

# CSE 401/M501 – Compilers

Dataflow Analysis

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# Administrivia (1)

- Codegen due next Tuesday (for 401)
  - CSE M 501 compilers with extensions due end of last week of classes. No separate codegen submission for M 501.
  - `System.out.println(17)` from main should be working now (no kidding!)
    - Well, maybe by tonight or tomorrow morning, but get going 😊
  - Easy enough to finish by the deadline if you ***don't*** put it off
- Once codegen is done we'll do an overall evaluation of your compiler, all phases, and rerun a comprehensive set of tests. This final evaluation is *the* major part of the project grade. So you need to fix any remaining bugs, all the way back to the scanner! 🐛🕷️🐜🦋🐞

# Administrivia (2)

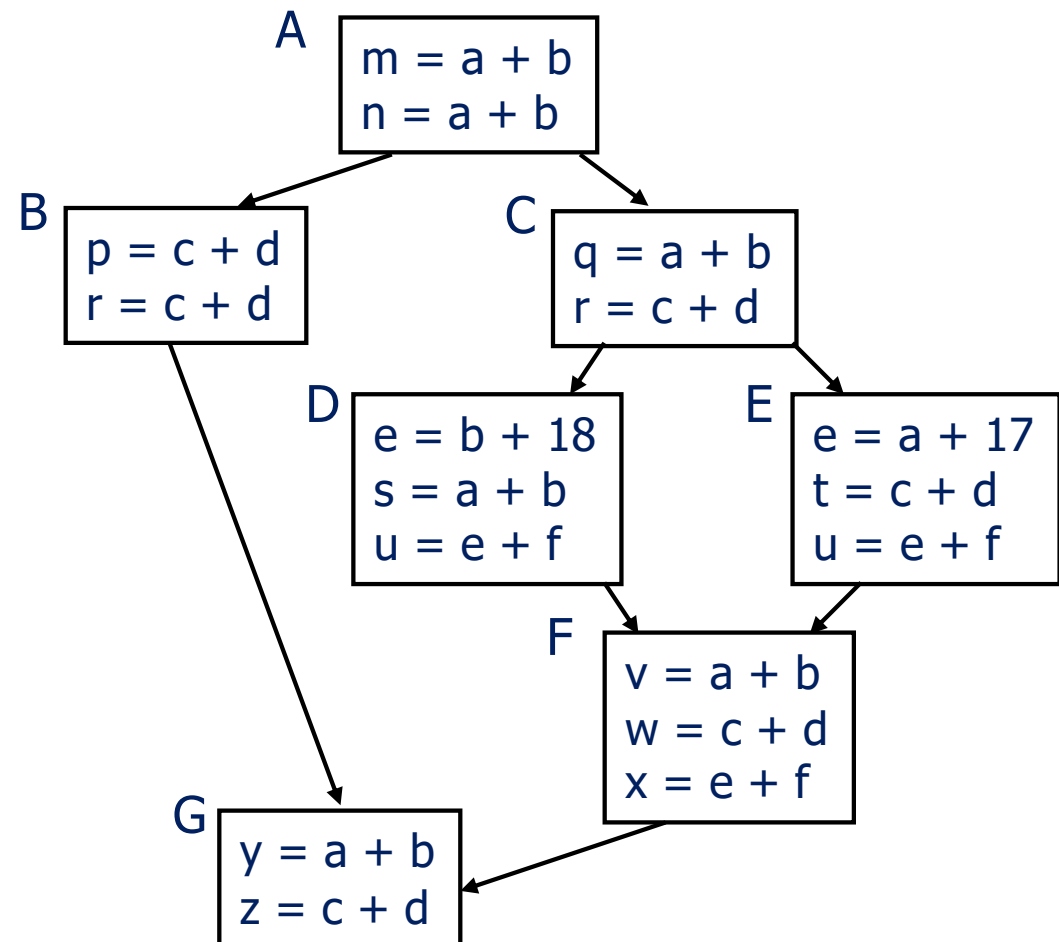
- CSE M 501 groups: please let instructor know what you plan to do for project extras if you haven't done that yet
  - Mail to `cse401-staff[at]cs` please
  - M501 groups turn in codegen + additions at end of quarter, not a separate codegen step
- Everyone: we're closing down regrades on old work shortly and will copy grades to canvas. If there are any lingering questions, send mail to `cse401-staff[at]cs` asap

# Agenda

- Dataflow analysis: a framework and algorithm for many common compiler analyses
- Initial example: dataflow analysis for common subexpression elimination
- Then: other analysis problems that work in the same framework
- Some of these are the same analysis and optimizations we've seen, but more formally and with details

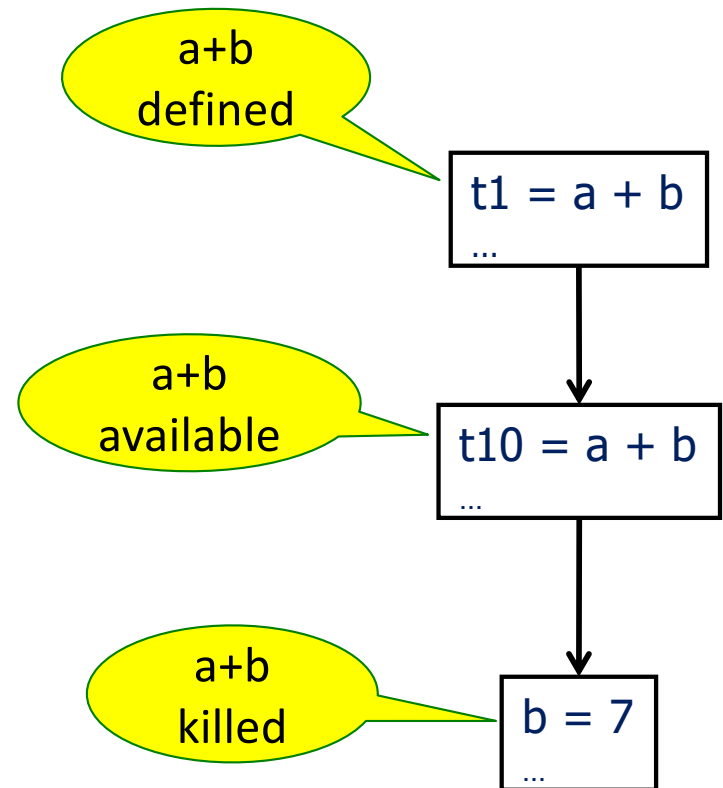
# Common Subexpression Elimination

- Goal: use dataflow analysis to find common subexpressions
- Idea: calculate *available expressions* at beginning of each basic block
- Avoid re-evaluation of an available expression – use a copy operation
  - Simple inside a single block; more complex dataflow analysis used across blocks



# “Available” and Other Terms

- An expression  $e$  is *defined* at point  $p$  in the CFG\* if its value is computed at  $p$ 
  - Sometimes called *definition site*
- An expression  $e$  is *killed* at point  $p$  if one of its operands is defined at  $p$ 
  - Sometimes called *kill site*
- An expression  $e$  is *available* at point  $p$  if every path leading to  $p$  contains a prior definition of  $e$  and  $e$  is not killed between that definition and  $p$



\*CFG = Control Flow Graph

# Available Expression Sets

- To compute available expressions, for each block  $b$ , define
  - $AVAIL(b)$  – the set of expressions available on entry to  $b$
  - $NKILL(b)$  – the set of expressions not killed in  $b$ 
    - i.e., all expressions in the program *except* for those killed in  $b$
  - $DEF(b)$  – the set of expressions defined in  $b$  and not subsequently killed in  $b$

# Computing Available Expressions

- $AVAIL(b)$  for block  $b$  is the set
$$AVAIL(b) = \bigcap_{x \in \text{preds}(b)} (\text{DEF}(x) \cup (\text{AVAIL}(x) \cap \text{NKILL}(x)))$$
  - $\text{preds}(b)$  is the set of  $b$ 's predecessors in the CFG
  - The set of expressions available on entry to  $b$  is the set of expressions that were available at the end of *every* predecessor basic block  $x$
  - The expressions available on exit from block  $b$  are those defined in  $b$  or available on entry to  $b$  and not killed in  $b$
- This gives a system of simultaneous equations – a dataflow problem



# Computing Available Expressions

- Big Picture
  - Build control-flow graph
  - Calculate initial local data –  $DEF(b)$  and  $NKILL(b)$ 
    - This only needs to be done once for each block  $b$  and depends only on the statements in  $b$
  - Iteratively calculate  $AVAIL(b)$  by repeatedly evaluating equations until nothing changes
    - Another fixed-point algorithm

# Computing DEF and NKILL (1)

- First, figure out which expressions are killed in each block (i.e., clobbered by some assignment later in that block)
- For each block  $b$  with operations  $o_1, o_2, \dots, o_n$

KILLED =  $\emptyset$       // variables killed (later) in  $b$ , not expressions

DEF( $b$ ) =  $\emptyset$

for  $k = n$  to 1    // note: working back to front

    assume  $o_k$  is “ $x = y + z$ ”

    add  $x$  to KILLED

    if ( $y \notin$  KILLED and  $z \notin$  KILLED)

        add “ $y + z$ ” to DEF( $b$ )    // i.e., neither  $y$  nor  $z$  killed  
        //    after this point in  $b$

...

