

Section 3: LR Parsing

CSE 401/M501

Adapted from Spring 2021

Announcements

- Scanner is due tonight
 - Be sure to test, push, and tag!
- Every person has 4 project late days and 4 assignment late days
 - Up to 2 can be used per assignment
 - Each late day gives an extra 24 hour chunk (including weekend days)
 - We recommend not using your late days this early if possible!
 - **Submitting project components a day late uses a project late day from each partner!**

12:00-13:00 OH (Bill) CSE2 150	06	12:00-13:00 OH (Larry) CSE2 150	07	12:00-13:00 OH (Bill) CSE2 150	08	Section <i>LR parser construction</i>	09	13:00-14:00 OH (Varun) CSE2 131	10
14:30-15:20 Lecture CSE2 G10 <i>LR parsing (concl.)</i>		13:00-15:00 OH (Karen) CSE2 150		14:30-15:20 Lecture CSE2 G10 <i>LR table construction (3.5) (start)</i> <i>slides</i>		16:00-17:00 OH (Larry) CSE2 150		14:30-15:20 Lecture CSE2 G10 <i>LR conflicts, first/follow (no new slides)</i>	
15:30-16:30 OH (Andy) CSE2 131				15:30-16:30 OH (Andy) CSE2 131		17:00-18:00 OH (Varun) Allen 4th floor breakout		15:30-17:00 OH (Rajat) CSE2 150	
						23:59 Project: <i>scanner due</i>			



Agenda

- (Fast) LR terminology review
- Worksheet

Get Your LR Jargon On

- Frontier
 - The upper “layer” of the current parse tree (held in the stack)

Get Your LR Jargon On

- Frontier
 - The upper “layer” of the current parse tree (held in the stack)
- Sentential Form
 - A string that can be generated at any point in a derivation (can be reached using any number of productions from the start symbol)

Get Your LR Jargon On

- Frontier
 - The upper “layer” of the current parse tree (held in the stack)
- Sentential Form
 - A string that can be generated at any point in a derivation (can be reached using any number of productions from the start symbol)
- Handle
 - An occurrence of the right side of a production in the frontier that is used in the rightmost derivation to arrive at the current string
 - Given the derivation ... $\Rightarrow$ aAbcde $\Rightarrow$ abbcde, using the production $A ::= b$:
 - The production ' $A ::= \underline{b}$ ' at index 1 would be a handle of abbcde

Get Your LR Jargon On - Example

Shift-Reduce Example

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

Frontier

The upper “layer” of
the current parse tree
(held in the stack)

Stack	Input	Action
\$	abbcde\$	<i>shift</i>
\$a	bbcd\$	<i>shift</i>
\$ab	bcde\$	<i>reduce</i>
\$aA	bcde\$	<i>shift</i>
\$aAb	cde\$	<i>shift</i>
\$aAbc	de\$	<i>reduce</i>
\$aA	de\$	<i>shift</i>
\$aAd	e\$	<i>reduce</i>
\$aAB	e\$	<i>shift</i>
\$aABe	\$	<i>reduce</i>
\$S	\$	<i>accept</i>

Get Your LR Jargon On - Example

Shift-Reduce Example

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

Sentential Forms

A string that can be generated at any point in a derivation (can be reached using any number of productions from the start symbol)

Stack	Input	Action
\$	abbcde\$	<i>shift</i>
\$a	bbcde\$	<i>shift</i>
\$ab	bcde\$	<i>reduce</i>
\$aA	bcde\$	<i>shift</i>
\$aAb	cde\$	<i>shift</i>
\$aAbc	de\$	<i>reduce</i>
\$aA	de\$	<i>shift</i>
\$aAd	e\$	<i>reduce</i>
\$aAB	e\$	<i>shift</i>
\$aABe	\$	<i>reduce</i>
\$S	\$	<i>accept</i>

Get Your LR Jargon On - Example

Shift-Reduce Example

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

	Stack	Input	Action
Handles	\$	abbcde\$	<i>shift</i>
	\$a	bbcd\$	<i>shift</i>
A ::= b at index 2	\$ab	bcde\$	<i>reduce</i>
	\$aA	bcde\$	<i>shift</i>
	\$aAb	cde\$	<i>shift</i>
A ::= Abc at index 4	\$aAbc	de\$	<i>reduce</i>
	\$aA	de\$	<i>shift</i>
B ::= d at index 3	\$aAd	e\$	<i>reduce</i>
	\$aAB	e\$	<i>shift</i>
S ::= aABe at index 4	\$aABe	\$	<i>reduce</i>
	\$S	\$	<i>accept</i>

A Little Bit More Jargon

- Viable Prefix
 - The prefixes of a right sentential form that do not extend beyond the end of its handle
 - Perhaps less confusing -> the set of prefixes of strings that can appear on the stack of a shift-reduce parser

A Little Bit More Jargon

- Viable Prefix
 - The prefixes of a right sentential form that do not extend beyond the end of its handle
 - Perhaps less confusing -> the set of prefixes of strings that can appear on the stack of a shift-reduce parser
- Item
 - A marked production (a production with a ‘.’ in it)
 - $[A ::= .XY], [A ::= X.Y], [A ::= XY.]$

Get Your LR Jargon On - Example

Shift-Reduce Example

$S ::= aABe$
 $A ::= Abc \mid b$
 $B ::= d$

Viable Prefix

The set of prefixes of strings that can appear on the stack of a shift-reduce parser

Stack	Input	Action
\$	abbcd e\$	<i>shift</i>
\$a	bbcd e\$	<i>shift</i>
\$ab	bcde\$	<i>reduce</i>
\$aA	bcde\$	<i>shift</i>
\$aAb	cde\$	<i>shift</i>
\$aAbc	de\$	<i>reduce</i>
\$aA	de\$	<i>shift</i>
\$aAd	e\$	<i>reduce</i>
\$aAB	e\$	<i>shift</i>
\$aABe	\$	<i>reduce</i>
\$S	\$	<i>accept</i>

LR (0)

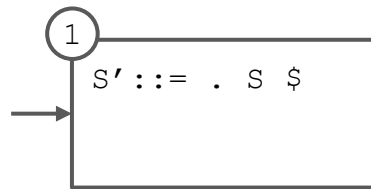


Problem 1 (On Worksheet)

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$

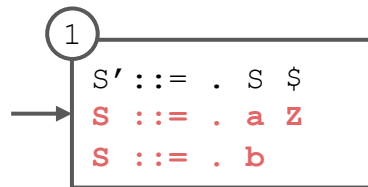
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



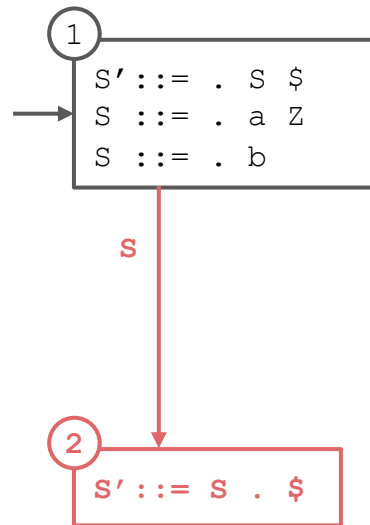
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



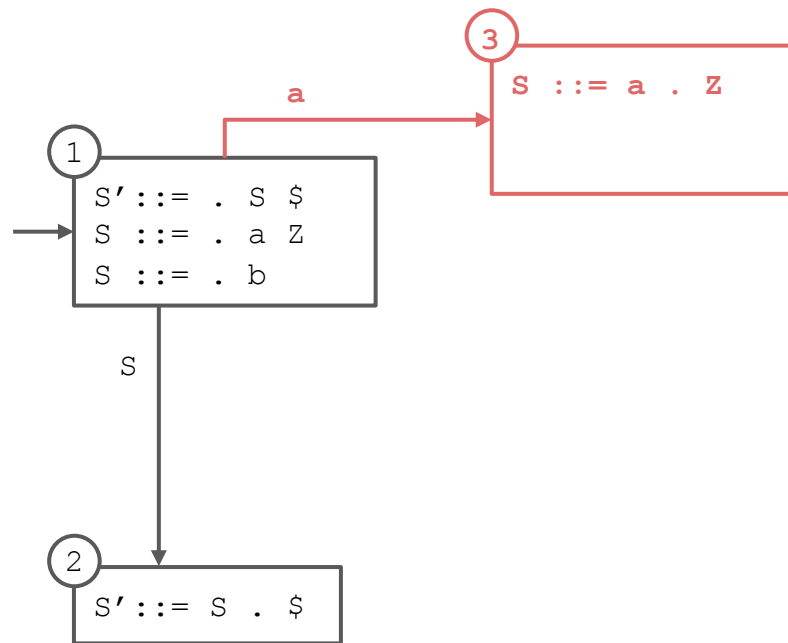
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



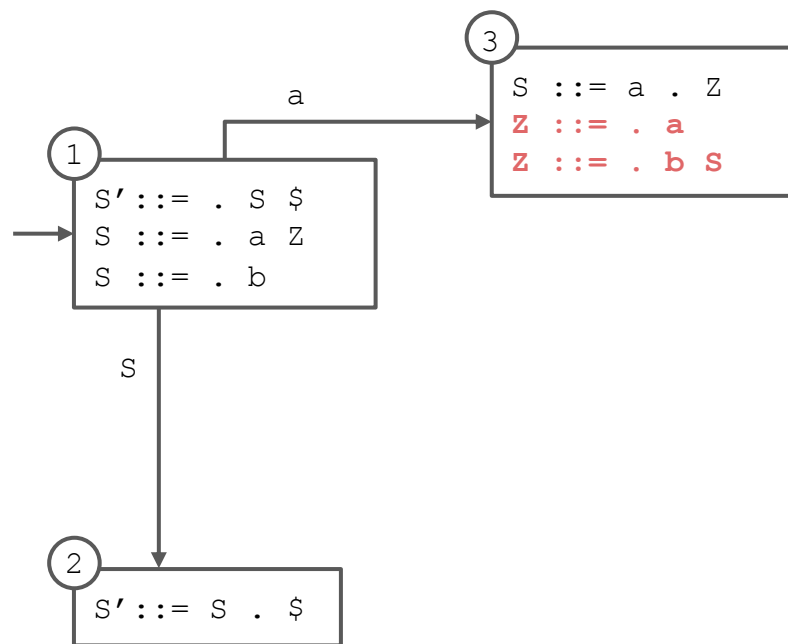
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



