
Grammar

CSE 413, Autumn 2002
Programming Languages

<http://www.cs.washington.edu/education/courses/413/02au/>

Recall: Programming Language Specs

- Syntax of every significant programming language is specified by a formal grammar
 - » BNF or some variation there on
- As language engineering has developed, formal methods have improved for defining useful grammars and tools for processing them

Productions

- The rules of a grammar are called *productions*
- Rules contain
 - » Nonterminal symbols: grammar variables (*program, statement, id, etc.*)
 - » Terminal symbols: concrete syntax that appears in programs: a, b, c, 0, 1, if, (, ...
- Meaning of
 - nonterminal* ::= <sequence of terminals and nonterminals>
 - In a derivation, an instance of *nonterminal* can be replaced by the sequence of terminals and nonterminals on the right of the production
- Often, there are two or more productions for a single nonterminal – can use either at different times

Grammar for D, a little language

program ::= *function-def* | *program function-def*
function-def ::= **int** *id* () { *statements* }
 | **int** *id* (*parameters*) { *statements* }
 | **int** *id* () { *declarations statements* }
 | **int** *id* (*parameters*) { *declarations statements* }
parameters ::= *parameter* | *parameters* , *parameter*
parameter ::= **int** *id*
declarations ::= *declaration* | *declarations declaration*
declaration ::= **int** *id* ;
statements ::= *statement* | *statements statement*
statement ::= *id* = *exp* ; | **return** *exp* ; | { *statements* }
 | **if** (*bool-exp*) *statement* | **if** (*bool-exp*) *statement* **else** *statement*
 | **while** (*bool-exp*) *statement*
bool-exp ::= *rel-exp* | ! (*rel-exp*)
rel-exp ::= *exp* == *exp* | *exp* > *exp*
exp ::= *term* | *exp* + *term* | *exp* - *term*
term ::= *factor* | *term* * *factor*
factor ::= *id* | *int* | (*exp*) | *id* () | *id* (*exps*)
exps ::= *exp* | *exps* , *exp*

Grammar for Java, a big language

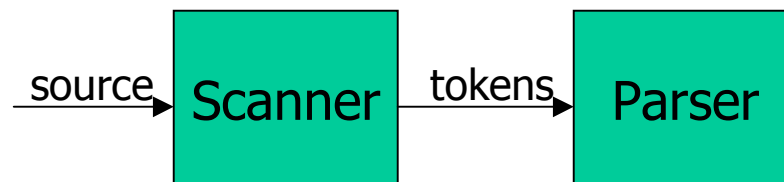
- The Java™ Language Specification, *Second Edition*
 - » *Entire document*
 - 500+ pages*
 - Grammar productions with explanatory text*
 - » *Chapter 18, Syntax*
 - 8 pages of grammar productions, presented in "BNF-style"*

Parsing

- Parsing: reconstruct the derivation (syntactic structure) of a program
- In principle, a single recognizer could work directly from the concrete, character-by-character grammar
 - » In practice this is never done

Parsing & Scanning

- In real compilers the recognizer is split into two phases
 - » Scanner: translate input characters to tokens
Also, report lexical errors like illegal characters and illegal symbols
 - » Parser: read token stream and reconstruct the derivation



