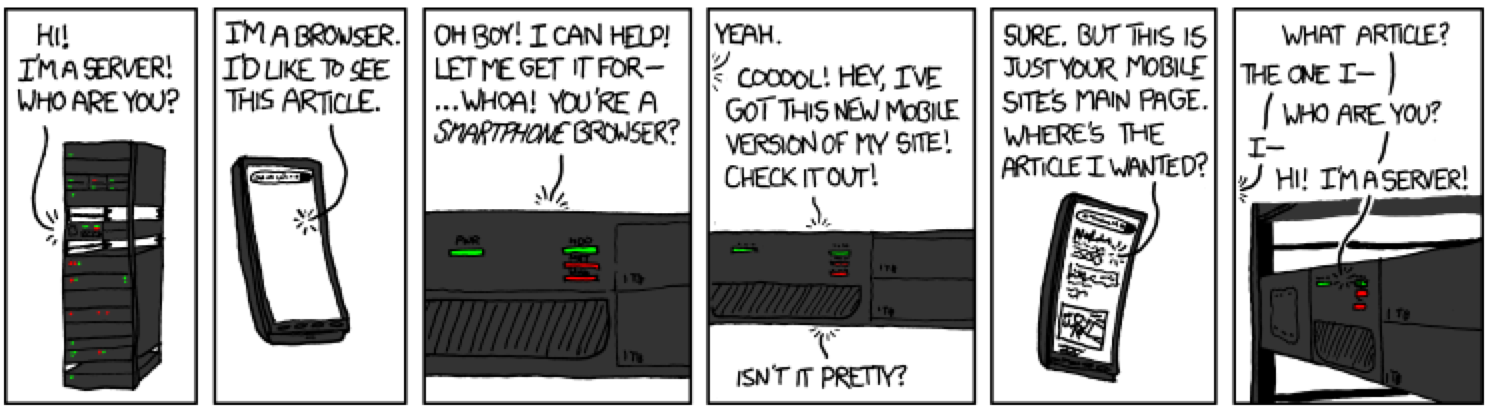
Reasoning about programs



Team member contribution #3

Assess contribution from Feb 11 through March 10th

- Customer exposure testing and
- Final assignment work periods

Surveys are open after class

Due by 11pm on Friday March 11th

Tips from #1 and #2

- Highlight your accomplishments but don't give yourself points
- Check your addition: values should sum to 100
- Average = $\frac{100}{team\ size - 1}$

Collaborative development survey

- 10 simple questions
- On catalyst
- Each participation points
- No wrong answers

- <u>Due by 11pm on Friday March 11th</u>

It’s the home stretch

Final (1.0) release due Thursday

presentation due in class

Second midterm

- Design patterns
- Testing
  - guest speaker: unit testing
- Debugging
- Consistent and complete specifications
- Reasoning about programs
  - guest speaker: language annotations

Wednesday, March 16, 2:30 PM – 4:20 PM, EEB 045

Ways to verify your code

- The hard way:
  - Make up some inputs
  - If it doesn't crash, ship it
  - When it fails in the field, attempt to debug
- The easier way:
  - Reason about possible behaviors and desired outcomes
  - Construct simple tests that exercise those behaviors
- Another way that can be easy
  - Prove that the system does what you want
    - Rep invariants are preserved
    - Implementation satisfies specification
  - Proof can be formal or informal (we will be informal)
  - Complementary to testing

Reasoning about code

- Determine what facts are true during execution
  - x > 0
  - for all nodes n: n.next.previous == n
  - array a is sorted
  - x + y == z
  - if x != null, then x.a > x.b
- Applications:
  - Ensure code is correct (via reasoning or testing)
  - Understand why code is incorrect

Forward reasoning

- You know what is true before running the code
  What is true after running the code?
- Given a precondition, what is the postcondition?

- Applications:
  Representation invariant holds before running code
  Does it still hold after running code?
- Example:

```
// precondition: x is even
x = x + 3;
y = 2x;
x = 5;
// postcondition:  ??
```

Backward reasoning

- You know what you want to be true after running the code
  What must be true beforehand in order to ensure that?
- Given a postcondition, what is the corresponding precondition?
- Application:
  (Re-)establish rep invariant at method exit: what's required?
  Reproduce a bug: what must the input have been?
- Example:
  ```
  // precondition: ??
  x = x + 3;
  y = 2x;
  x = 5;
  // postcondition: y > x
  ```
- How did you (informally) compute this?