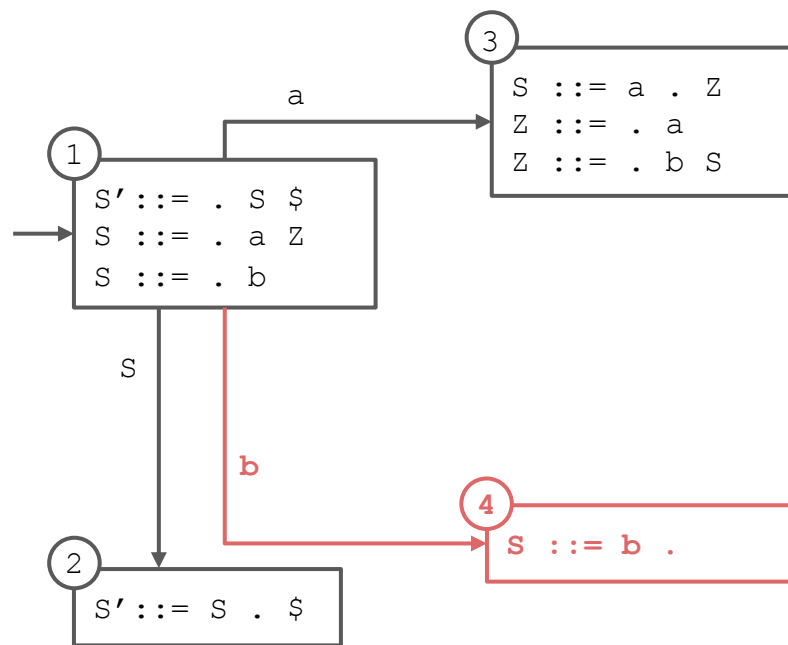
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



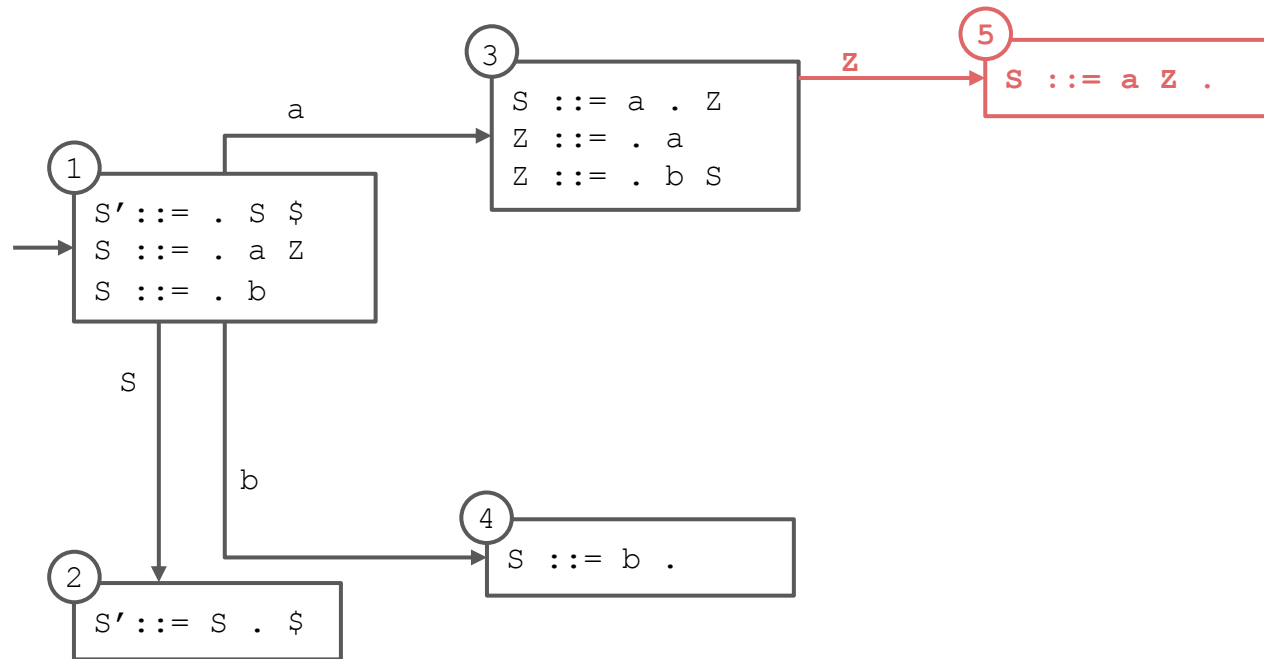
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



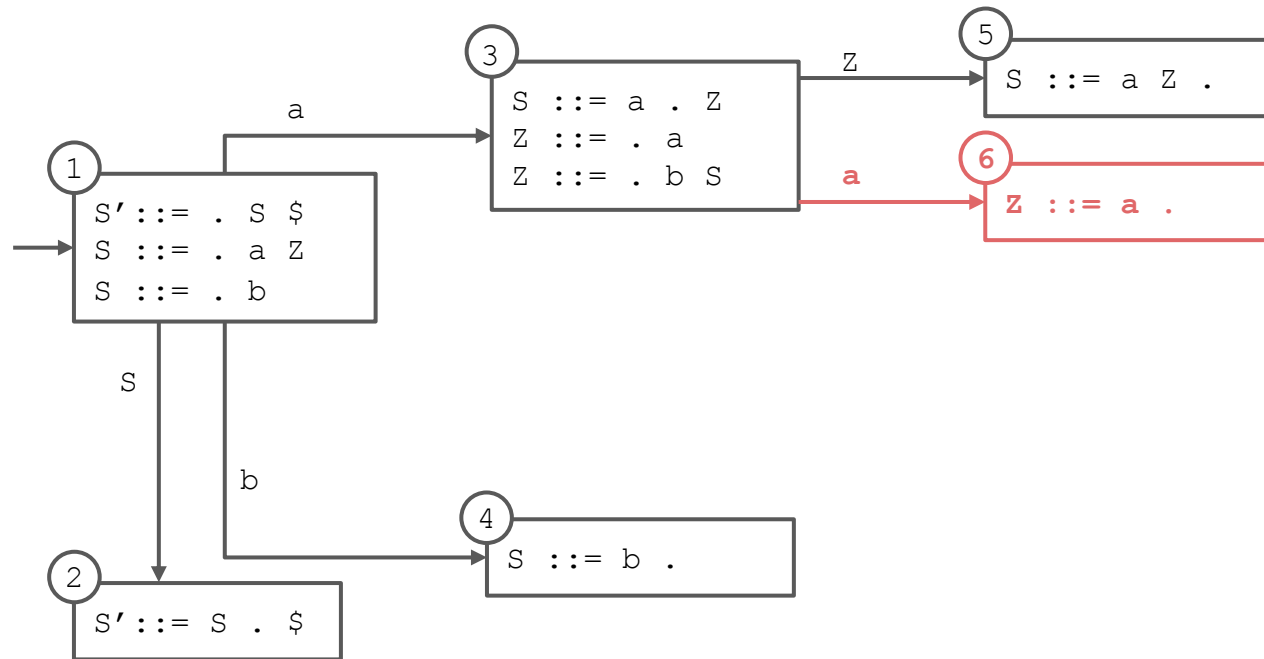
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



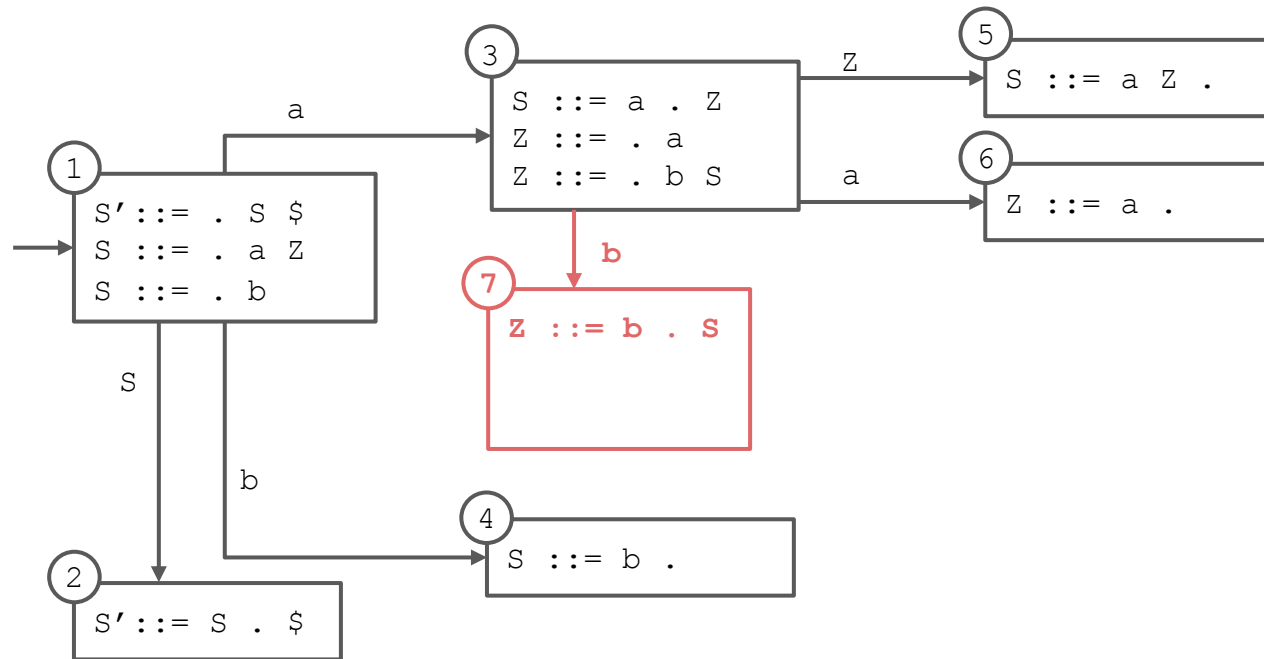
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