# Computing DEF and NKILL (2)

- After computing DEF and KILLED for a block  $b$ , compute set of all expressions in the program not killed in  $b$

$NKILL(b) = \{ \text{all expressions} \}$

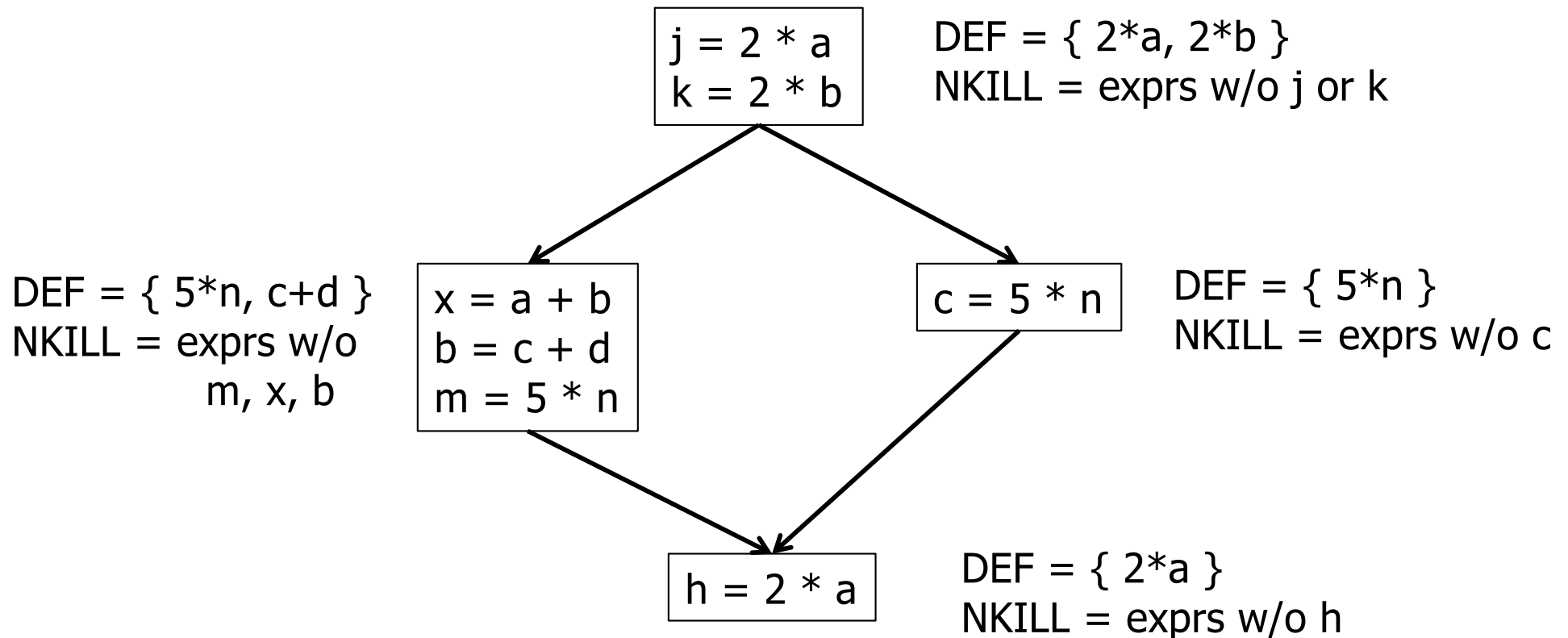
for each expression  $e$

for each variable  $v \in e$

if  $v \in KILLED$  then

$NKILL(b) = NKILL(b) - e$

# Example: Compute DEF and NKILL



# Computing Available Expressions

Once  $DEF(b)$  and  $NKILL(b)$  are computed for all blocks  $b$

Worklist = { all blocks  $b_k$  }

while (Worklist  $\neq \emptyset$ )

    remove a block  $b$  from Worklist

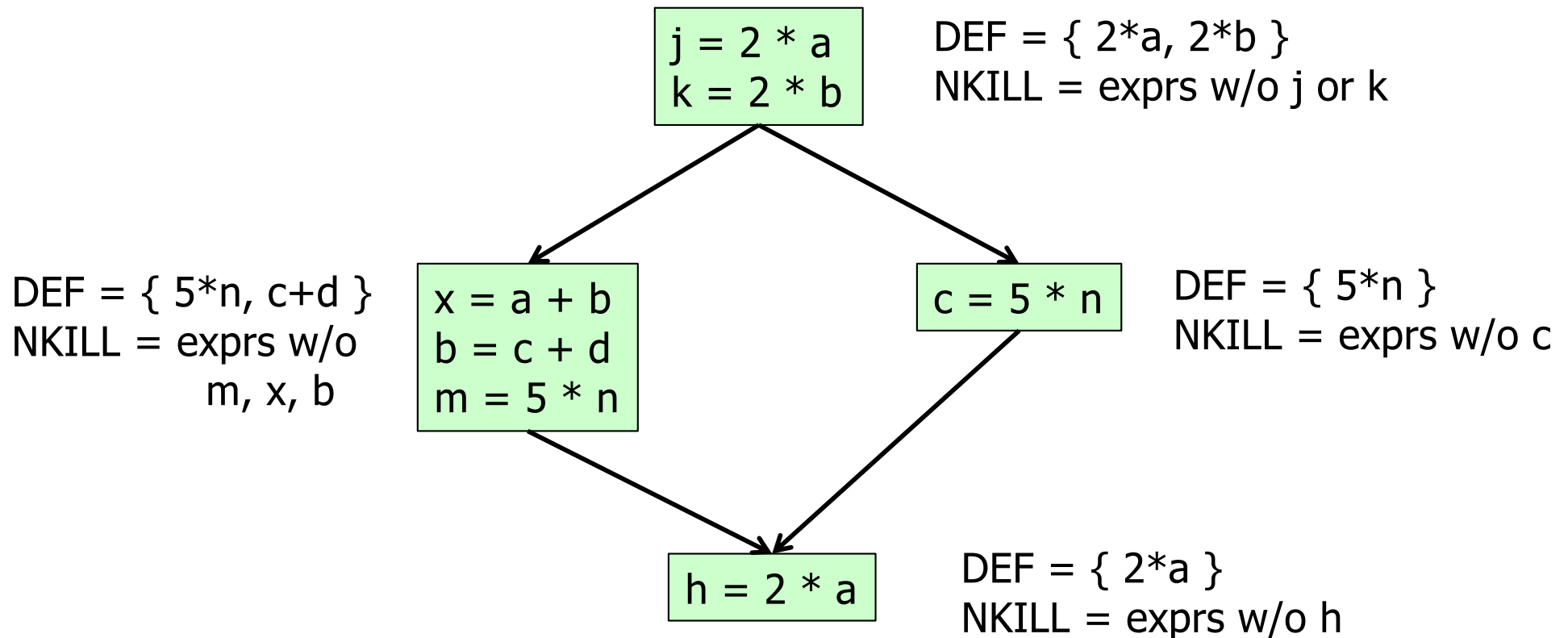
    recompute  $AVAIL(b)$


    if  $AVAIL(b)$  changed


        Worklist = Worklist  $\cup$  successors( $b$ )

# Example: Find Available Expressions

$$AVAIL(b) = \bigcap_{x \in \text{preds}(b)} (\text{DEF}(x) \cup (\text{AVAIL}(x) \cap \text{NKILL}(x)))$$