Forward vs. backward reasoning

- Forward reasoning is more intuitive for most people
  - Helps understand what will happen (simulates the code)
  - Introduces facts that may be irrelevant to goal
    - Set of current facts may get large
  - Takes longer to realize that the task is hopeless
- Backward reasoning is usually more helpful
  - Helps you understand what should happen
  - Given a specific goal, indicates how to achieve it
  - Given an error, gives a test case that exposes it

Forward reasoning example

```
assert x >= 0;
i = x;
    // x ≥ 0  &  i = x
z = 0;
    // x ≥ 0  &  i = x  &  z = 0
while (i != 0) {
  z = z + 1;
  i = i –1;
}
    // x ≥ 0  &  i = 0  &  z = x
assert x == z;
```

⇐ What property holds here?

⇐ What property holds here?

Backward reasoning

Technique for backward reasoning:

- Compute the weakest precondition (“wp”)
- There is a wp rule for each statement in the programming language
- Weakest precondition yields strongest specification for the computation (analogous to function specifications)

Assignment

```
// precondition: ??
x = e;
// postcondition: Q
```

Precondition = Q with all (free) occurrences of x replaced by e

- Example:

```
// assert:  ??
x = x + 1;
// assert x > 0
```

Precondition = (x+1) > 0

Method calls

```
// precondition: ??
x = foo();
// postcondition: Q
```

- If the method has no side effects: just like ordinary assignment
- If it has side effects: an assignment to every variable in modifies

Use the method specification to determine the new value

If statements

// precondition: ??

if (b) S1 else S2

// postcondition: Q

Essentially case analysis:

wp("if (b) S1 else S2", Q) =

($\quad$ b $\Rightarrow$ wp("S1", Q)

$\wedge$ $\neg$ b $\Rightarrow$ wp("S2", Q))

If: an example

```
// precondition: ??
if (x == 0) {
   x = x + 1;
} else {
   x = (x/x);
}
// postcondition:  x ≥ 0
```

Precondition:

wp("if (x==0) {x = x+1} else {x = x/x}", $x \geq 0$)

= ($x = 0 \Rightarrow$ wp("x = x+1", $x \geq 0$)

& $x \neq 0 \Rightarrow$ wp("x = x/x", $x \geq 0$))

= $(x = 0 \Rightarrow x + 1 \geq 0)$ & $(x \neq 0 \Rightarrow x/x \geq 0)$

= $1 \geq 0$ & $1 \geq 0$

= true

Reasoning About Loops

- A loop represents an unknown number of paths
  - Case analysis is problematic
  - Recursion presents the same issue
- Cannot enumerate all paths
  - That is what makes testing and reasoning hard

Loops: values and termination

```
// assert x ≥ 0 & y = 0
while (x != y) {
    y = y + 1;
}
// assert x = y
```

1) Pre-assertion guarantees that $x \geq y$

2) Every time through loop

   $x \geq y$ holds and, if body is entered, $x > y$

   y is incremented by 1

   x is unchanged

   Therefore, y is closer to x (but $x \geq y$ still holds)

3) Since there are only a finite number of integers between x and y, y will eventually equal x

4) Execution exits the loop as soon as $x = y$

Understanding loops by induction

- We just made an inductive argument
  - Inducting over the number of iterations
- Computation induction
  - Show that conjecture holds if zero iterations
  - Assume it holds after n iterations and show it holds after n+1
- There are two things to prove:
  - Some property is preserved (known as “partial correctness”)
    - loop invariant is preserved by each iteration
  - The loop completes (known as “termination”)
    - The “decrementing function” is reduced by each iteration

Loop invariant for the example

```
// assert x ≥ 0 & y = 0
while (x != y) {
     y = y + 1;
}
// assert x = y
```

- So, what is a suitable invariant?
- What makes the loop work?

$LI = x \geq y$

1) $x \geq 0 \;\; \& \;\; y = 0 \Rightarrow LI$
2) $LI \;\; \& \;\; x \neq y$ {`y = y+1;`} LI
3) $(LI \;\; \& \;\; \neg(x \neq y)) \;\; \Rightarrow \;\; x = y$

In practice

I don’t routinely write loop invariants

I do write them when I am unsure about a loop and when I have evidence that a loop is not working

– Add invariant and decrementing function if missing
– Write code to check them
– Understand why the code doesn't work
– Reason to ensure that no similar bugs remain