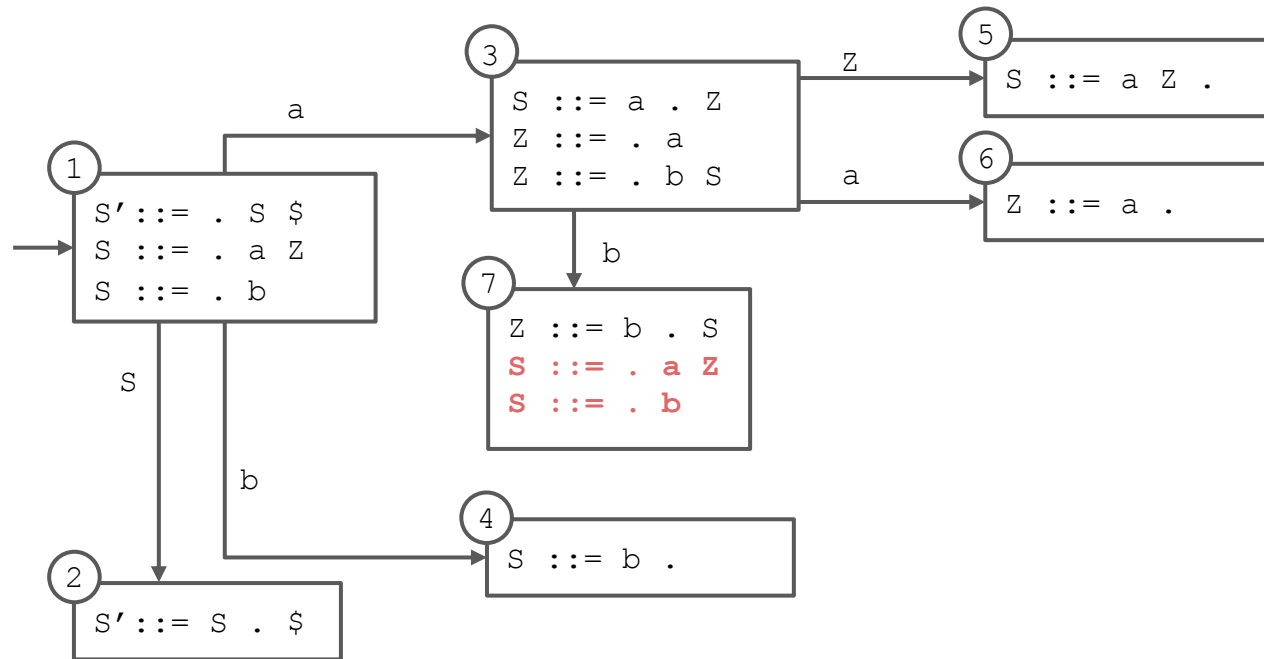
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



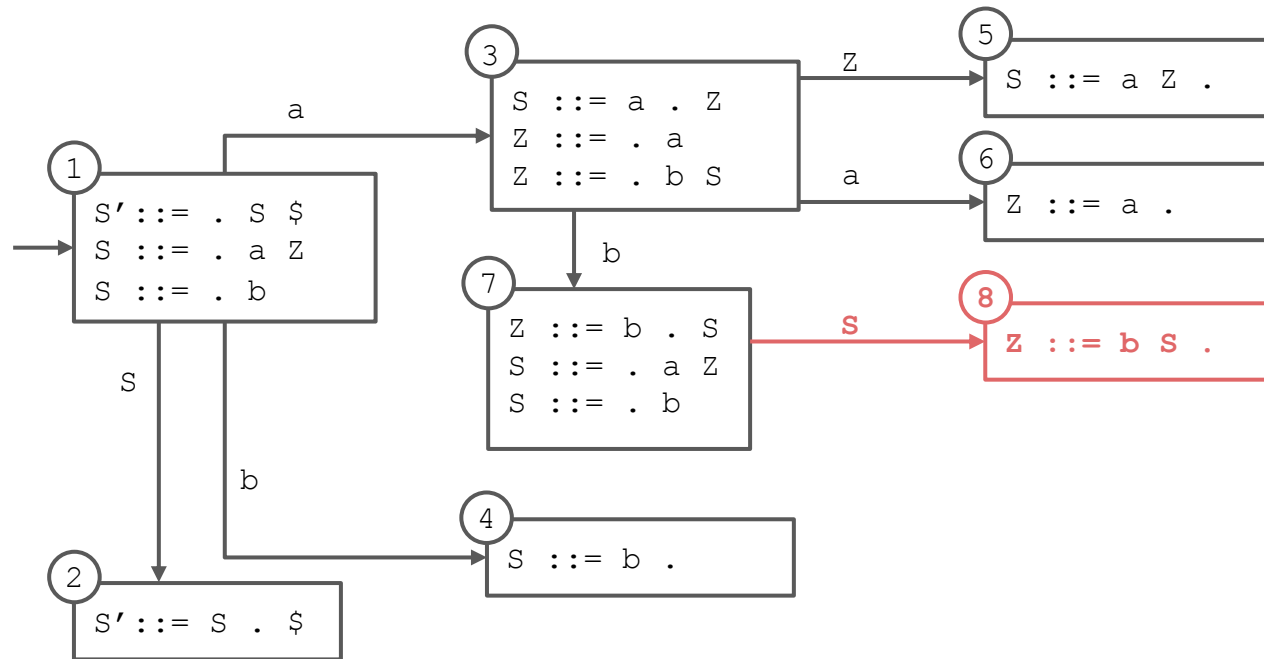
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



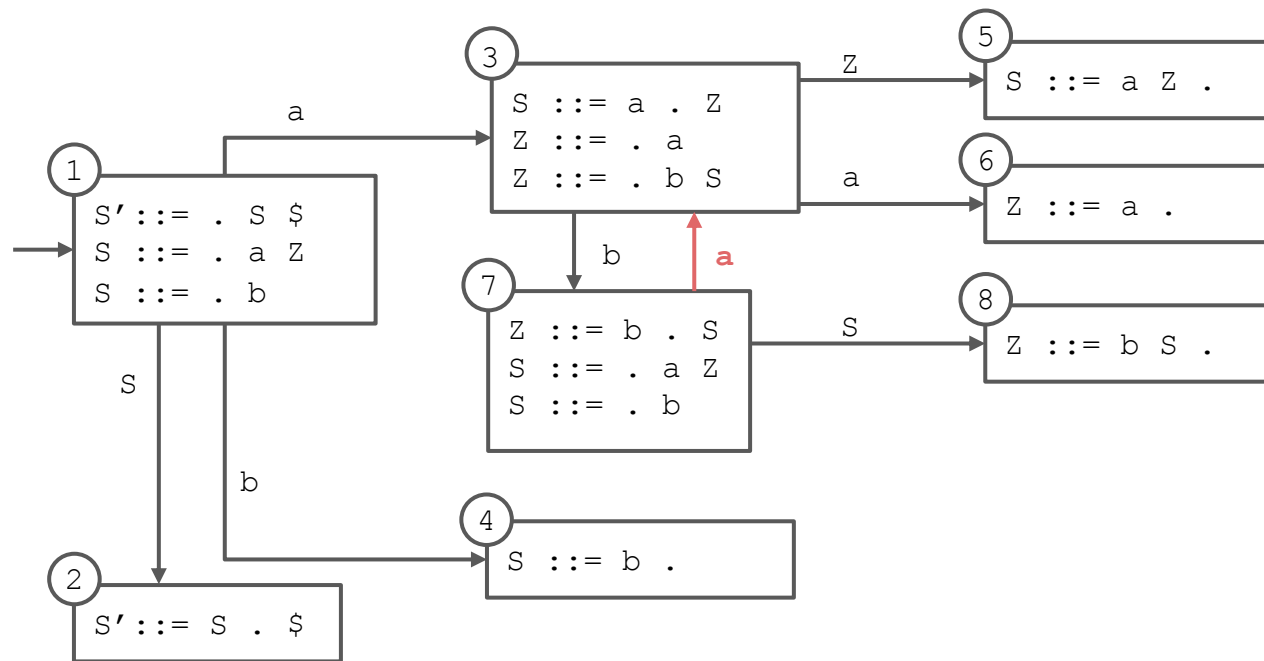
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



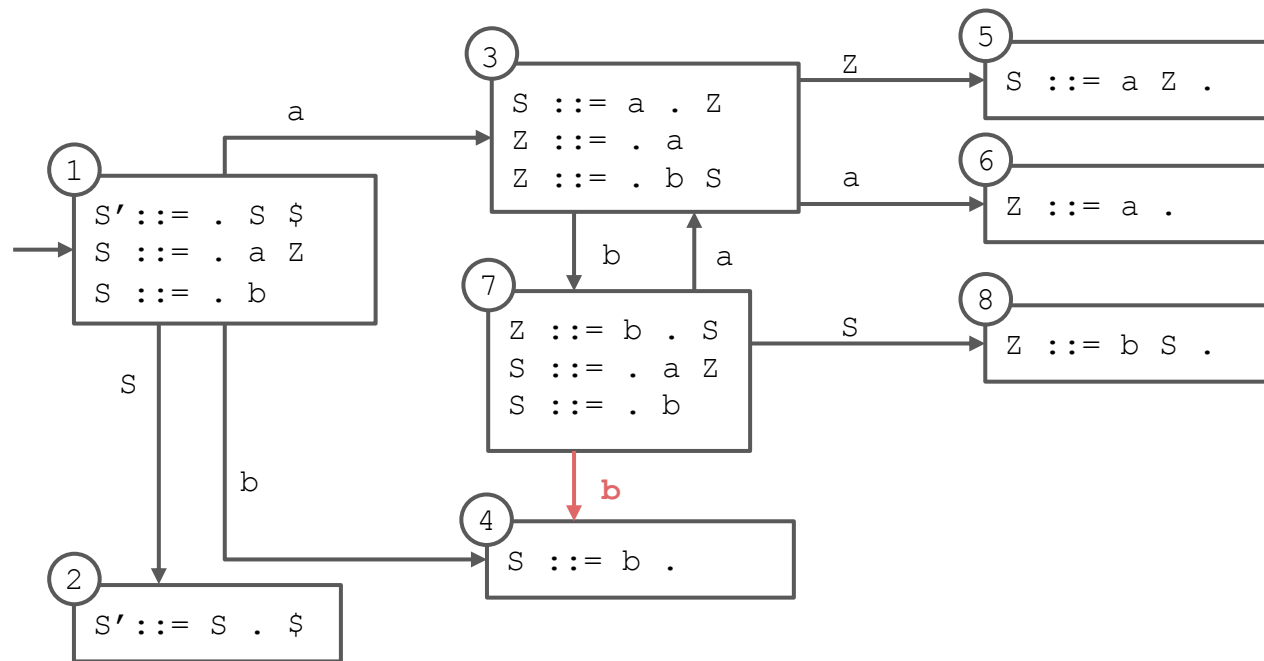
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