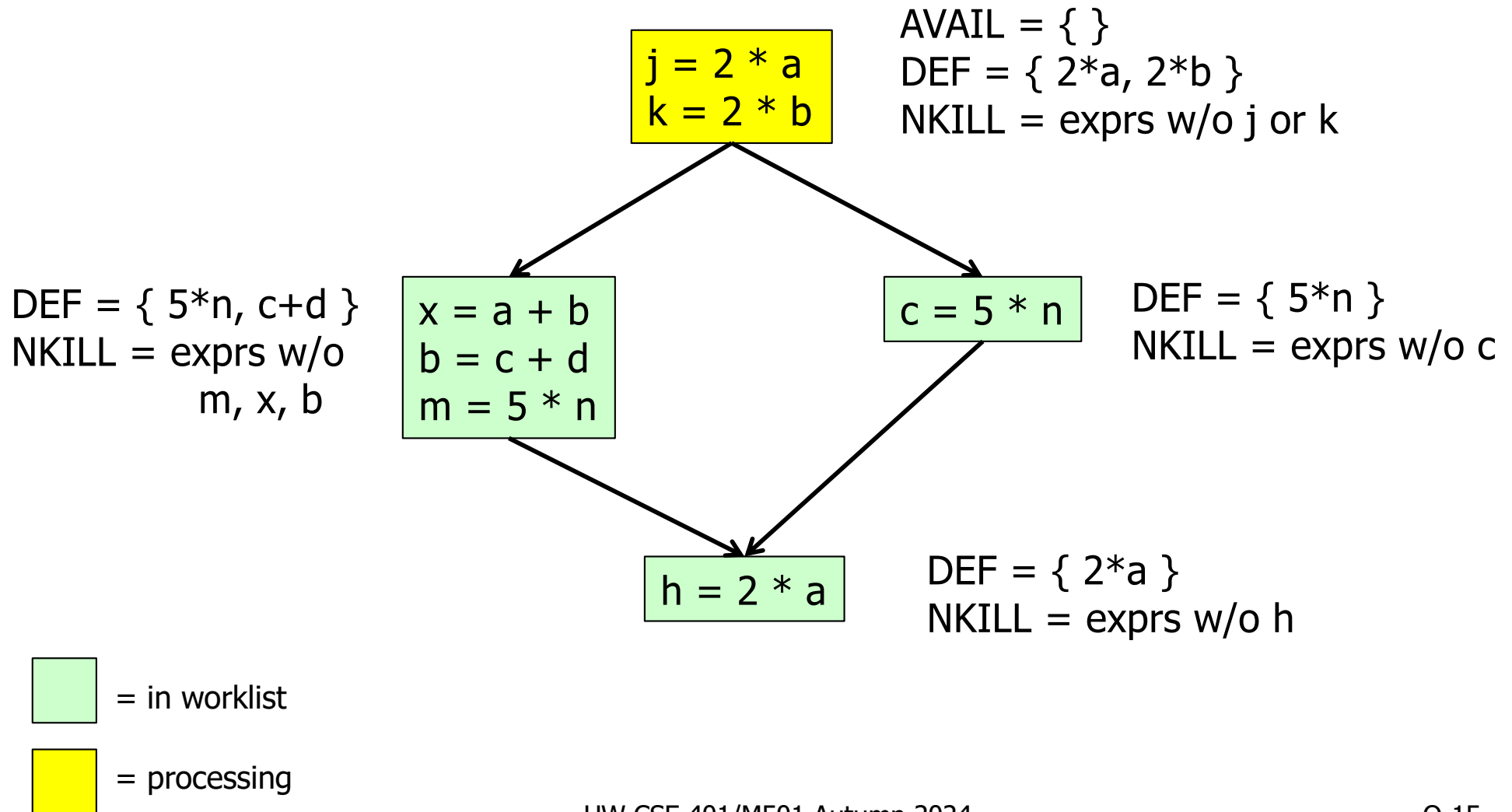


 = in worklist

 = processing

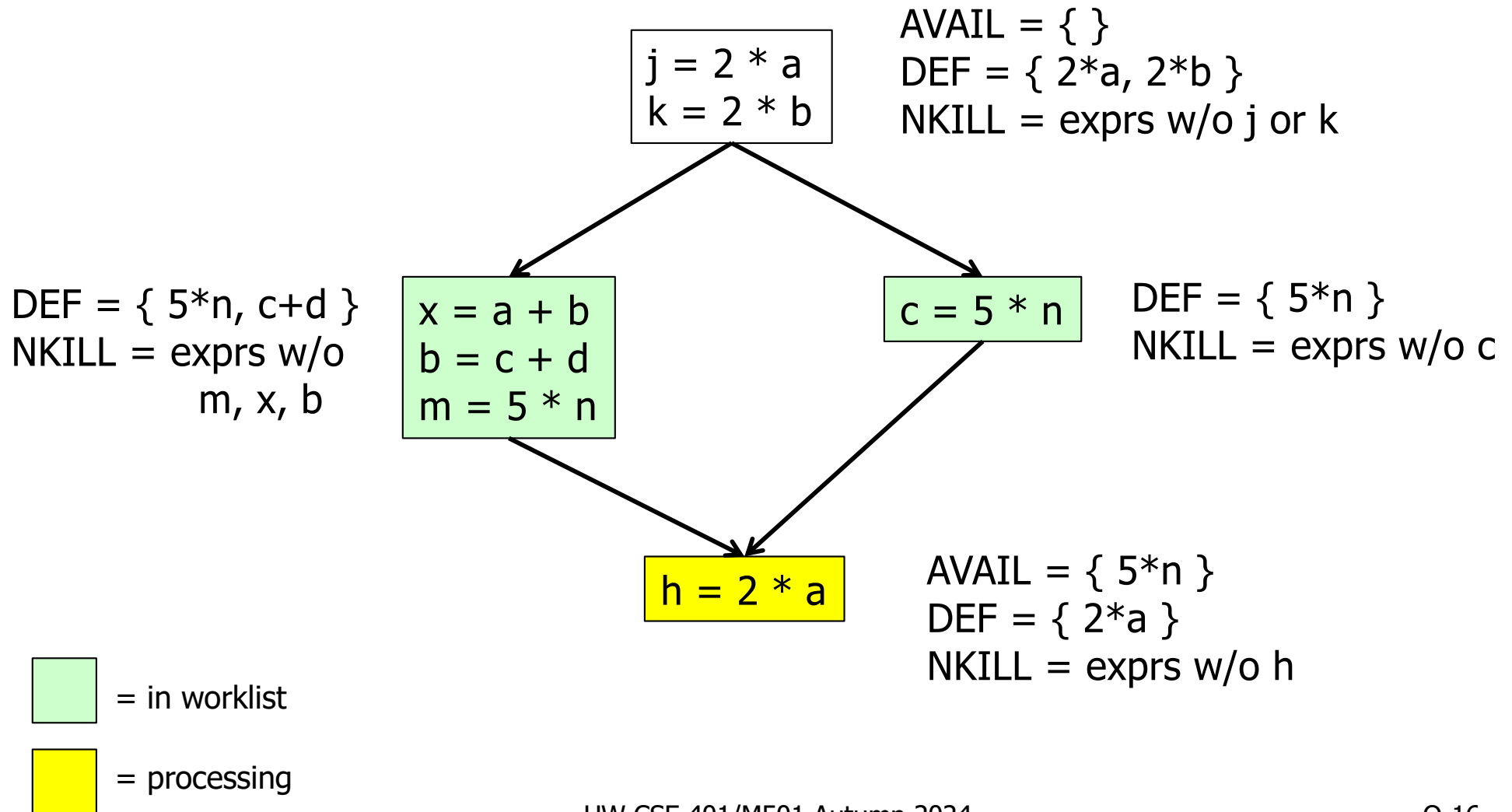
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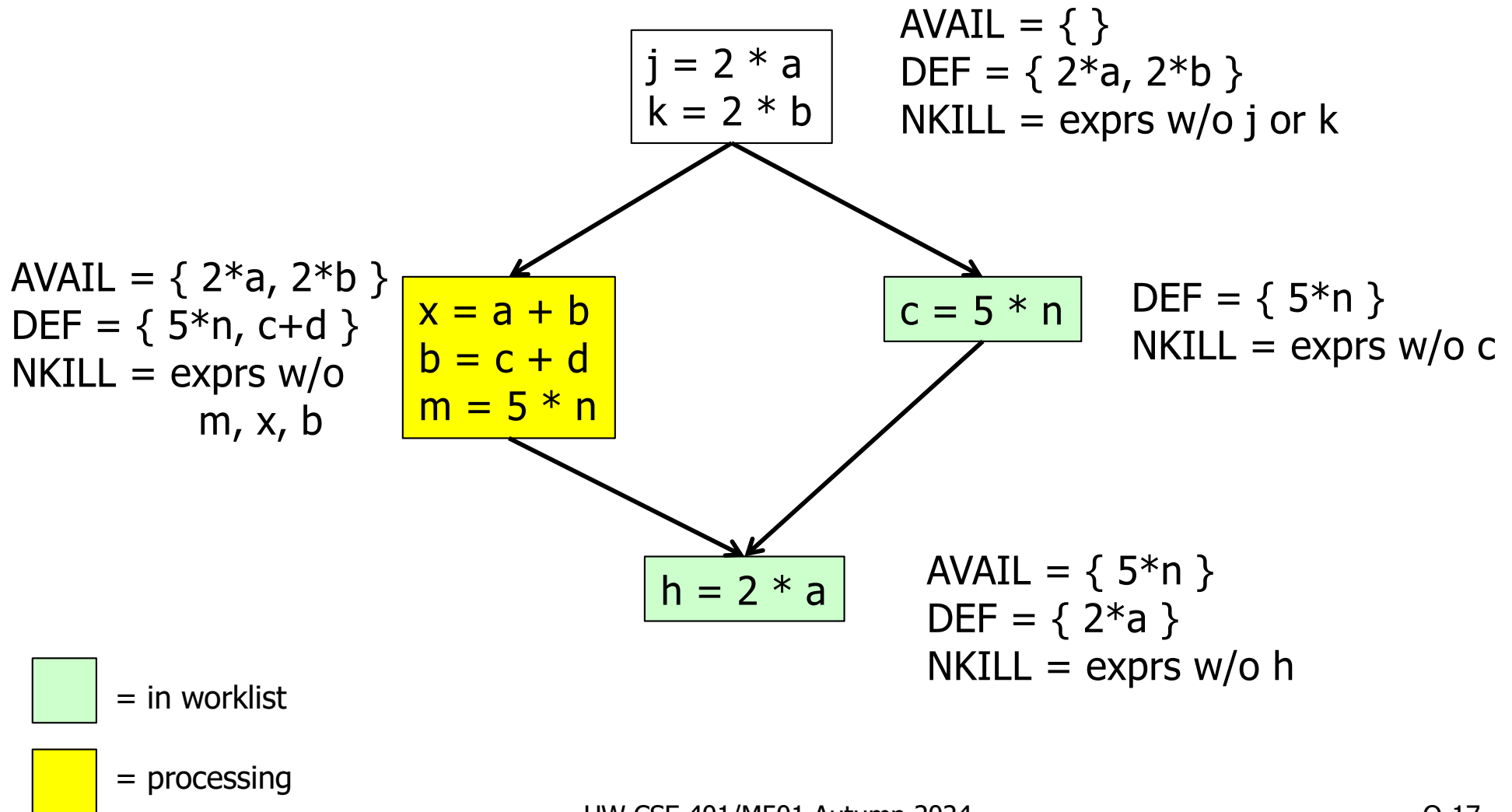
