Hierarchical Structures

CSE 413, Autumn 2002

Programming Languages

http://www.cs.washington.edu/education/courses/413/02au/

Readings and References

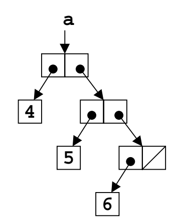
- Reading
  - » Section 2.2.2, *Structure and Interpretation of Computer Programs*, by Abelson, Sussman, and Sussman
- Other References

Lists are a basic abstraction

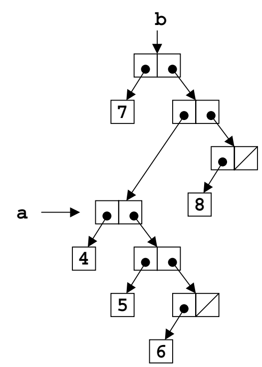
- Using `list` to build lists, we can build data structures of increasing complexity
- Nested lists
  - » one or more of the elements of the list are themselves lists
  - » **`(list 1 2 (list 3 4) 5)`**

List structure

`(define a (list 4 5 6))`



`(define b (list 7 a 8))`



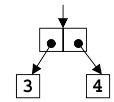
car = "this element"
cdr = "rest of the elements"

Printed representation of a list

- Lists are so fundamental to Scheme that the interpreter assumes that any data structure that uses pairs is probably a list
- The printed representation of a pair uses a "." to separate the car and the cdr elements
  - » `(cons 3 4) => (3 . 4)`
- But when printing a list, the complexity of the pair is suppressed for clarity when possible
  - » `(cons 3 '()) => (3)`

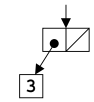
Printing pairs and lists

`(cons 3 4) => (3 . 4)`



this is a valid data structure, but it is not a well formed list

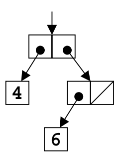
`(cons 3 '()) => (3)`



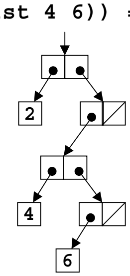
this is a well formed list

List structure

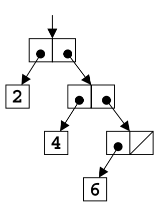
`(list 4 6) => (4 6)`



`(list 2 (list 4 6)) => (2 (4 6))`

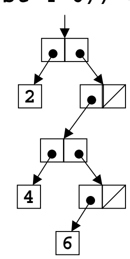


`(list 2 4 6) => (2 4 6)`

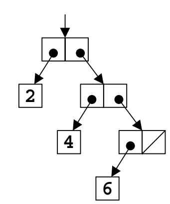


List structure and cons

`(list 2 (list 4 6)) => (2 (4 6))`



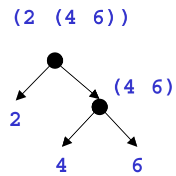
`(cons 2 (list 4 6)) => (2 4 6)`



Recursive tree structure

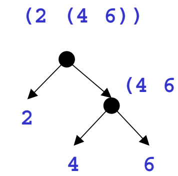
```
(list 2 (list 4 6)) => (2 (4 6))
```

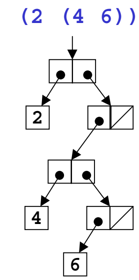
- This list has two elements
  - » the literal 2 and the list (4 6)
- The sublist also has two elements
  - » the literals 4 and 6
- We can think of lists, and lists of lists, as tree structures
  - » all the elements in one list are siblings



(depth x)

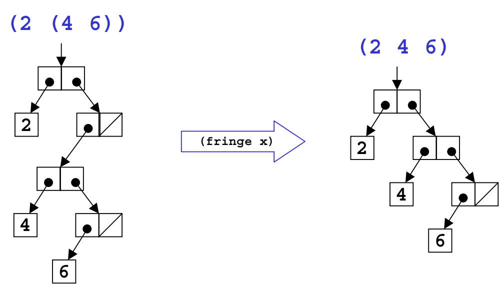
```
; x is a tree node.  It is defined by a
; list that contains the node at this entry,
; plus all the the sibling tree nodes to the
; right of this node.
; The value at this node is (car x).
; The list of siblings to the right is (cdr x).

(define (depth x)
  (cond ((null? x) 0)
        ((not (pair? x)) 0)
        (else (max (+ 1 (depth (car x)))
                   (depth (cdr x))))))
```





(fringe x)

```
; pick the leaves off a tree defined as lists of lists
(define (fringe m)
  (cond
    ((null? m) m)
    ((not (pair? m)) (list m) )
    (else (append (fringe (car m)) (fringe (cdr m))))))
```



Further abstraction

- The more we can map into the problem domain the better
- A layer of abstraction can hide much or all of the messy details of implementation
  - » easier to understand
  - » easier to replace the implementation
- Lists are an abstraction of a pair structure
- Trees are an abstraction of a list structure

Expression trees

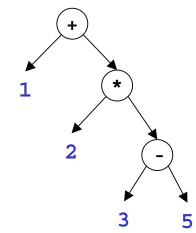
- In Scheme, we often use constructors and accessors to abstract away the underlying representation of data (which is usually a list)
- For example, consider arithmetic expression trees
- A binary expression is
  - » an operator: +, -, *, / and two operands
- An operand is
  - » a number or another expression

Expression tree example

infix notation `(1 + ( 2 * ( 3 - 5 )))`

Scheme expression `(+ 1 (* 2 (- 3 5)))`

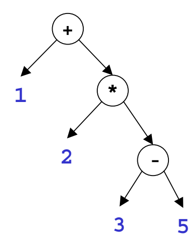
expression tree



Represent expression with a list

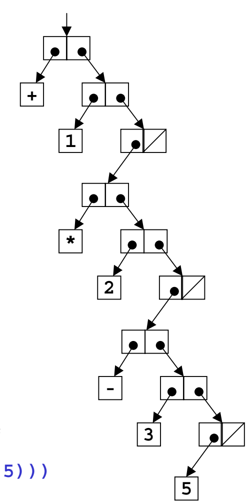
- For this example, we are restricting the type of expression somewhat
  - » Operators in the tree are all binary
  - » All of the leaves (operands) are numbers
- Each node is represented by a 3-element list
  - » (operator left-operand right-operand)
- Recall that the operands can be
  - » numbers (explicit values)
  - » other expressions (lists)

Expressions as trees, trees as lists



logical expression tree

`(1+(2*(3-5)))`



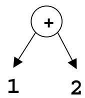
our data structure

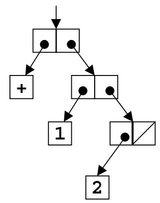
`(list + 1 (list * 2 (list - 3 5)))`

Constructors and accessors

```
(define (make-exp op left right)
  (list op left right))

(define (operator exp)
  (car exp))

(define (left exp)
  (cadr exp))

(define (right exp)
  (caddr exp))
```

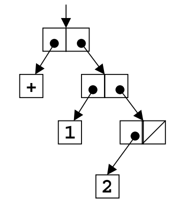
`(define a (make-exp + 1 2))`





Evaluator

`(eval-expr (make-exp + 1 2))`




```
(define (eval-expr exp)
  (if (not (pair? exp))
      exp
      ((operator exp)
       (eval-expr (left exp))
       (eval-expr (right exp)))))
```

```
; note that this code expects the operators
; to be the actual functions, not text symbols
```