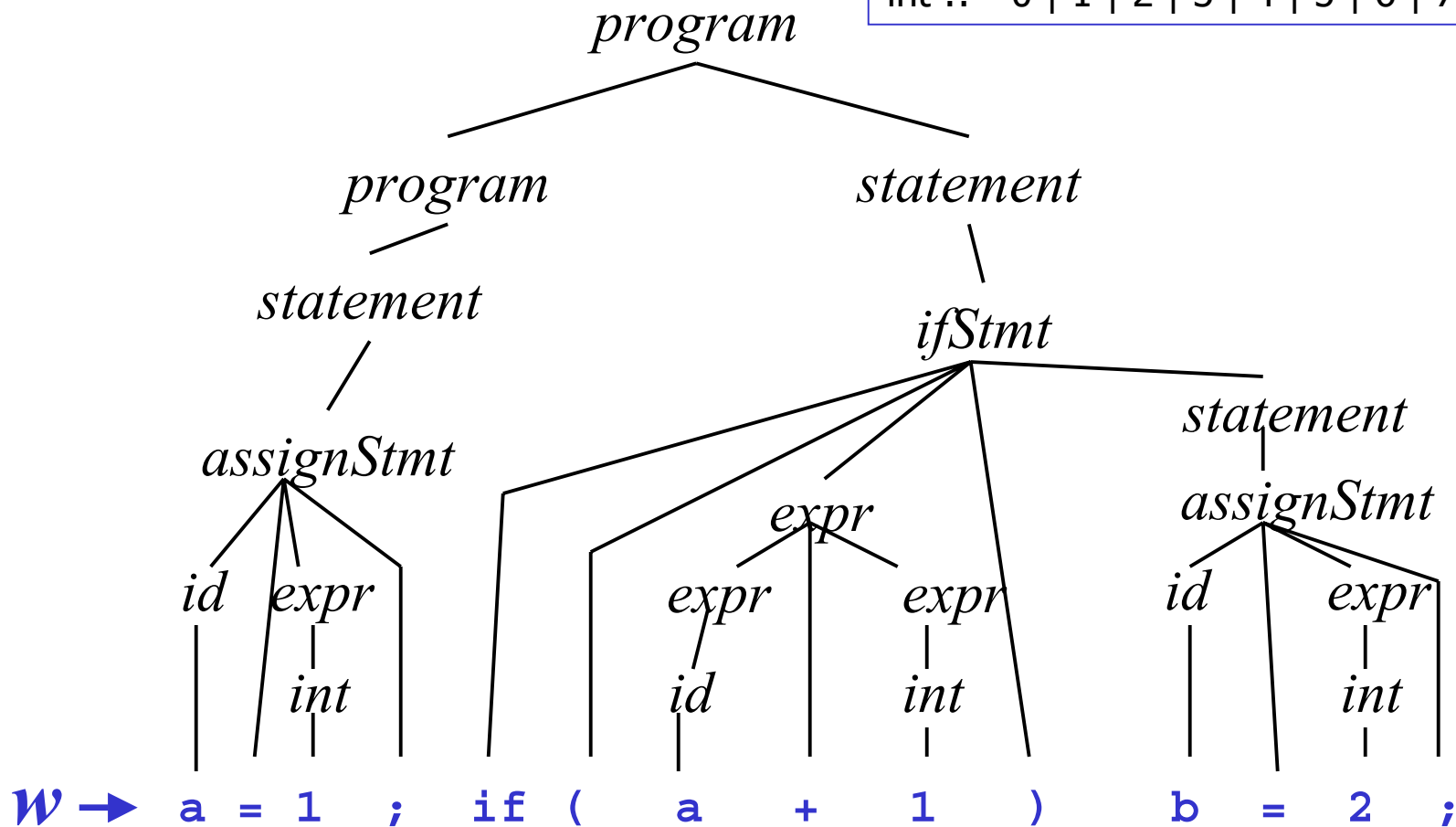
Parsing

- The syntax of most programming languages can be specified by a *context-free grammar* (CFG)
- Parsing
 - » Given a grammar G and a sentence w in $L(G)$, traverse the derivation (parse tree) for w in some *standard order* and do *something useful* at each node
 - » The tree might not be produced explicitly, but the control flow of a parser corresponds to a traversal