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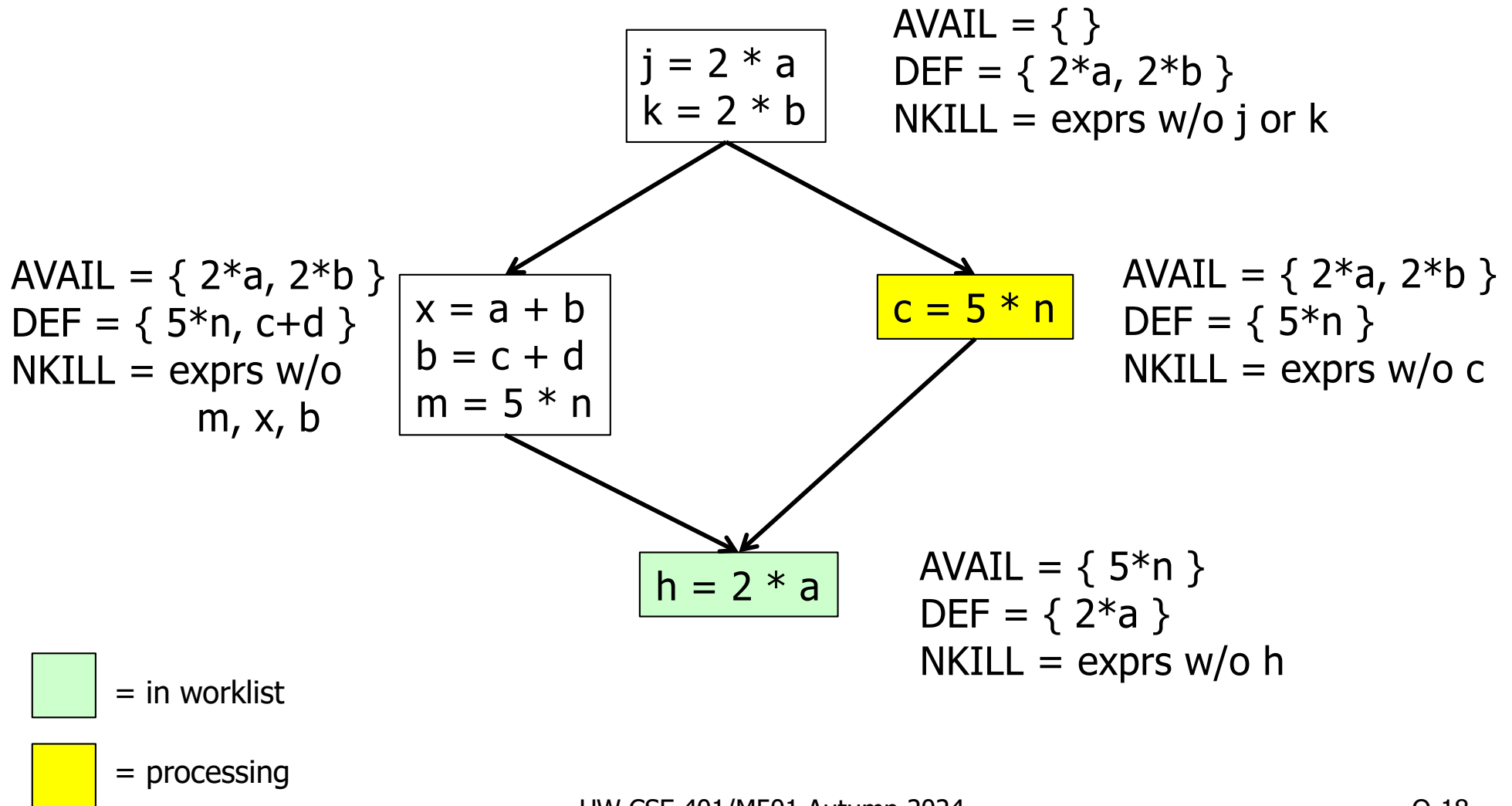
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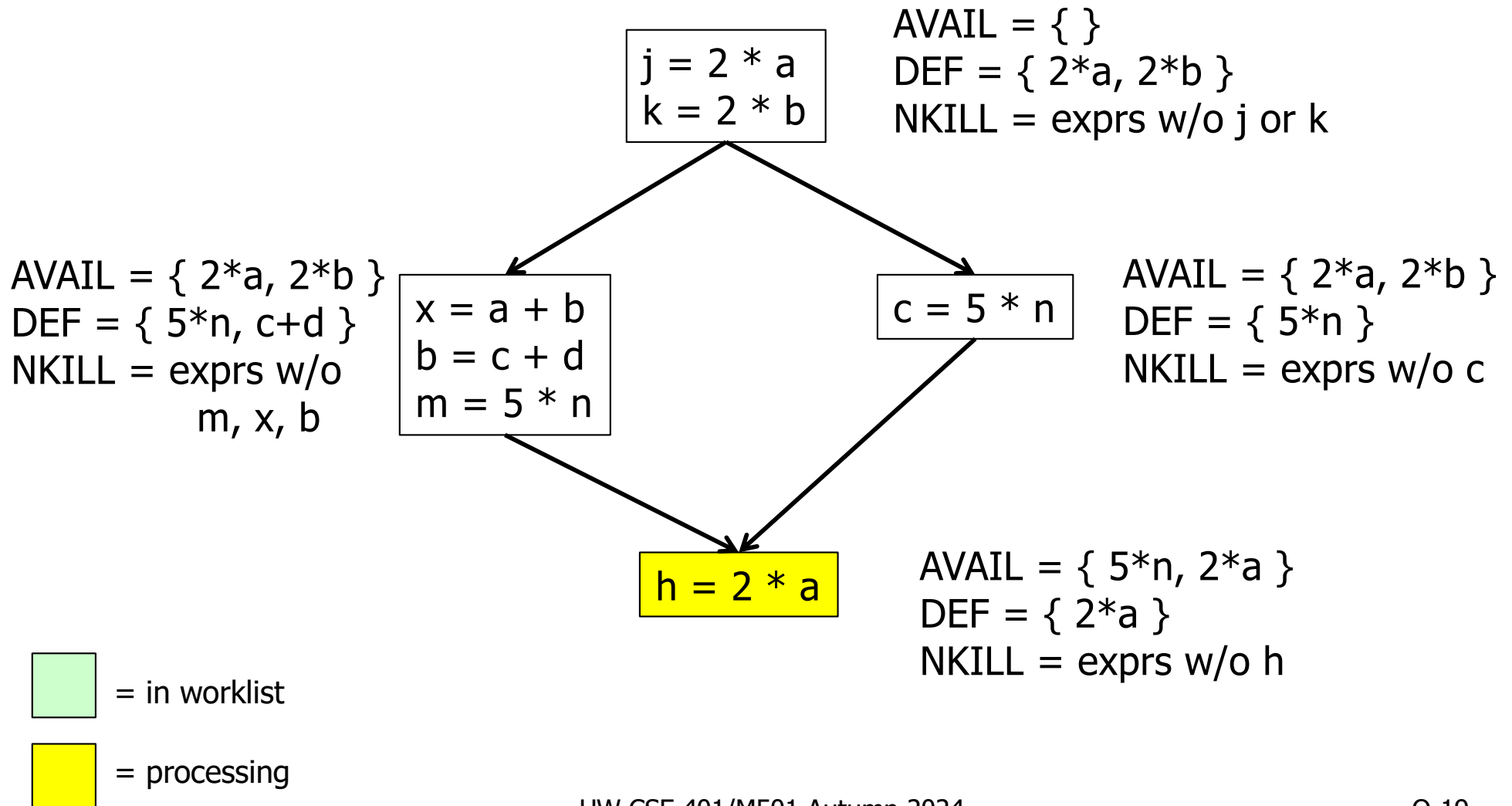
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