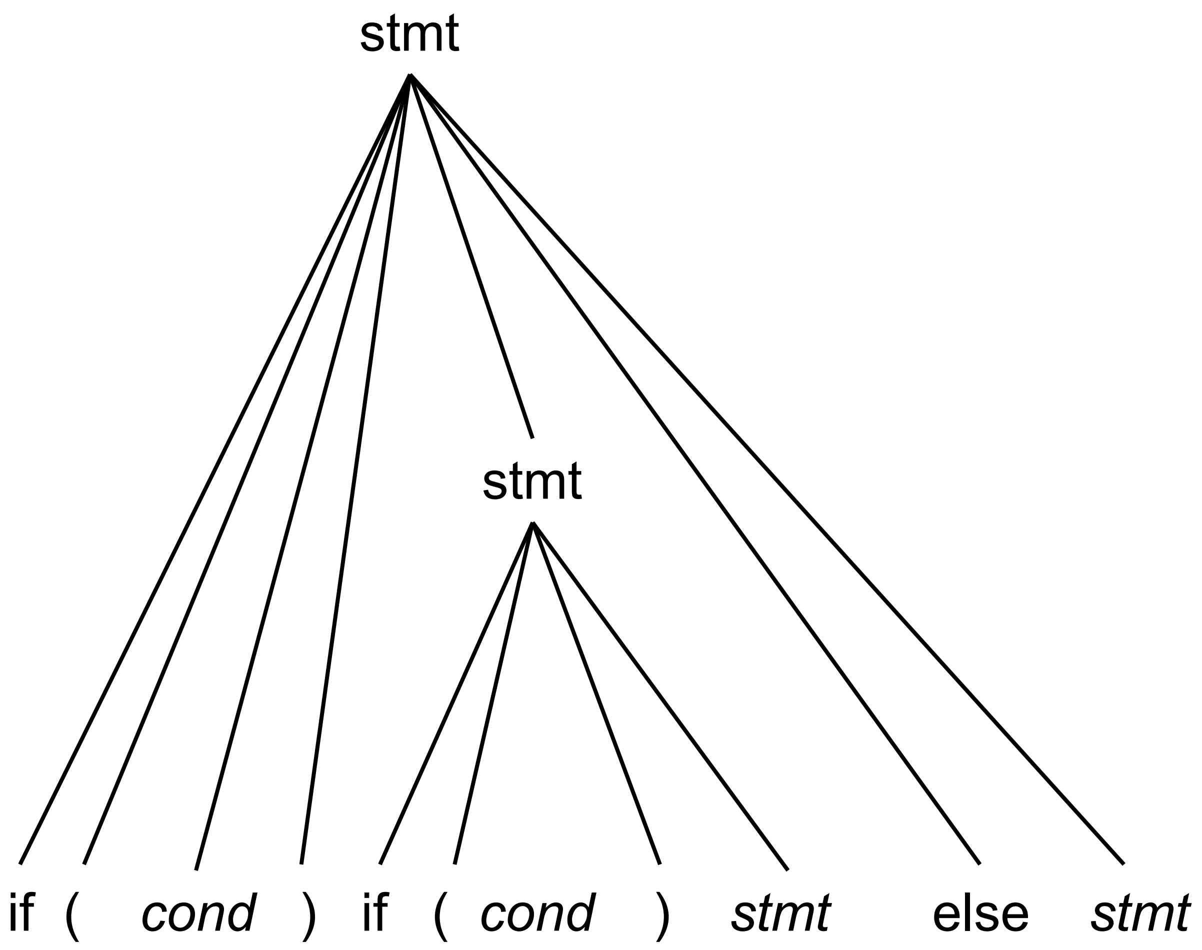
One Derivation



```
if (cond)
    if (cond)
        stmt
    else
        stmt
```

Another Derivation

stmt ::= if (*cond*) *stmt*
  | if (*cond*) *stmt* else *stmt*
  | *assign*



```
if (cond)
    if (cond)
        stmt
else
    stmt
```

Solving if Ambiguity

- Fix the grammar to separate `if` statements with `else` from if statements with no `else`
  - Done in original Java reference grammar
  - Adds lots of non-terminals
    - Need productions for things like “while statement that contains an unmatched if” and “while statement with only matched ifs”, etc. etc. etc.
- Use some ad-hoc rule in parser
  - “else matches closest unpaired if”

Parser Tools and Operators

- Most parser tools can cope with ambiguous grammars
  - Makes life simpler if used with discipline
- Typically one can specify operator precedence & associativity
  - Allows simpler, ambiguous grammar with fewer nonterminals as basis for generated parser, without creating problems

Parser Tools and Ambiguous Grammars

- Possible rules for resolving other problems
  - Earlier productions in the grammar preferred to later ones
  - Longest match used if there is a choice
- Parser tools normally allow for this
  - But be sure that what the tool does is really what you want
    - (Order in the input is particularly error-prone – reordering the input lines can change the meaning! ☹)

Or…

- If the parser is hand-written, either fudge the grammar or the parser, or cheat a little where it helps.

to be continued…