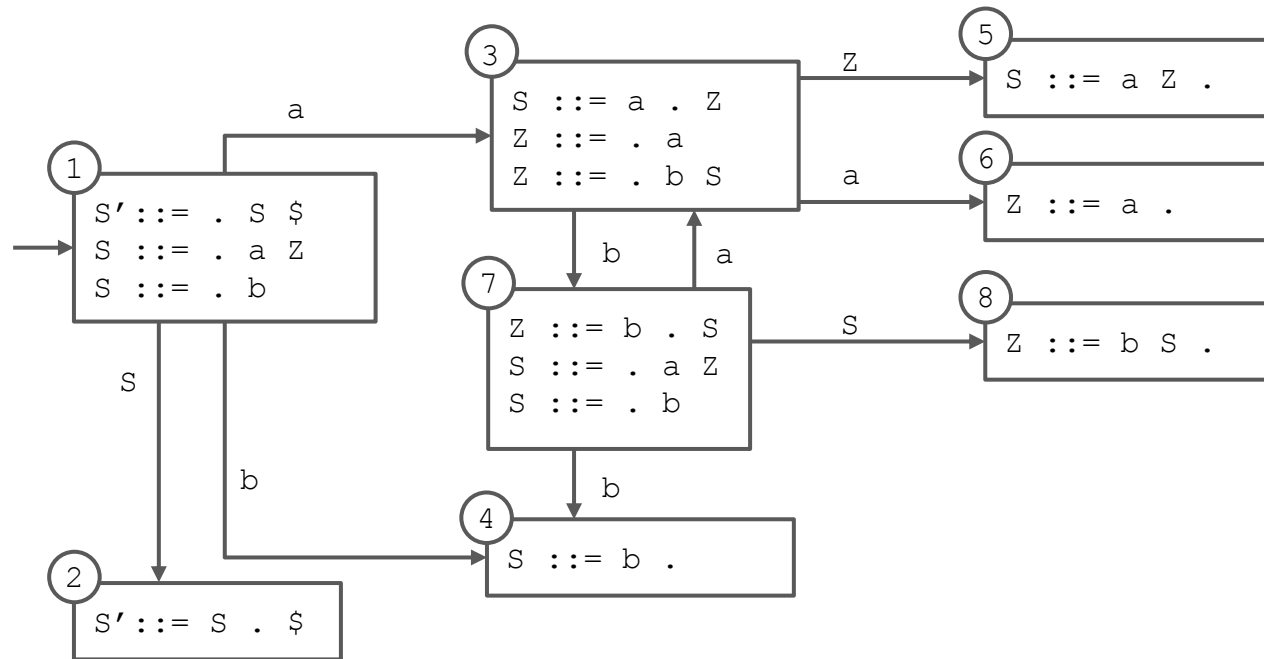
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



Completed State Diagram

- 0. $S' ::= S \$$
- 1. $S ::= a Z$
- 2. $S ::= b$
- 3. $Z ::= a$
- 4. $Z ::= b S$



Converted to Table

s# means “shift and enter state #”

- occurs when there is a transition on a terminal

r# means “reduce using production #”

- occurs when a state contains an item with the location at the end of the right-hand side

g# means “go to state #”

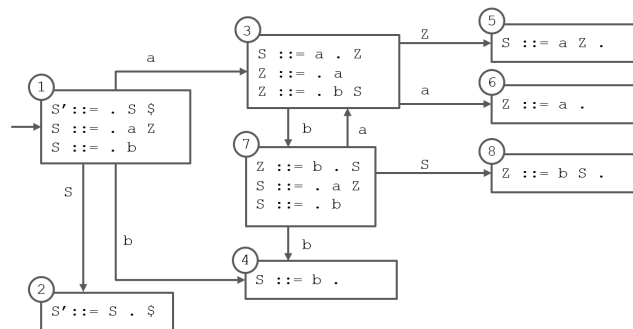
- occurs when there is a transition on a nonterminal

acc means “accept”

- occurs when the start symbol (S here) has been completed and there is no more input

STATE	ACTION			GOTO	
	a	b	\$	S	Z
1	s3	s4		g2	
2			acc		
3	s6	s7			g5
4	r2	r2	r2		
5	r1	r1	r1		
6	r3	r3	r3		
7	s3	s4		g8	
8	r4	r4	r4		

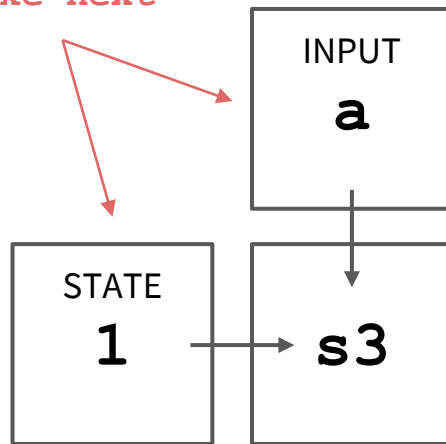
Parse Trace



STACK	INPUT	ACTION
\$ 1	a b a b b \$	

Parse Trace

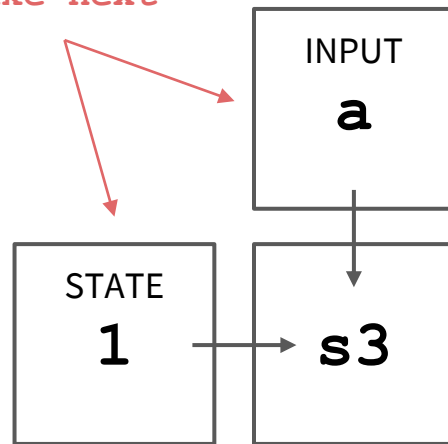
Row and column of table to
look up: decides what action
to take next



STACK	INPUT	ACTION
\$ 1 \$ 1 a	a b a b b \$ b a b b \$	SHIFT

Parse Trace

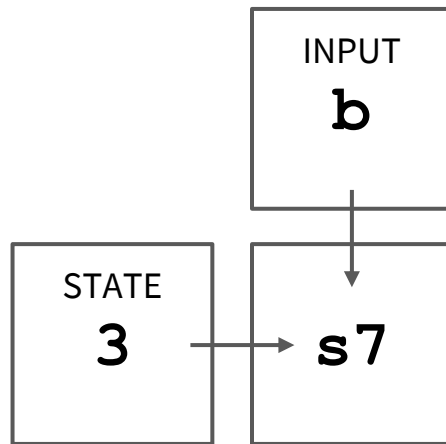
Row and column of table to
look up: decides what action
to take next



Shift, and enter state 3

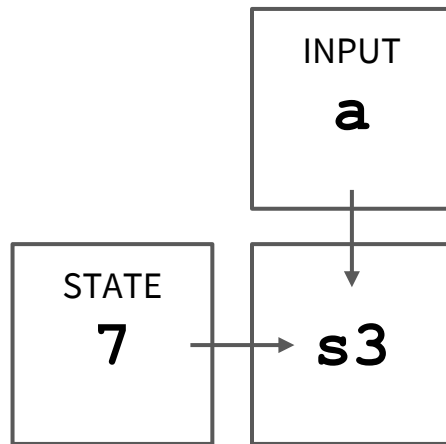
STACK	INPUT	ACTION
\$ 1 \$ 1 a 3	a b a b b \$ b a b b \$	SHIFT

Parse Trace



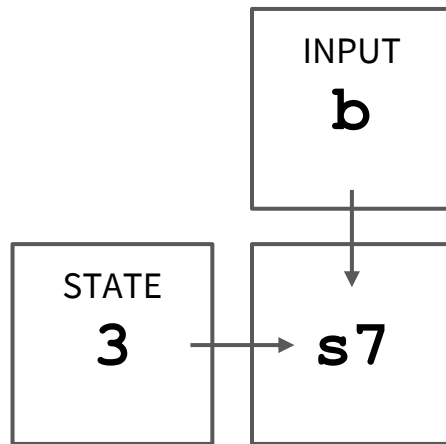
STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	

Parse Trace



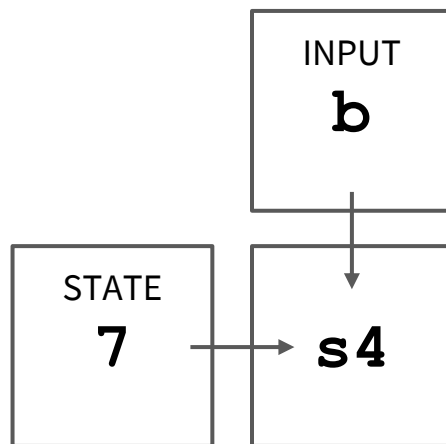
STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	

Parse Trace



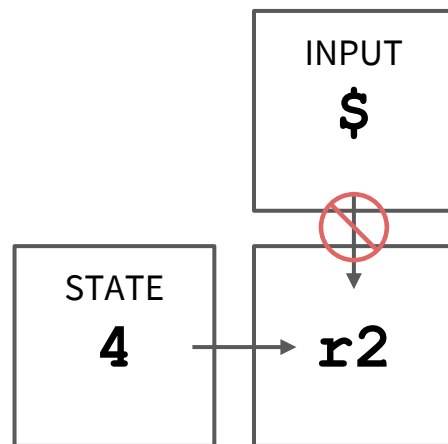
STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	