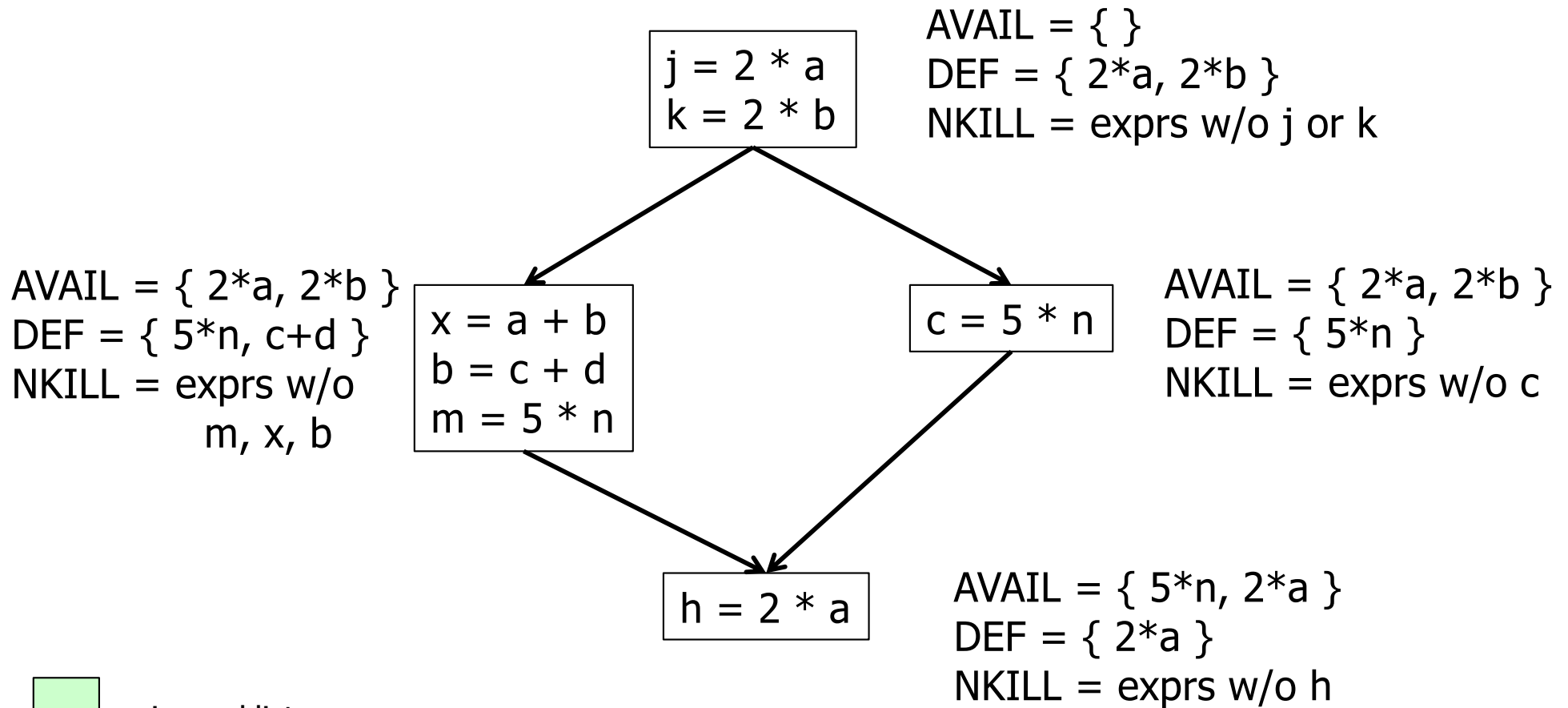
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
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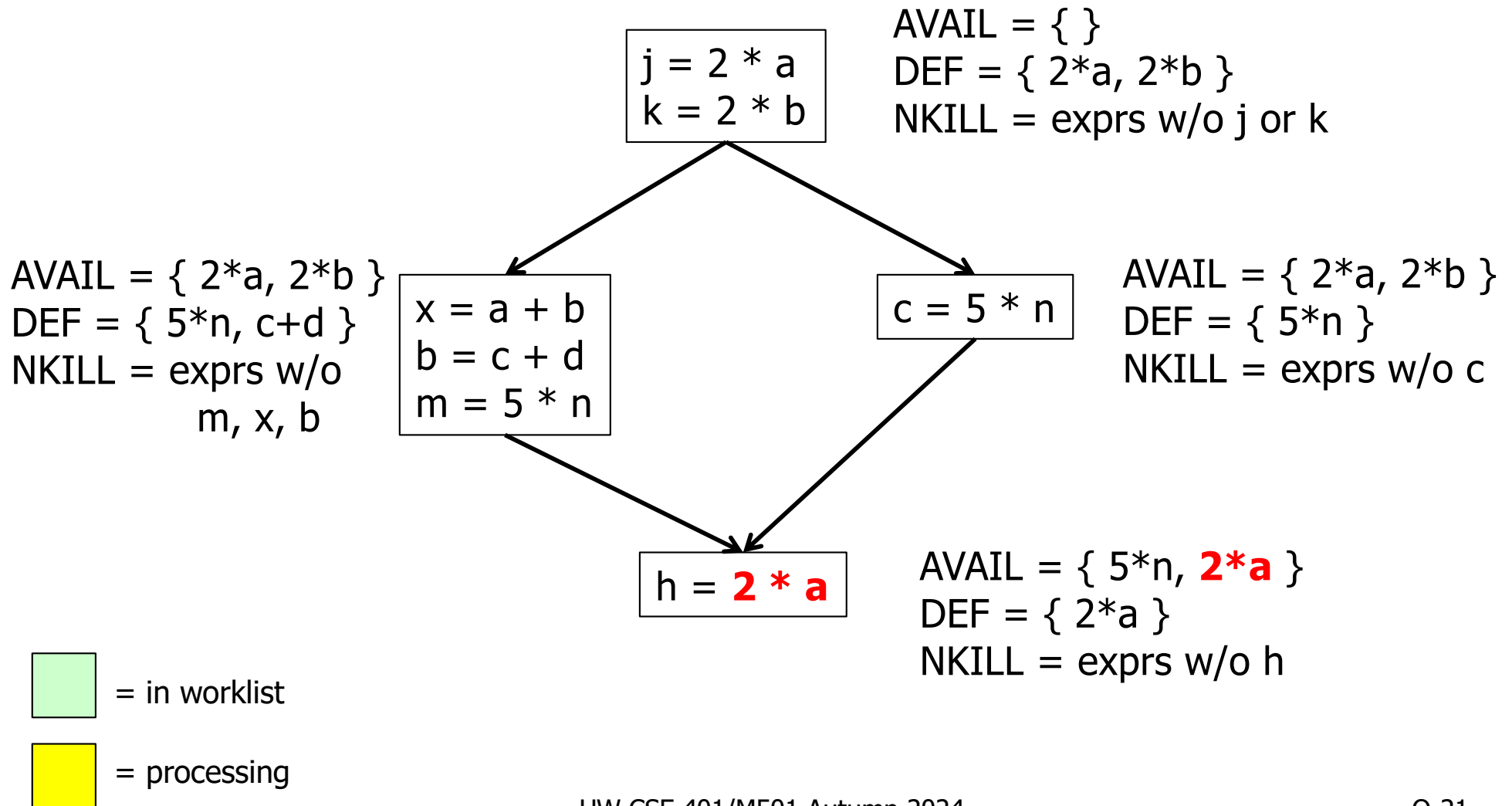
 = in worklist