Parse Tree Example

G

```
program ::= statement | program statement
statement ::= assignStmt | ifStmt
assignStmt ::= id = expr ;
ifStmt ::= if ( expr ) stmt
expr ::= id | int | expr + expr
Id ::= a | b | c | i | j | k | n | x | y | z
int ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```



“Standard Order”

- For practical reasons we want the parser to be *deterministic* (no backtracking), and we want to examine the source program from *left to right*.
 - » parse the program in linear time in the order it appears in the source file

Common Orderings

- Top-down
 - » Start with the root
 - » Traverse the parse tree depth-first, left-to-right (leftmost derivation)
 - » LL(k)
- Bottom-up
 - » Start at leaves and build up to the root
 - Effectively a rightmost derivation in reverse
 - » LR(k) and subsets (LALR(k), SLR(k), etc.)

“Something Useful”

- At each point (node) in the traversal, perform some *semantic action*
 - » Construct nodes of full parse tree (rare)
 - » Construct abstract syntax tree (common)
 - » Construct linear, lower-level representation (more common in later parts of a modern compiler)
 - » Generate target code on the fly (1-pass compiler; not common in production compilers – can’t generate very good code in one pass)

Context-Free Grammars

- Formally, a grammar G is a tuple $\langle N, \Sigma, P, S \rangle$ where
 - » N a finite set of non-terminal symbols
 - » Σ a finite set of terminal symbols
 - » P a finite set of productions
 - A subset of $N \times (N \cup \Sigma)^*$
 - » S the *start symbol*, a distinguished element of N
 - If not specified otherwise, this is usually assumed to be the non-terminal on the left of the first production

Standard Notations

a, b, c	elements of Σ	<i>terminals</i>
w, x, y, z	elements of Σ^*	<i>strings of terminals</i>
A, B, C	elements of N	<i>non-terminals</i>
X, Y, Z	elements of $N \cup \Sigma$	<i>grammar symbols</i>
α, β, γ	elements of $(N \cup \Sigma)^*$	<i>strings of symbols</i>
$A \rightarrow \alpha$ or $A ::= \alpha$ if $\langle A, \alpha \rangle$ in P		

"non-terminal A can take the form α "

Derivation Relations

- $\alpha A \gamma \Rightarrow \alpha \beta \gamma$ iff $A ::= \beta$ in P
 - » " \Rightarrow " is read "derives"
- $A \Rightarrow^* w$ if there is a chain of productions starting with A that generates w
 - » transitive closure

Derivation Relations

- $w A \gamma \Rightarrow_{\text{lm}} w \beta \gamma$ iff $A ::= \beta$ in P
 » derives leftmost
- $\alpha A w \Rightarrow_{\text{rm}} \alpha \beta w$ iff $A ::= \beta$ in P
 » derives rightmost
- We will only be interested in leftmost and rightmost derivations – not random orderings

Languages

- For A in N , $L(A) = \{ w \mid A \Rightarrow^* w \}$
- If S is the start symbol of grammar G , define $L(G) = L(S)$
 - » The language derived by G is the language derived by the start symbol S

Reduced Grammars

- Grammar G is *reduced* iff for every production $A ::= \alpha$ in G there is a derivation

$$S \Rightarrow^* x A z \Rightarrow x \alpha z \Rightarrow^* xyz$$

» i.e., no production is useless

- Convention: we will use only reduced grammars

Ambiguity

- Grammar G is *unambiguous* iff every w in $L(G)$ has a unique leftmost (or rightmost) derivation
 - » Fact: unique leftmost or unique rightmost implies the other
- A grammar without this property is *ambiguous*
 - » Note that other grammars that generate the same language may be unambiguous
- We need unambiguous grammars for parsing

Ambiguous Grammar for Expressions

$$\begin{aligned} \text{expr} ::= & \text{expr} + \text{expr} \mid \text{expr} - \text{expr} \\ & \mid \text{expr} * \text{expr} \mid \text{expr} / \text{expr} \mid \text{int} \end{aligned}$$
$$\text{int} ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

- Show that this is ambiguous
 - » How? Show two different leftmost or rightmost derivations for the same string
 - » Equivalently: show two different parse trees for the same string

Example Derivation

$$\begin{aligned} \text{expr} ::= & \text{expr} + \text{expr} \mid \text{expr} - \text{expr} \\ & \mid \text{expr} * \text{expr} \mid \text{expr} / \text{expr} \mid \text{int} \\ \text{int} ::= & 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

Give a leftmost derivation of $2+3*4$ and show the parse tree

Another Derivation

$$\begin{aligned} \text{expr} &::= \text{expr} + \text{expr} \mid \text{expr} - \text{expr} \\ &\quad \mid \text{expr} * \text{expr} \mid \text{expr} / \text{expr} \mid \text{int} \\ \text{int} &::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

Give a different leftmost derivation of $2+3*4$ and show the parse tree

Another Example

$$\begin{aligned} \text{expr} ::= & \text{expr} + \text{expr} \mid \text{expr} - \text{expr} \\ & \mid \text{expr} * \text{expr} \mid \text{expr} / \text{expr} \mid \text{int} \\ \text{int} ::= & 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

Give two different derivations of $5+6+7$

What's going on here?

- The grammar has no notion of precedence or associativity
- Solution
 - » Create a non-terminal for each level of precedence
 - » Isolate the corresponding part of the grammar
 - » Force the parser to recognize higher precedence subexpressions first

Classic Expression Grammar

$expr ::= expr + term \mid expr - term \mid term$

$term ::= term * factor \mid term / factor \mid factor$

$factor ::= int \mid (expr)$

$int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$

Derive $2 + 3 * 4$

$expr ::= expr + term \mid expr - term \mid term$
 $term ::= term * factor \mid term / factor \mid factor$
 $factor ::= int \mid (expr)$
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$

Derive $5 + 6 + 7$

$expr ::= expr + term \mid expr - term \mid term$
 $term ::= term * factor \mid term / factor \mid factor$
 $factor ::= int \mid (expr)$
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$

Derive $5 + (6 + 7)$

$expr ::= expr + term \mid expr - term \mid term$
 $term ::= term * factor \mid term / factor \mid factor$
 $factor ::= int \mid (expr)$
 $int ::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7$

Another Classic Example

- Grammar for conditional statements

$$\begin{aligned} ifStmt ::= & \mathbf{if} (cond) stmt \\ & | \mathbf{if} (cond) stmt \mathbf{else} stmt \end{aligned}$$

- » Exercise: show that this is ambiguous
How?

One Derivation

$$\begin{aligned} \text{ifStmt} ::= & \text{if} (\text{cond}) \text{stmt} \\ & | \text{if} (\text{cond}) \text{stmt} \text{ else } \text{stmt} \end{aligned}$$

$\text{if} (\text{cond}) \text{if} (\text{cond}) \text{stmt} \text{ else } \text{stmt}$

Another Derivation

$$\begin{aligned} \text{ifStmt} ::= & \text{if} (\text{cond}) \text{stmt} \\ & | \text{if} (\text{cond}) \text{stmt} \text{ else } \text{stmt} \end{aligned}$$

$\text{if} (\text{cond}) \text{if} (\text{cond}) \text{stmt} \text{ else } \text{stmt}$

Solving if Ambiguity

- Fix the grammar to separate if statements with else clause and if statements with no else
 - » Done in Java reference grammar
 - » Adds lots of non-terminals
- Use some ad-hoc rule in parser
 - » “else matches closest unpaired if”