Question 1. (18 points) Regular expressions. For each of the following, (i) give a regular expression that generates the set of strings described, and (ii) draw a DFA that accepts that set of strings. There is lots of blank space for your answers – don’t worry if you don’t need nearly this much room.

Fine print: You may use basic regular expressions (sequences rs, choice r | s, repetition r*, and parentheses for grouping). You may also use + (one or more) and ? (zero or one), and character classes like [ax-z] and[^abc]. You also may use named abbreviations like “vowels ::= [aeiou]” if these help. You may not use additional regular expression operators that might be found in various programming language libraries or software tools.

(a) (6 points) A valid CSE course number like CSE413 consists of the letters ‘CSE’ followed by exactly 3 digits. The first digit of the number must be a decimal number in the range 1-6; the remaining numbers can be any decimal number in the range 0-9.

\[
\text{C S E [1-6] [0-9] [0-9]} 
\]

(b) (12 points) An ISO-8601 date has the format YYYY-MM-DD, where YYYY are the characters of a 4-digit year (0000 to 9999), MM are the two characters of the month in the range 01 to 12, and DD are the two characters of the day in the range 01 to 31. (For example, the date of this exam is 2016-12-12, and tomorrow will be 2016-12-13.) You do not need to worry about restricting dates to smaller ranges based on the month. For example, if the month is 02, the actual date will be no greater than 29, but you do not need to account for that. But you should be sure that no month is greater than 12 and no day is greater than 31.

\[
[0-9] [0-9] [0-9] [0-9] [0-9] [0-9] - (0[1-9] | 1[0-2]) - (0[1-9] | [12][0-9] | 3[01]) 
\]
Question 2. (12 points) Consider the following grammar:

\[ S ::= (S)S \mid \varepsilon \]

(Recall that \( \varepsilon \) is the empty string.)

(a) (8 points) Draw the parse tree for \((())()\)

(b) (4 points) Describe in English the set of strings generated by this grammar.

All strings with sequences of nested sets of balanced parentheses, including the empty string.
Question 3. (16 points) Ruby programming. Write a Ruby program that reads text from standard input and, after reading the entire input, prints all of the words that occur more than once in the input. Each word that is printed should be printed exactly once, but the order in which the words are printed is not specified. Each word should be printed on a separate line. For example, if the input is:

```
how now brown cow
don’t have a cow
the cow jumped over the moon
```

then the output should consist of the words “cow” and “the”, since they are the only words that occur more than once in the input.

You should assume that words are any non-blank sequences of characters in an input line that are separated by one or more blanks. Upper- and lower-case characters (‘A’ and ‘a’) are different – you should not convert or transform the input characters. There might, or might not, be leading or trailing blanks at the beginning or end of a line.

For full credit you should use Ruby iterators like `each` to process the contents of any containers like arrays or hashes. Your solution should process the input in linear time – i.e., a solution that reads all of the words into a giant string and then compares every word to every other, which takes $O(n^2)$ time, would not receive full credit. You also should avoid reading and storing the entire input file before processing it – process each line of input as you read it.

A couple of possibly useful facts about strings:

- If `s` is a string, `s.length` is the number of characters in it.
- If `s` is a string, `s.trim` is a copy of `s` with any leading or trailing blanks omitted.
- The string `split` method returns an array of the words in a string. Example: “one two three”.split returns [“one”, “two”, “three”]. If the entire string consists of blanks or has no characters in it, `split` will return an empty array [].

Write your code below or on the next page.

(See solution on next page)
Question 3. (cont.) Additional space for your Ruby code, if needed.

There are, of course, many ways to solve the problem. This solution simply counts the number of occurrences of each distinct word, then loops through the table and prints ever word with a count greater than one.

```ruby
freq = { } # {word=>frequency} pairs (Hash.new also works)

# read input and count number of occurrences of each word read
while line = gets
  words = line.split
  words.each do | w |
    # blocks with { } are also ok if done right
    if freq[w]
      freq[w] += 1
    else
      freq[w] = 1
    end
  end
end

# go through the table and print all words that occur more than once
freq.each do | word, n |
  if n > 1
    puts word
  end
end
```
Question 4. (12 points) Ruby inheritance and mixins. Consider the following code that consists of four Ruby classes and an additional “mixin” module. (Recall that if we “include” a mixin module in a class, it incorporates the methods from the module in the current class.)

class Apple
def m1
  puts "A-a"
end
def m2
  puts "A-aa"
self.m1()
end
end

module Mango
def m1
  puts "M-m"
end
def m3
  self.m1()
end
end

class Banana < Apple
  include Mango
def m4
    puts "B-b"
  end
end

class Citrus < Apple
  def m2
    super
    puts "C-cc"
  end
end

class Durian < Banana
  include Mango
def m2
    puts "D-d"
  end
  m3()
edend

class Durian < Banana
  include Mango
def m4
    super
end
end

For each of the following, write down the output produced by executing that line of code, or, if an error occurs, explain what happens.

a) Apple.new.m1
   A-a

b) Banana.new.m2
   A-aa
   M-m

c) Citrus.new.m1
   A-a

d) Citrus.new.m2
   A-aa

e) Citrus.new.m3
   undefined method ‘m3’

f) Durian.new.m4
   B-b
The next few questions concern the calculator language from the last two assignments. If you recall, the grammar for the calculator language was as follows:

```
program ::= statement | program statement
statement ::= exp | id = exp | clear id | list | quit | exit
exp ::= term | exp + term | exp - term
term ::= power | term * power | term / power
power ::= factor | factor ** power
factor ::= id | number | ( exp ) | sqrt ( exp )
```

We would like to add relational operators to the language. The idea is that an operator like < compares the values of two expressions and evaluates to the value 1 if the relation is true or the value 0 if it is false. So, for instance, 3<4 evaluates to 1, while 1<0 evaluates to 0.