 = processing

And the common subexpression is???

# Example: Find Available Expressions

$$AVAIL(b) = \cap_{x \in \text{preds}(b)} (\text{DEF}(x) \cup (\text{AVAIL}(x) \cap \text{NKILL}(x)))$$



# Example: Find Available Expressions

$$\text{AVAIL}(b) = \bigcap_{x \in \text{preds}(b)} (\text{DEF}(x) \cup (\text{AVAIL}(x) \cap \text{NKILL}(x)))$$

- Termination?
  - Always
  - $\text{AVAIL}(b)$  initially empty for all blocks  $b$
  - In equation above, DEF & NKILL are unchanging, and adding to  $\text{AVAIL}(x)$  can't shrink  $\text{AVAIL}(b)$
  - Only a finite number of exprs in the program, so the alg is again climbing a finite n-cube; can't climb forever
- Order of worklist removals?
  - Any will work
  - Some are faster than others; e.g., if CFG is a DAG, then go in topological order (which would have been faster on the previous example)

# Dataflow analysis

- *Available expressions* is an example of a *dataflow analysis* problem
- Many similar problems can be expressed in a similar framework
- Only the first part of the story – once we've discovered facts, we then need to use them to improve code

# Characterizing Dataflow Analysis

- All of these algorithms involve sets of facts about each basic block  $b$ 
  - $IN(b)$  – facts true on entry to  $b$
  - $OUT(b)$  – facts true on exit from  $b$
  - $GEN(b)$  – facts created and not killed in  $b$
  - $KILL(b)$  – facts killed in  $b$
- These are related by the equation
$$OUT(b) = GEN(b) \cup (IN(b) - KILL(b))$$
  - Solve this iteratively for all blocks
  - Sometimes information propagates forward; sometimes backward
    - But will reach correct solution (fixed point) regardless of order in which blocks are considered



# Example: Live Variable Analysis

- A variable  $v$  is *live* at point  $p$  iff there is *any* path from  $p$  to a use of  $v$  along which  $v$  is not redefined
- Some uses:
  - Register allocation – only live variables need a register
  - Eliminating useless stores – if a variable is not live at the store location, then the stored variable will never be used
  - Detecting uses of uninitialized variables – if live at declaration (before initialization) then it might be used uninitialized
  - Improve SSA construction – only need  $\Phi$ -functions for variables that are live in a block (later)

# Liveness Analysis Sets

- For each block  $b$ , define
  - $\text{use}[b]$  = variable used in  $b$  before any def
  - $\text{def}[b]$  = variable defined in  $b$
  - $\text{in}[b]$  = variables live on entry to  $b$
  - $\text{out}[b]$  = variables live on exit from  $b$

# Equations for Live Variables

- Given the preceding definitions, we have

$$\text{in}[b] = \text{use}[b] \cup (\text{out}[b] - \text{def}[b])$$

$$\text{out}[b] = \bigcup_{s \in \text{succ}[b]} \text{in}[s]$$

- Algorithm

- Set  $\text{in}[b] = \text{out}[b] = \emptyset$
- Update in, out until no change

# Example (1 stmt per block)

- Code

`a := 0`

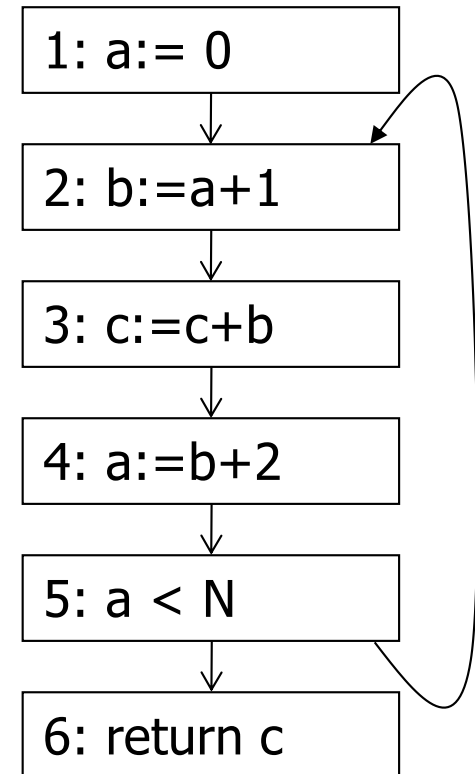
`L: b := a+1`

`c := c+b`

`a := b*2`

`if a < N goto L`     *N assumed const*

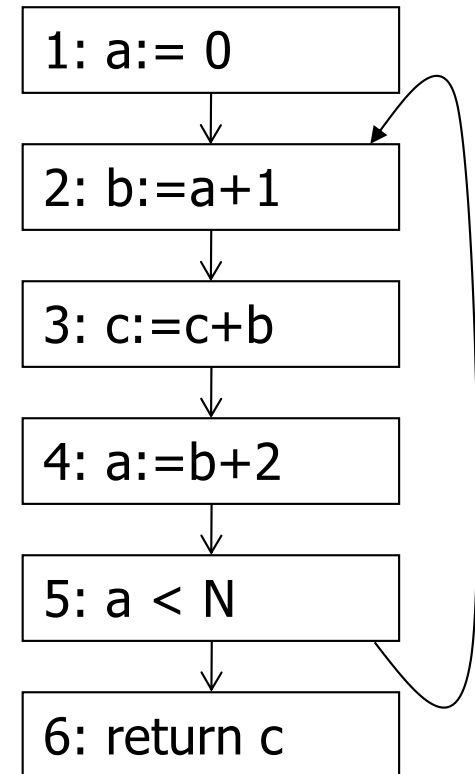
`return c`



$$\begin{aligned} \text{in}[b] &= \text{use}[b] \cup (\text{out}[b] - \text{def}[b]) \\ \text{out}[b] &= \bigcup_{s \in \text{succ}[b]} \text{in}[s] \end{aligned}$$

# Calculation

	I		II		III			
block	use	def	out	in	out	in	out	in
6								
5								
4								
3								
2								
1								