Parse Trace



STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	

Parse Trace

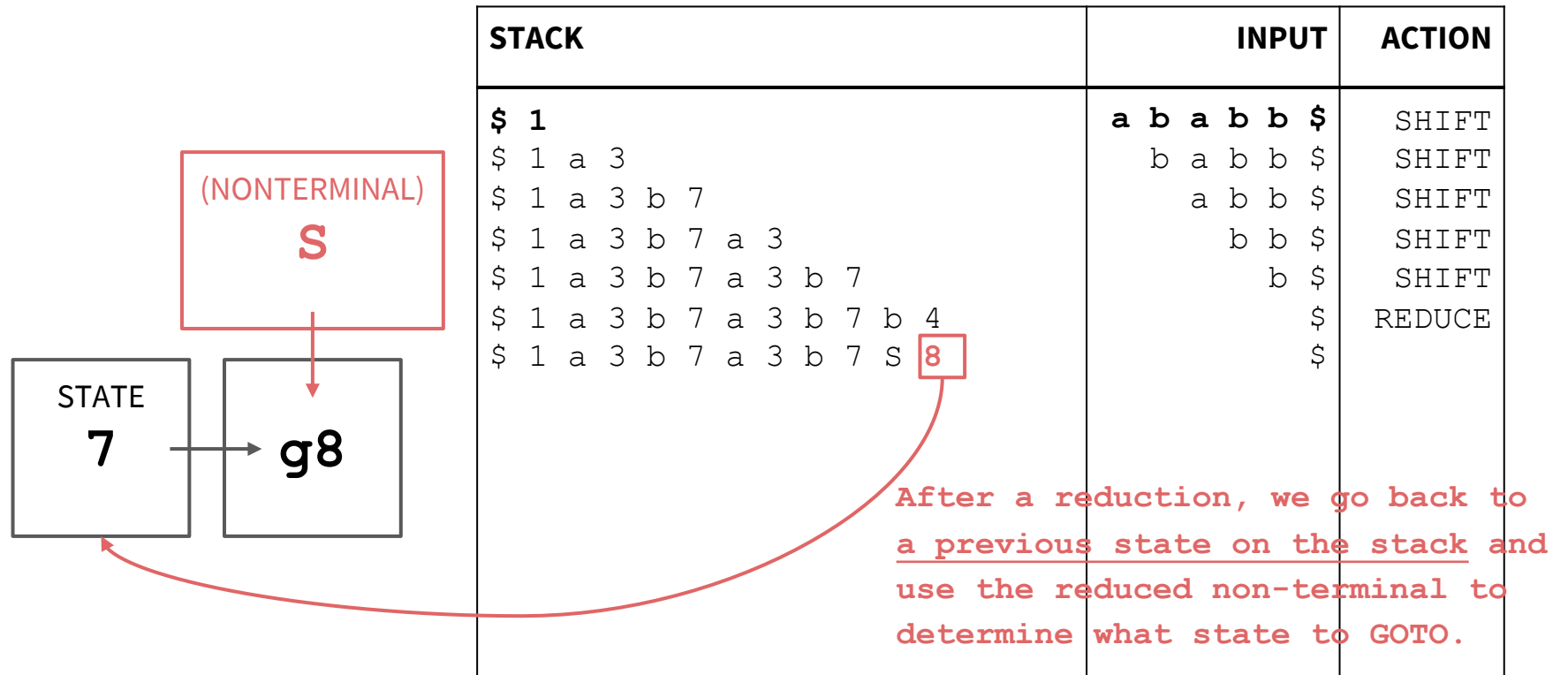


2. $S ::= b$

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S	\$	

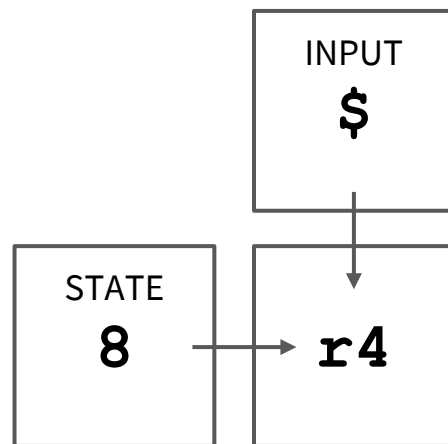
For LR(0), the input doesn't technically matter here

Parse Trace



This allows the parser to run in $O(n)$ time, since it doesn't have to re-evaluate the entire stack!

Parse Trace



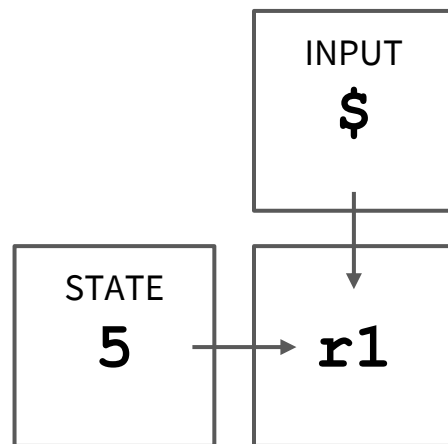
4. $Z ::= b S$

(and GOTO step:

s3 & Z $\rightarrow$ g5)

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 z 5	\$	

Parse Trace



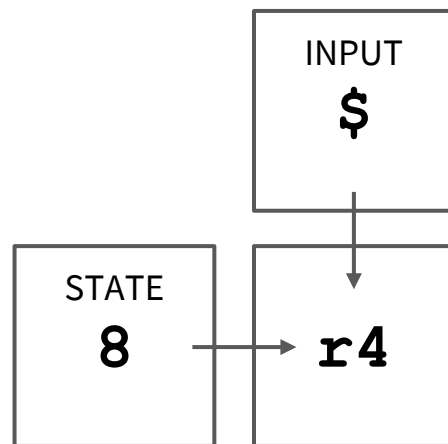
1. $S ::= a Z$

(and GOTO step:

s7 & S $\rightarrow$ g8)

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 Z 5	\$	REDUCE
\$ 1 a 3 b 7 S 8	\$	

Parse Trace



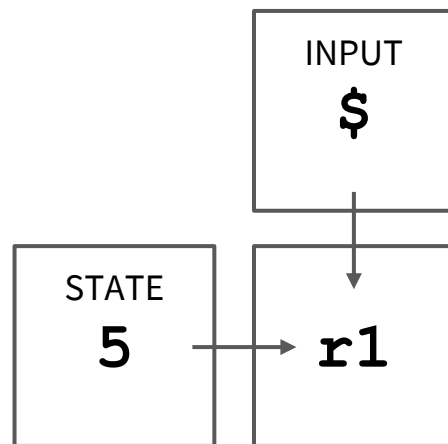
4. $Z ::= b S$

(and GOTO step:

s3 & Z -> g5)

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 Z 5	\$	REDUCE
\$ 1 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 Z 5	\$	REDUCE

Parse Trace



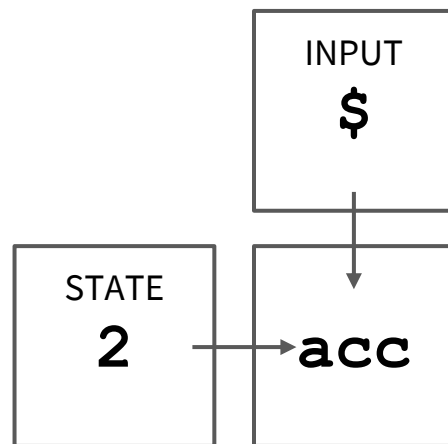
1. $S ::= a Z$

(and GOTO step:

s1 & S $\rightarrow$ g2)

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 Z 5	\$	REDUCE
\$ 1 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 Z 5	\$	REDUCE
\$ 1 S 2	\$	

Parse Trace



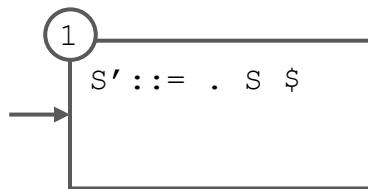
STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 Z 5	\$	REDUCE
\$ 1 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 Z 5	\$	REDUCE
\$ 1 S 2	\$	ACCEPT

Problem 2 (On Worksheet)

- 0. $S' ::= S \$$
- 1. $S ::= a X a$
- 2. $S ::= b X$
- 3. $X ::= c$
- 4. $X ::= S c$

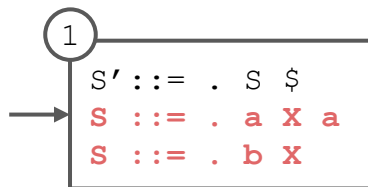
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a X a$
- 2. $S ::= b X$
- 3. $X ::= c$
- 4. $X ::= S c$



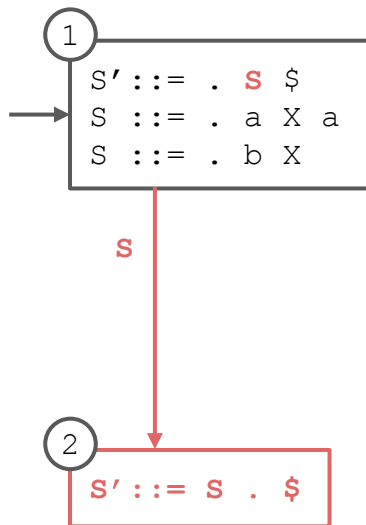
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a X a$
- 2. $S ::= b X$
- 3. $X ::= c$
- 4. $X ::= S c$



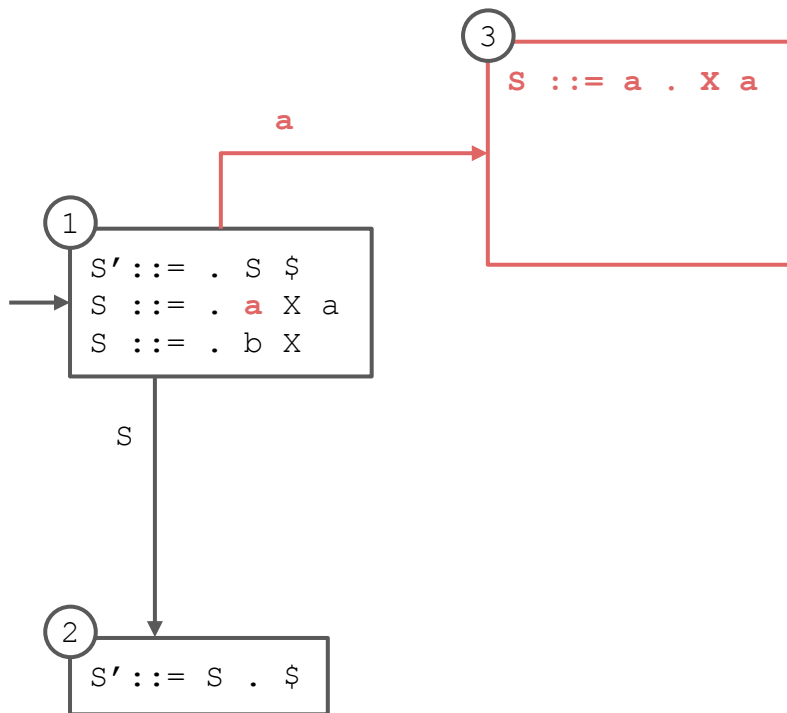
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a X a$
- 2. $S ::= b X$
- 3. $X ::= c$
- 4. $X ::= S c$