Relational operators should have lower precedence than any of the other arithmetic operators. So 2+1<2 means the same as (2+1)<2, which evaluates to 0. Relational operators are left associative binary operators just like +, and −, so 3<1<2 is interpreted as (3<1)<2, which evaluates to 0<2 or 1.

The rest of the calculator language remains the same, with numeric constants and variables; expressions involving +, −, *, /, **, and parentheses; the sqrt function; assignment statements id = exp; and the keywords clear, list, quit, and exit. (Most of this information about the existing calculator language is not needed to answer the following questions.)

**Question 5.** (10 points) Suppose that we have added to the language the full set of six relational operators: <, <=, ==, !=, >=, and >. After these changes to the language, consider the following input:

```
xvii = 17
```

```
clear a<=b!=1+0>2plus3
```

```
sqrt(1<2)>3==exit42
```

Show how the calculator scanner would divide these input characters into tokens by drawing a box around each sequence of characters that make up a single token. Boxes on the first line are drawn for you. You do not need to show any “end of line” or “end of file” tokens. (Remember that we’re only asking how the scanner would divide the input characters into tokens, not whether the resulting token sequence makes any sense or is a legal calculator program.)
Question 6. (12 points) We need to add the relational operators to the grammar. To keep things simple for this question, we will only deal with the \(<\) operator. The others would all be handled similarly, but handling only \(<\) is enough here.

In an attempt to add \(<\) to the language of expressions, one of our summer interns modified the rule for \(exp\) as follows:

\[
exp ::= term \mid exp + term \mid exp - term \mid exp < exp
\]

(a) (6 points) Show that this grammar is ambiguous.

The ambiguity is due to the \(exp ::= exp < exp\) rule. Two partial leftmost derivations of \(1 < 2 < 3\) are enough to show the problem (it would also be fine to draw the corresponding parse trees instead):

\[
\begin{align*}
exp &\Rightarrow exp < exp \Rightarrow* 1 < 2 < 3 \\
exp &\Rightarrow exp < exp \Rightarrow* 1 < exp \Rightarrow 1 < exp < exp \Rightarrow* 1 < 2 < 3
\end{align*}
\]

(b) (6 points) Give a different grammar that will add the \(<\) operator to expressions but that is unambiguous, gives \(<\) lower precedence than the other arithmetic operators, and ensures that \(<\) is left-associative. You only need to rewrite or add the rules (productions) needed to make this change – you do not need to copy down other rules that remain unchanged.

Changing the \(exp ::= exp < exp\) rule to \(exp ::= exp < term\) would fix the ambiguity, but it doesn’t solve the precedence problem, since it gives \(<\) the same precedence as \(+\) and \(\_\_\). To get everything right we need to introduce a new non-terminal so that relations and the addition operators are generated by separate rules. Here is one way to fix things:

\[
\begin{align*}
exp &::= exp2 \mid exp < exp2 \\
exp2 &::= term \mid exp2 + term \mid exp2 - term
\end{align*}
\]
Some short questions on memory management.

**Question 7.** (8 points) Two of the strategies we looked at for reclaiming memory automatically were reference counting and mark-sweep garbage collection. A claim made in class was that reference counting did not always do as complete a job as garbage collection in reclaiming unreachable (not-in-use) memory.

Give an example of a dynamically allocated data structure that would not be reclaimed by a memory manager using reference counting, but would be successfully reclaimed by a mark-sweep collector. Give a brief and to-the-point explanation of why this is the case.

Any linked data structure that contain a cycle, like a circular list or a double-linked list, won’t be reclaimed by reference counting. In such data structures every node can be reached from other nodes, which means that all nodes always have a positive reference count even if no other variables point to any of the nodes. Even if the entire circular data structure is unreachable, none of the nodes will have a reference count of 0, so they will never be reclaimed.

A mark-sweep garbage collector has no problem with circular, unreachable data structures. They would not be marked during the mark phase, and would be reclaimed during the sweep phase.

**Question 8.** (6 points) In traditional languages like C or Java, the *automatic storage* used for function (method) local variables is allocated on a stack. That makes it efficient to allocate storage when a function is called and release it quickly when the function exits. The question is, can this always be done for a functional language like Racket? Why or why not? If it cannot be done, what needs to be done instead? (It should be possible to answer this in a couple of sentences.)

Not always. In languages like Racket, function closures contain pointers to an environment, and the lifetime of a closure can be longer than the lifetime of the function where the closure was created. That means we can’t use a last-called, first-deleted set of stack frames for environments that might be reachable from a closure after a function returns.

A common solution is to allocate environments reachable from function closures on the heap along with the rest of the dynamic data.

**Question 9.** (6 points) A *generational garbage collector* performs frequent garbage collection on part of the heap, but does not collect the entire heap nearly as often. Explain why this is done in a couple of sentences: which part of the heap is collected frequently and what does it contain? And why is this an effective strategy?

A generational garbage collector performs frequent collections on the part of the heap containing recently allocated objects, and does less frequent collections on the remainder of the heap. The rationale is that most functional and object-oriented languages tend to allocate many small, short-lived objects, so collecting the memory space containing recently allocated objects is likely to reclaim a much higher percentage of the storage examined during the collection compared to a collection of the full heap.

*Have a great winter break and best wishes for the new year!*

*The CSE 413 staff*