CSE 401/M501 – Compilers

SSA

Hal Perkins

Fall 2023

Administrivia

- Welcome back! Hope everyone had a great / relaxing / productive Thanksgiving weekend
- Project reminders:
  - Codegen is due this Thursday for 401 students (M501 codegen is due at the end of the quarter with project extensions – still need to hear from some M501 groups!)
  - Remember to fix any remaining bugs, all the way back to the scanner, for the overall review at the end! 🐛🕷🐜🦂🐞
- HW4 out today (dataflow, SSA, etc.). Due next Thursday and we'll spend time in sections this week on it.
- This Wed. 11/29, 4-6 pm, Allen atrium: Allen School Research Night open house. Great chance to connect with research groups and find interesting projects.

Administrivia (added Wed.)

- Project reminders:
  - Codegen is "officially" due tomorrow for 401 groups (M501 codegen is due at the end of the quarter with project extensions – still need to hear from some M501 groups!)
    - But: all 401 groups can use 2 late days for this. Must finish by Sat. 11 pm.
  - Remember to fix any remaining bugs, all the way back to the scanner, for the overall review at the end! 🐛🕷🐜🦂🐞
- Project report writeup posted now. For 401: due Mon. 11 pm, **no late reports.** (M501 report due with project end of next week, also no late reports)
- HW4 due next Thursday and we'll spend time in sections on it tomorrow
- Tonight! 4-6 pm, Allen atrium: Allen School Research Night open house. Great chance to connect with research groups and find interesting projects.

Agenda

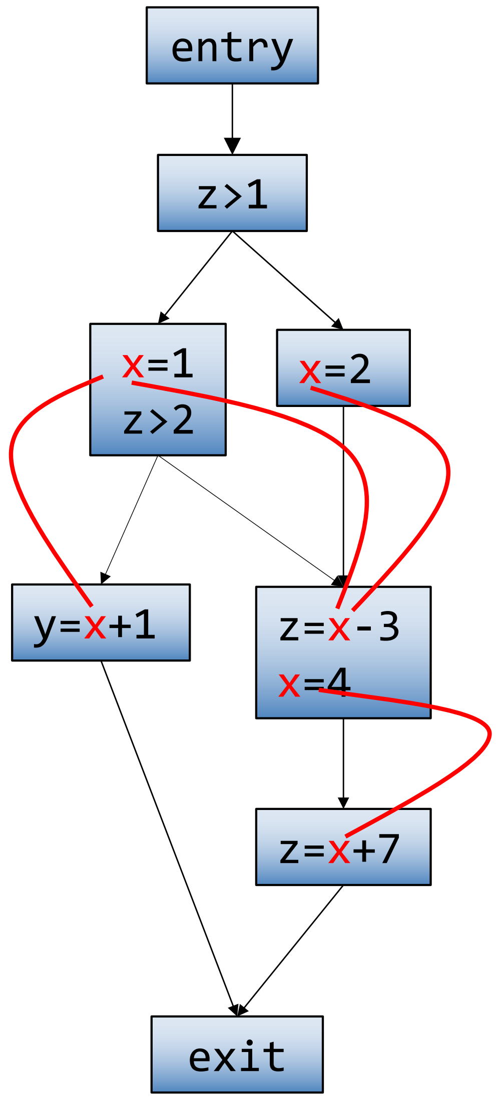
- Overview of SSA IR
  - Constructing SSA graphs
  - Sample of SSA-based optimizations
  - Converting back from SSA form

- Sources: Appel ch. 19, also an extended discussion in Cooper-Torczon sec. 9.3

Def-Use (DU) Chains

- Common dataflow analysis problem: Find all sites where a variable is used, or find the definition site(s) of a variable used in an expression
- Traditional solution: def-use chains – additional data structure on top of the dataflow graph
  - Link each statement defining a variable to all statements that use it
  - Link each use of a variable to its (single) definition

Def-Use (DU) Chains



In this example, two DU chains intersect

DU-Chain Drawbacks

- Expensive: if a typical variable has N uses and M definitions, the total cost *per-variable* is O(N * M), i.e., $O(n^2)$
  - Would be nice if cost were proportional to the size of the program
- Unrelated uses of the same variable are mixed together
  - Complicates analysis – variable looks live across all uses even if unrelated