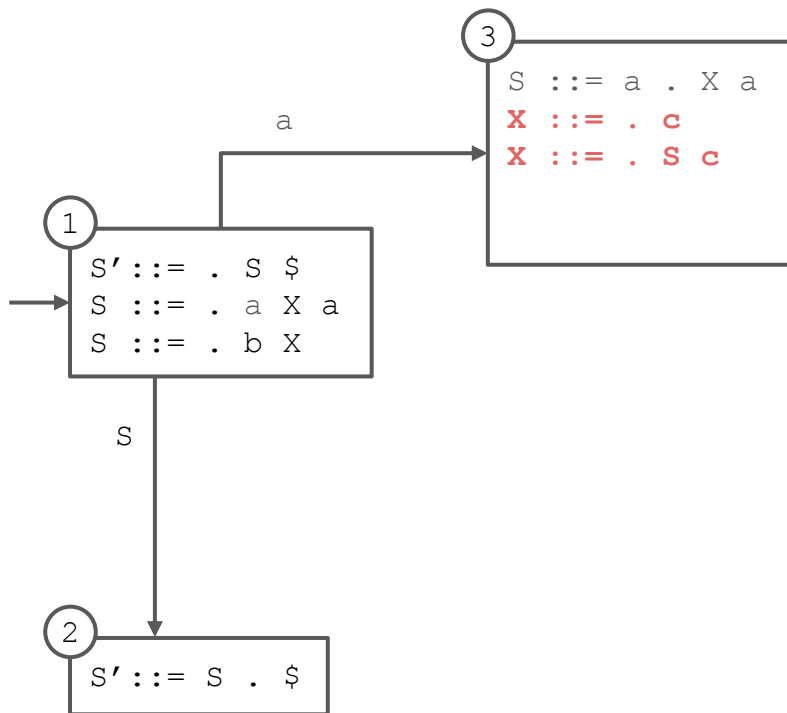
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a X a$
- 2. $S ::= b X$
- 3. $X ::= c$
- 4. $X ::= S c$



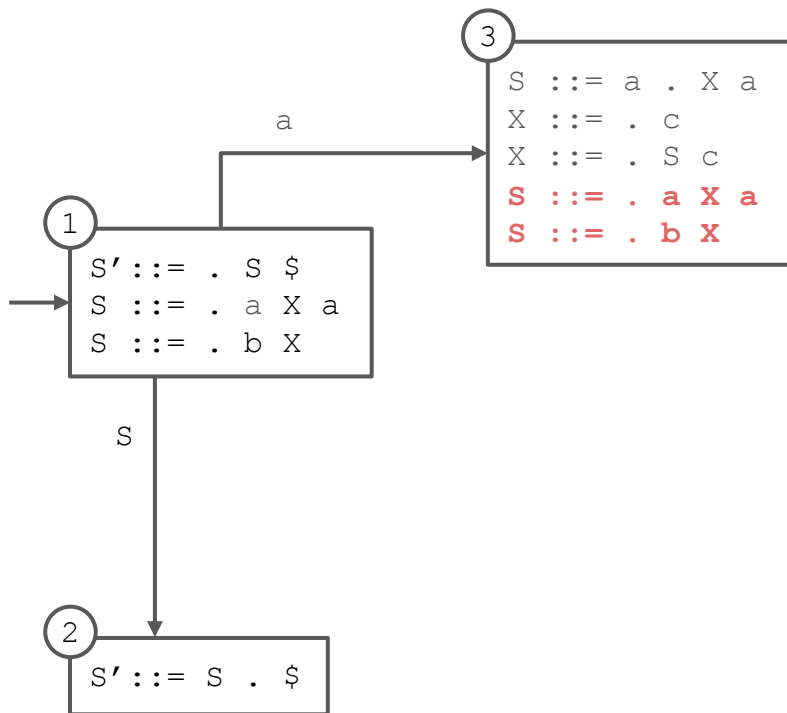
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a X a$
- 2. $S ::= b X$
- 3. $X ::= c$
- 4. $X ::= S c$



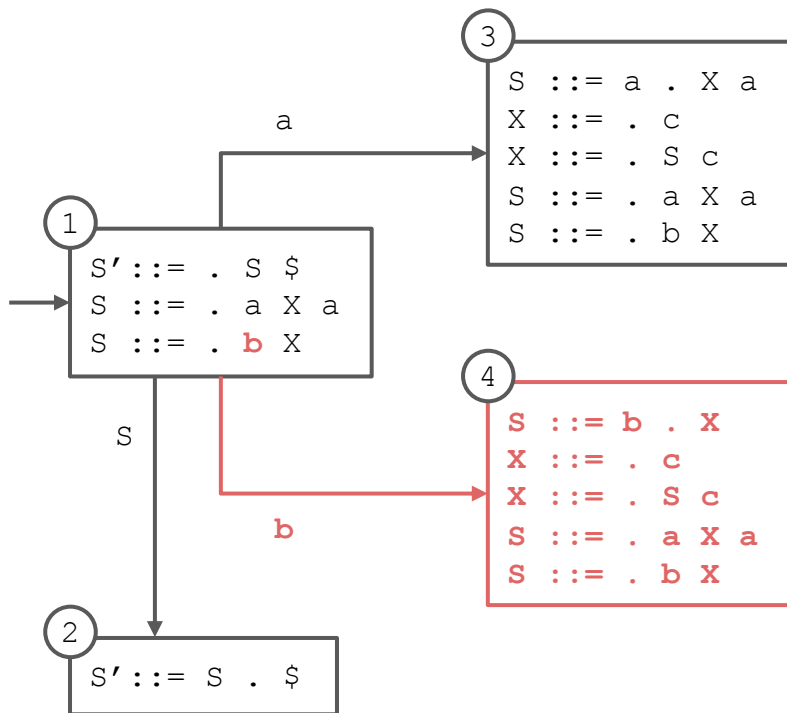
State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a X a$
- 2. $S ::= b X$
- 3. $X ::= c$
- 4. $X ::= S c$

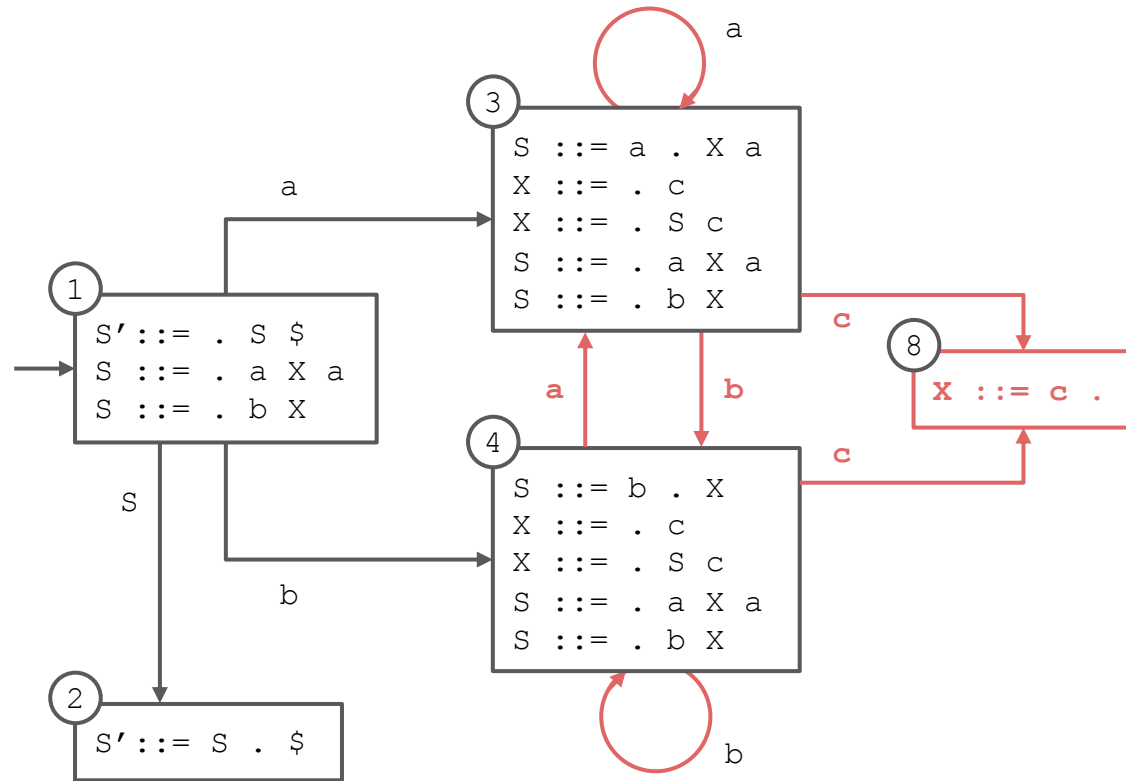


State Diagram Construction

- 0. $S' ::= S \$$
- 1. $S ::= a X a$
- 2. $S ::= b X$
- 3. $X ::= c$
- 4. $X ::= S c$