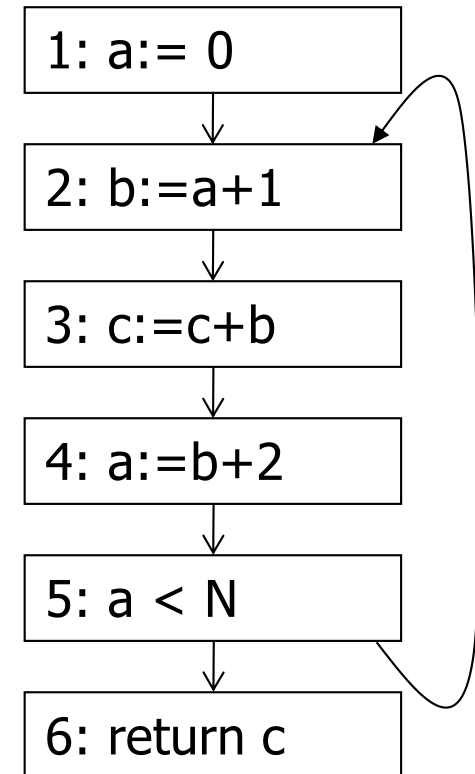


$$\text{in}[b] = \text{use}[b] \cup (\text{out}[b] - \text{def}[b])$$

$$\text{out}[b] = \bigcup_{s \in \text{succ}[b]} \text{in}[s]$$

# Calculation

			I		II		III	
block	use	def	out	in	out	in	out	in
6	c	--	--	c	--	c		
5	a	--	c	a,c	a,c	a,c		
4	b	a	a,c	b,c	a,c	b,c		
3	b,c	c	b,c	b,c	b,c	b,c		
2	a	b	b,c	a,c	b,c	a,c		
1	--	a	a,c	c	a,c	c		



$$\text{in}[b] = \text{use}[b] \cup (\text{out}[b] - \text{def}[b])$$

$$\text{out}[b] = \bigcup_{s \in \text{succ}[b]} \text{in}[s]$$

# Equations for Live Variables v2

- Many problems have more than one formulation. For example, Live Variables...
- Sets
  - $USED(b)$  – variables used in  $b$  before being defined in  $b$
  - $NOTDEF(b)$  – variables not defined in  $b$
  - $LIVE(b)$  – variables live on *exit* from  $b$
- Equation

$$LIVE(b) = \bigcup_{s \in succ(b)} USED(s) \cup (LIVE(s) \cap NOTDEF(s))$$

# Efficiency of Dataflow Analysis

- The algorithms eventually terminate, but the expected time needed can be reduced by picking a good order to visit nodes in the CFG
  - Forward problems – reverse postorder
  - Backward problems – postorder



# Example: Reaching Definitions

- A definition  $d$  of some variable  $v$  *reaches* operation  $i$  iff  $i$  reads the value of  $v$  and there is a path from  $d$  to  $i$  that does not define  $v$
- Uses
  - Find all of the possible definition points for a variable in an expression

# Equations for Reaching Definitions

- Sets
  - $\text{DEFOUT}(b)$  – set of definitions in  $b$  that reach the end of  $b$  (i.e., variable not subsequently redefined in  $b$ )
  - $\text{SURVIVED}(b)$  – set of all definitions not obscured by a definition in  $b$
  - $\text{REACHES}(b)$  – set of definitions that reach  $b$
- Equation
$$\text{REACHES}(b) = \bigcup_{p \in \text{preds}(b)} \text{DEFOUT}(p) \cup (\text{REACHES}(p) \cap \text{SURVIVED}(p))$$

# Example: Very Busy Expressions

- An expression  $e$  is considered *very busy* at some point  $p$  if  $e$  is evaluated and used along every path that leaves  $p$ , and evaluating  $e$  at  $p$  would produce the same result as evaluating it at the original locations
- Uses
  - Code hoisting – move  $e$  to  $p$  (reduces code size; no effect on execution time)

# Equations for Very Busy Expressions

- Sets
  - $USED(b)$  – expressions used in  $b$  before they are killed
  - $KILLED(b)$  – expressions redefined in  $b$  before they are used
  - $VERYBUSY(b)$  – expressions very busy on exit from  $b$
- Equation

$$VERYBUSY(b) = \bigcap_{s \in \text{succ}(b)} USED(s) \cup (VERYBUSY(s) - KILLED(s))$$

# Using Dataflow Information

- A few examples of possible transformations...

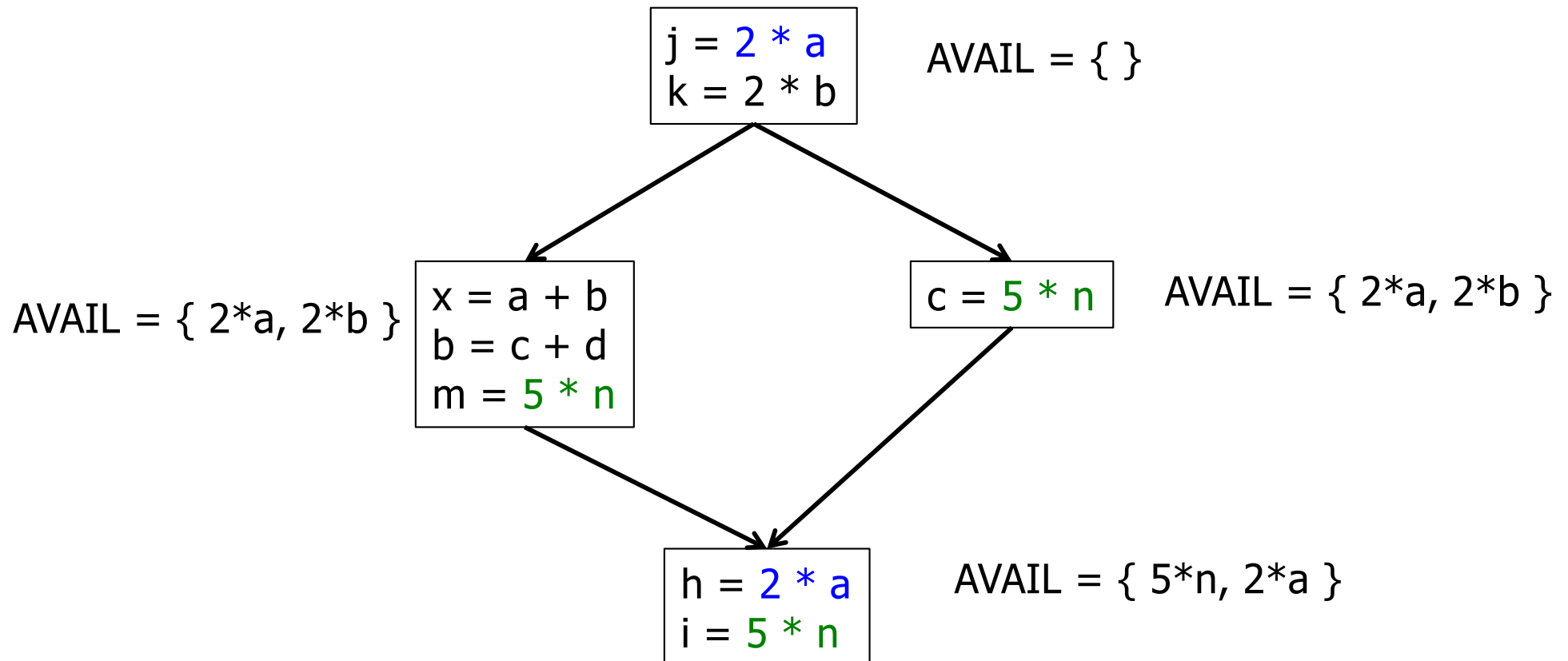
# Classic Common-Subexpression Elimination (CSE)

- In a statement  $s: z := x \text{ op } y$ , if  $x \text{ op } y$  is *available* at  $s$  then it need not be recomputed
- Analysis: compute *reaching expressions* i.e., statements  $n: v := x \text{ op } y$  such that the path from  $n$  to  $s$  does not compute  $x \text{ op } y$  or define  $x$  or  $y$ . (How? Like reaching definitions, but for expressions.)
  - Then an expression is available at  $s$  if it reaches  $s$  on all possible paths to  $s$