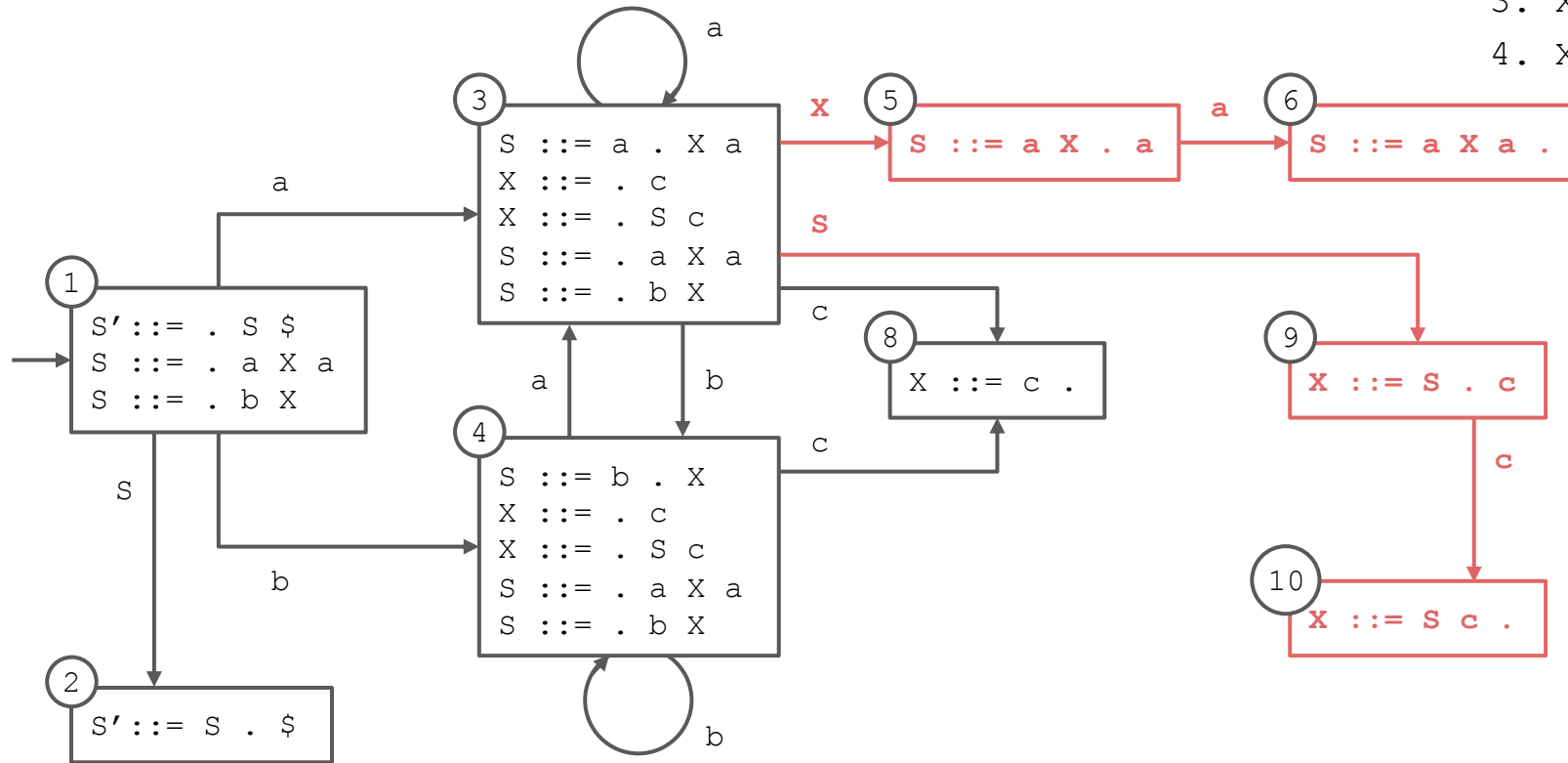
State Diagram Construction



0. $S' ::= S \$$
1. $S ::= a X a$
2. $S ::= b X$
3. $X ::= c$
4. $X ::= S c$

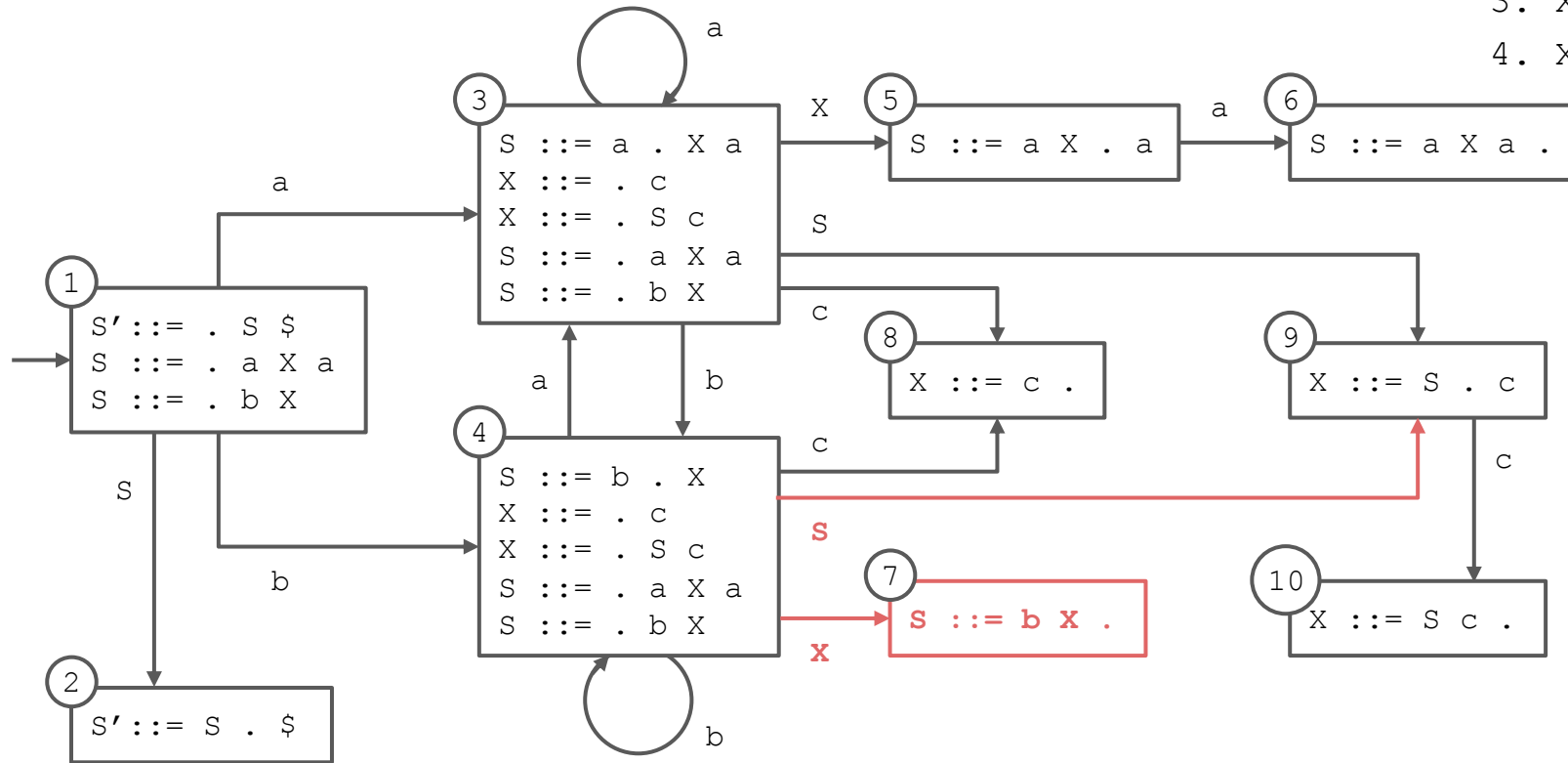
State Diagram Construction

0. $S' ::= S \$$
1. $S ::= a X a$
2. $S ::= b X$
3. $X ::= c$
4. $X ::= S c$



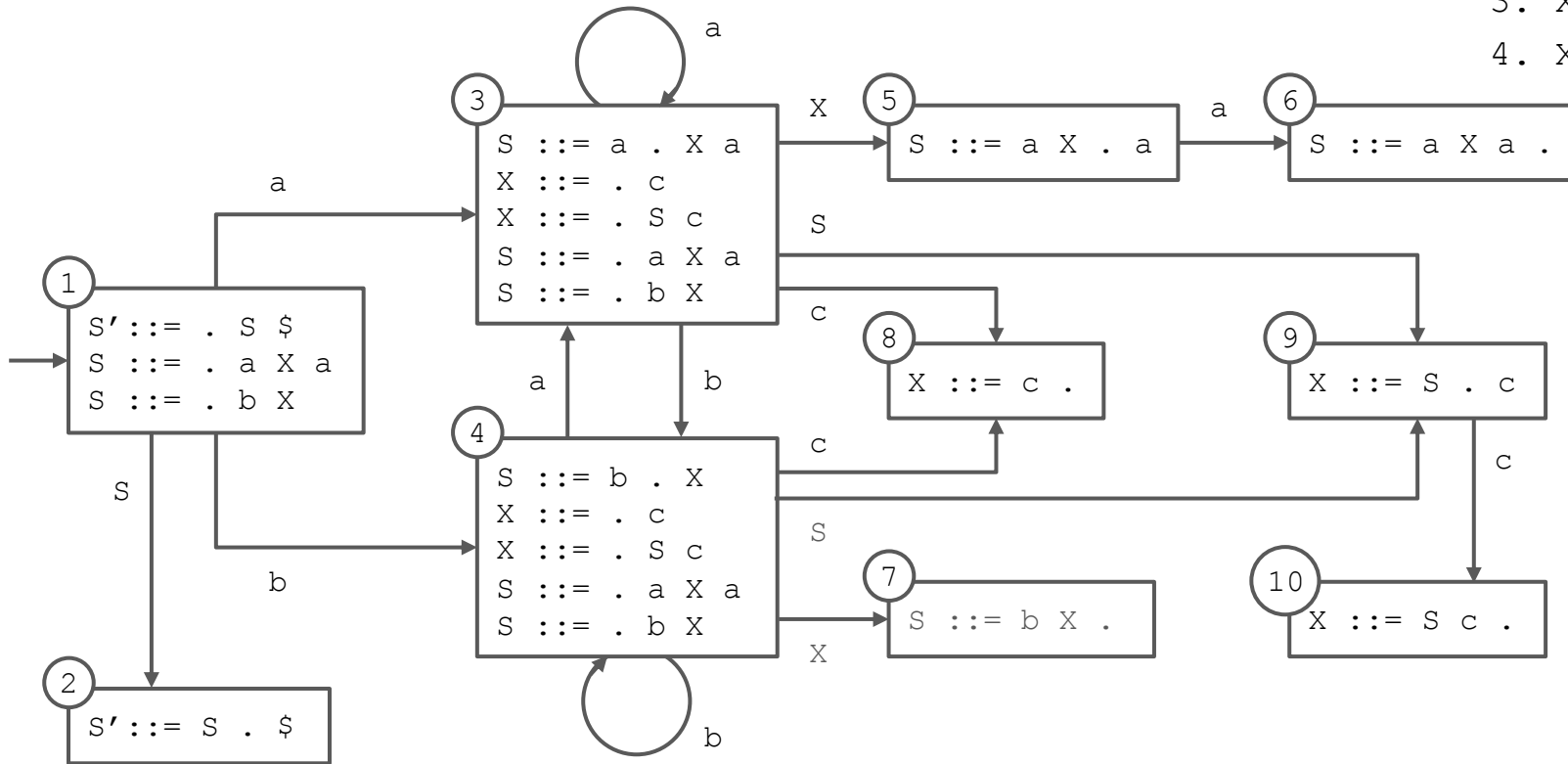
State Diagram Construction

0. $S' ::= S \$$
1. $S ::= a X a$
2. $S ::= b X$
3. $X ::= c$
4. $X ::= S c$



Completed State Diagram

0. $S' ::= S \$$
1. $S ::= a X a$
2. $S ::= b X$
3. $X ::= c$
4. $X ::= S c$



Converted to Table

s# means “shift and enter state #”

- occurs when there is a transition on a terminal

r# means “reduce using production #”

- occurs when a state contains an item with the location at the end of the right-hand side

g# means “go to state #”

- occurs when there is a transition on a nonterminal

acc means “accept”

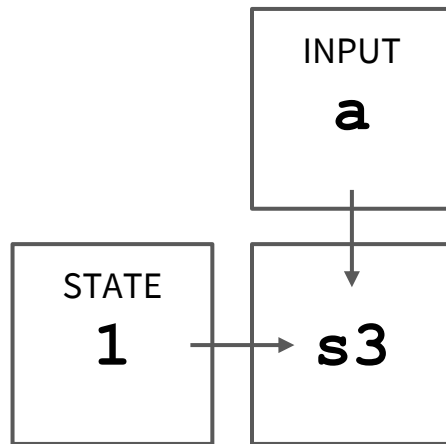
- occurs when the start symbol (S here) has been completed and there is no more input

STATE	ACTION				GOTO	
	a	b	c	\$	S	X
1	s3	s4			g2	
2				acc		
3	s3	s4	s8		g9	g5
4	s3	s4	s8		g9	g7
5	s6					
6	r1	r1	r1	r1		
7	r2	r2	r2	r2		
8	r3	r3	r3	r3		
9			s10			
10	r4	r4	r4	r4		

Parse Trace

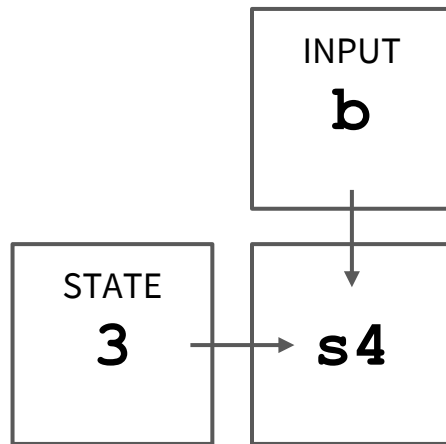
STACK	INPUT	ACTION
\$ 1	a b c c a \$	

Parse Trace



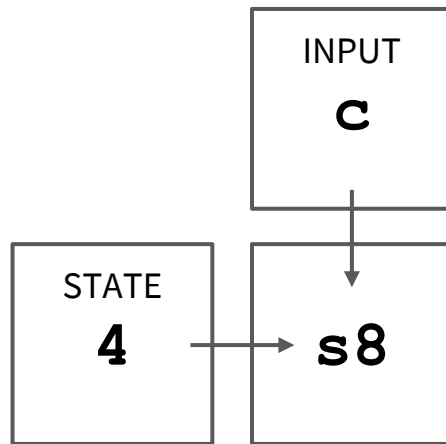
STACK	INPUT	ACTION
\$ 1 \$ 1 a 3	a b c c a \$ b c c a \$	SHIFT

Parse Trace



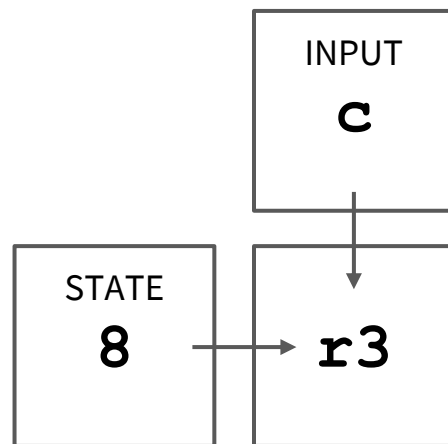
STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	

Parse Trace



STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	

Parse Trace



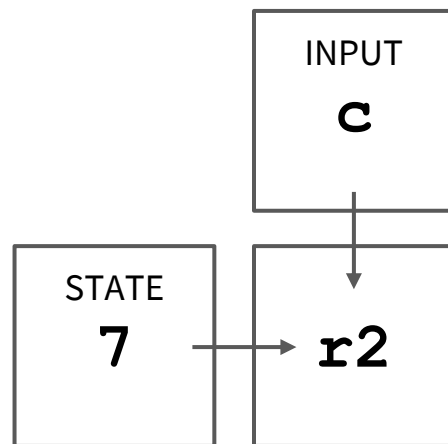
3. **X ::= c**

(and GOTO step:

s4 & X -> g7)

STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	

Parse Trace



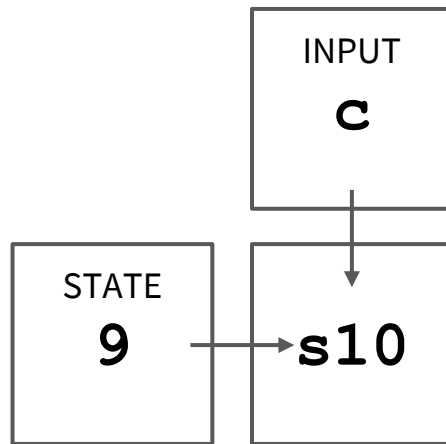
2. $S ::= b X$

(and GOTO step:

s3 & S $\rightarrow$ g9)

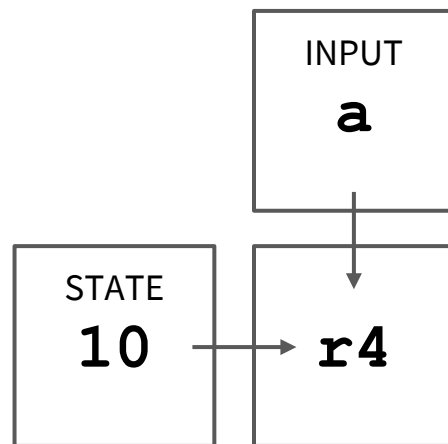
STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	

Parse Trace



STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	

Parse Trace



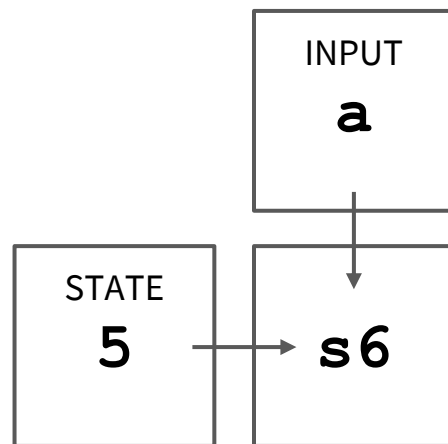
4. **X ::= S c**

(and GOTO step:

s3 & X -> g5)

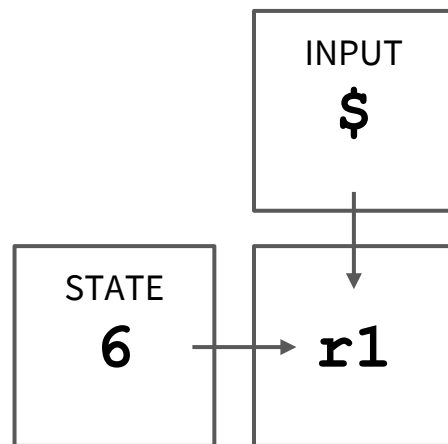
STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	REDUCE
\$ 1 a 3 X 5	a \$	

Parse Trace



STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	REDUCE
\$ 1 a 3 X 5	a \$	SHIFT
\$ 1 a 3 X 5 a 6	\$	

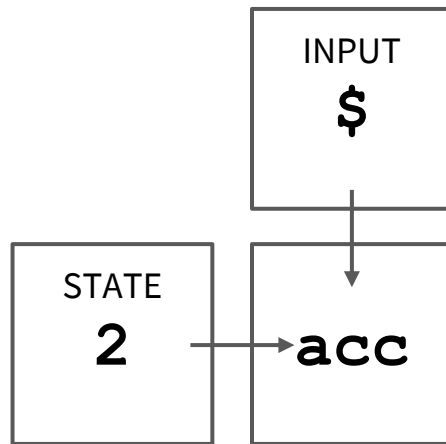
Parse Trace



1. $S ::= a X a$
 (and GOTO step:
 s1 & S $\rightarrow$ g2)

STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	REDUCE
\$ 1 a 3 X 5	a \$	SHIFT
\$ 1 a 3 X 5 a 6	\$	REDUCE
\$ 1 S 2	\$	

Parse Trace



STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	REDUCE
\$ 1 a 3 X 5	a \$	SHIFT
\$ 1 a 3 X 5 a 6	\$	REDUCE
\$ 1 S 2	\$	ACCEPT