# Classic CSE Transformation

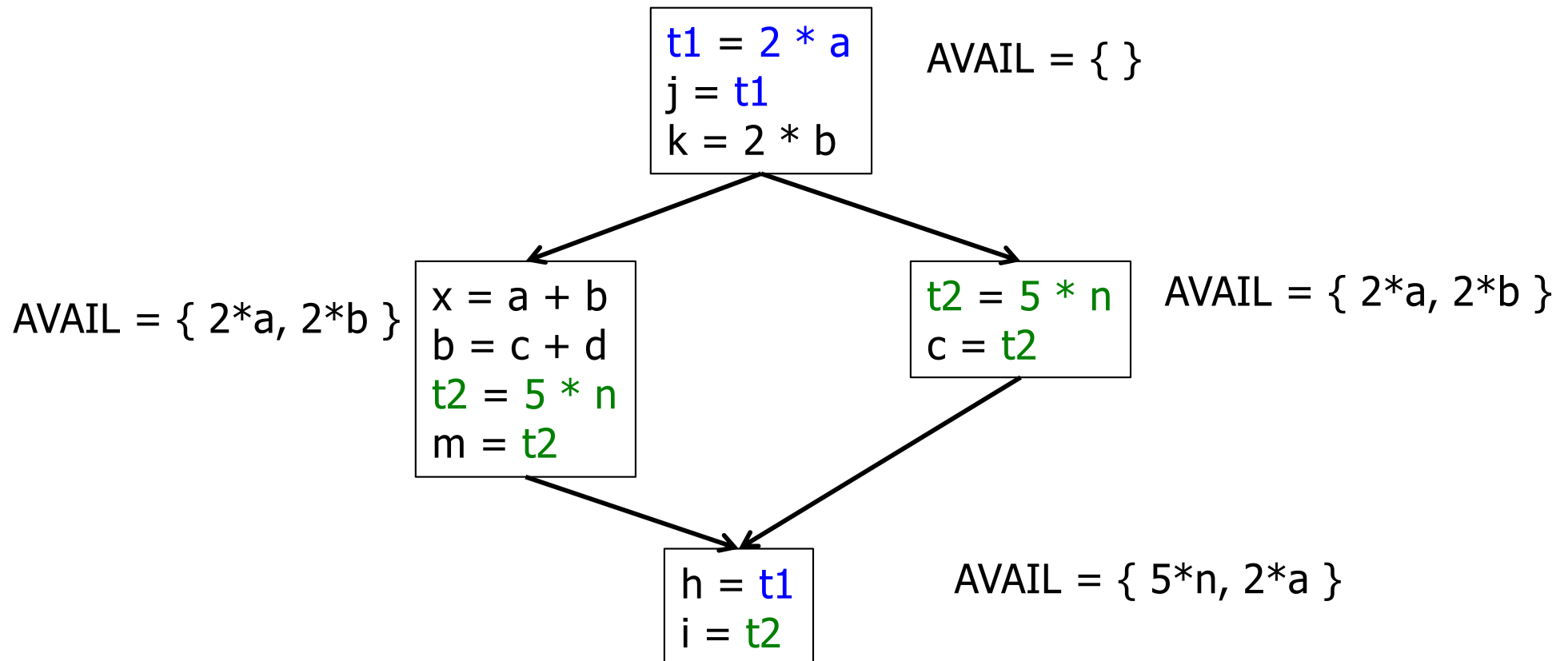
- If  $x \text{ op } y$  is defined at  $n$  and reaches  $s$ 
  - Create new temporary  $t_i$
  - Rewrite  $n: v := x \text{ op } y$  as
$$n: t_i := x \text{ op } y \quad // \text{ } t_i \text{ is a new temporary}$$
$$n': v := t_i$$
  - Rewrite statement  $s: z := x \text{ op } y$  to be
$$s: z := t_i$$
  - (Rely on copy propagation to remove extra assignments that are not really needed)

# Revisiting Example (w/small change)

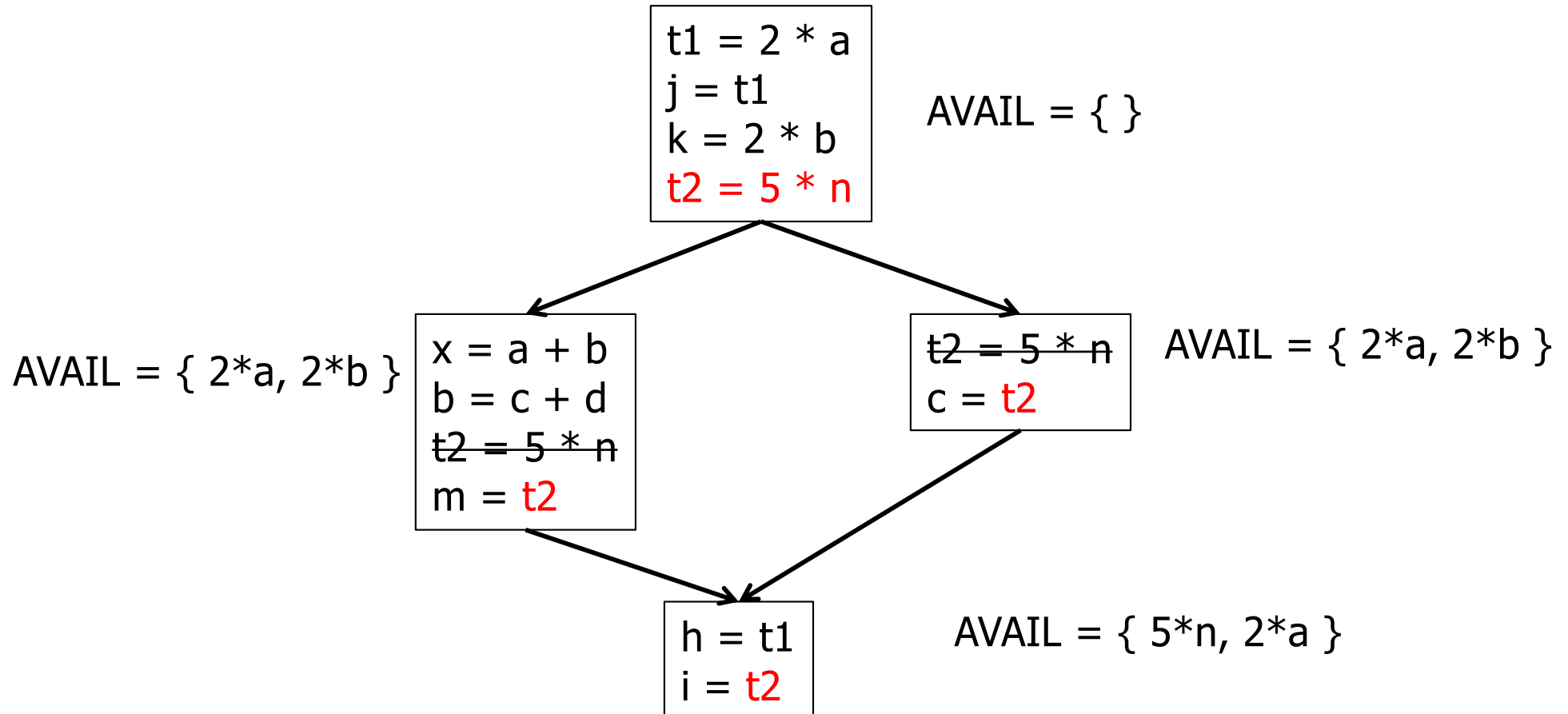




# Revisiting Example (w/small change)



# Then Apply Very Busy...



# Constant Propagation

- Suppose we have
  - Statement  $d$ :  $t := c$ , where  $c$  is constant
  - Statement  $n$  that uses  $t$
- If  $d$  reaches  $n$  and no other definitions of  $t$  reach  $n$ , then rewrite  $n$  to use  $c$  instead of  $t$

# Copy Propagation

- Similar to constant propagation
- Setup:
  - Statement  $d$ :  $t := z$
  - Statement  $n$  uses  $t$
- If  $d$  reaches  $n$  and no other definition of  $t$  reaches  $n$ , and there is no definition of  $z$  on any path from  $d$  to  $n$ , then rewrite  $n$  to use  $z$  instead of  $t$ 
  - Recall that this can help remove dead assignments

# Copy Propagation Tradeoffs

- Downside is that this can increase the lifetime of variable  $z$  and increase need for registers or memory traffic
- But it can expose other optimizations, e.g.,

$a := y + z$

$u := y$

$c := u + z$       // copy propagation makes this  $y + z$

- After copy propagation we can recognize the common subexpression

# Dead Code (Assignment) Elimination

- If we have an instruction

$s: a := b \text{ op } c$

and  $a$  is not live-out after  $s$ , then  $s$  can be eliminated

- Provided it has no implicit side effects that are visible (output, exceptions, etc.)
  - If  $b$  or  $c$  are function calls, they have to be assumed to have unknown side effects unless the compiler can prove otherwise

# Dataflow...

- General framework for discovering facts about programs
  - Although not the only possible story
- And then: facts open opportunities for code improvement
- Next time: SSA (static single assignment) form – transform program to a new form where each variable has only *one* single definition
  - Can make many optimizations/analysis more efficient