SSA: Static Single Assignment

- IR where each variable has only one definition in the program text
  - This is a single *static* definition, but that definition can be in a loop, function, or other code that is executed dynamically many times
- Makes many analyses (and related optimizations) more efficient
- Separates values from memory storage locations
- Complementary to CFG/DFG – better for some things, but cannot do everything

SSA in Basic Blocks

Idea: for each original variable v, create a new variable v_n at the n^{th} definition of the original v. Subsequent uses of v use v_n until the next definition point.

- Original

  a := x + y

  b := a – 1

  a := y + b

  b := x * 4

  a := a + b

- SSA

  a_1 := x + y

  b_1 := a_1 – 1

  a_2 := y + b_1

  b_2 := x * 4

  a_3 := a_2 + b_2

Merge Points

- The issue is how to handle merge points

```
if (...)
  a = x;
else
  a = y;
b = a;
```

→

if (...)
  a_1 = x;
else
  a_2 = y;
b_1 = ??;

Merge Points

- The issue is how to handle merge points

if (…)
  $a = x;$
else
  $a = y;$
$b = a;$

if (…)
  $a_1 = x;$
else
  $a_2 = y;$
$a_3 = \Phi(a_1, a_2);$
$b_1 = a_3;$

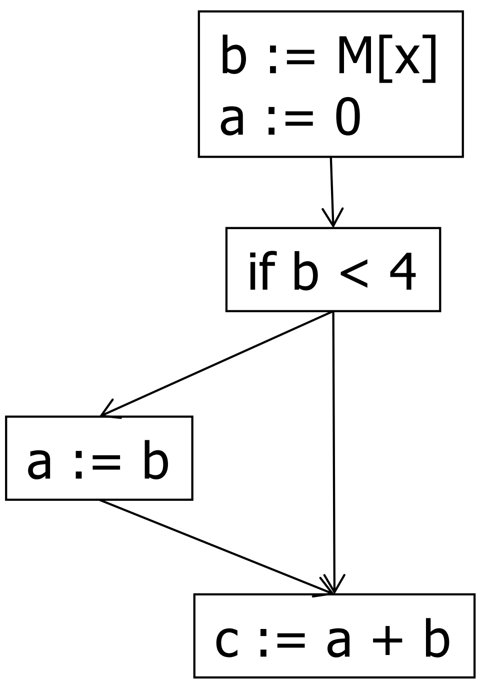
- Solution: introduce a Φ-function

  $a_3 := \Phi(a_1, a_2)$

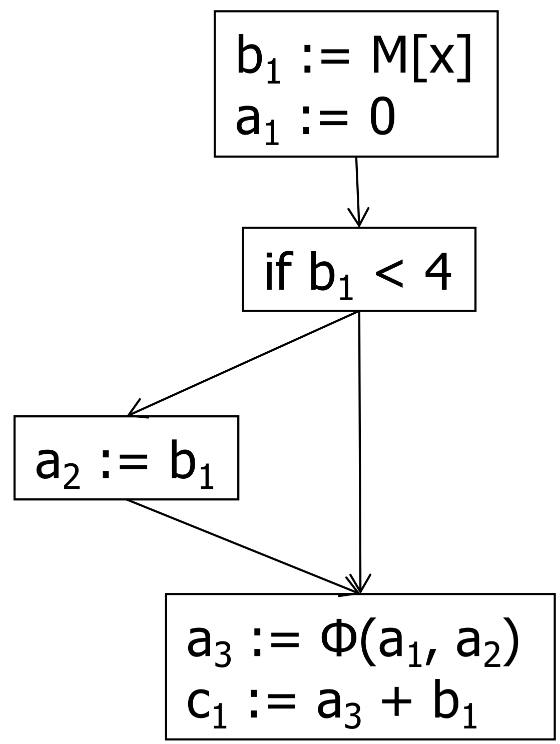
- Meaning: a_3 is assigned either a_1 or a_2 depending on which control path is used to reach the Φ-function

Another Example

Original



SSA

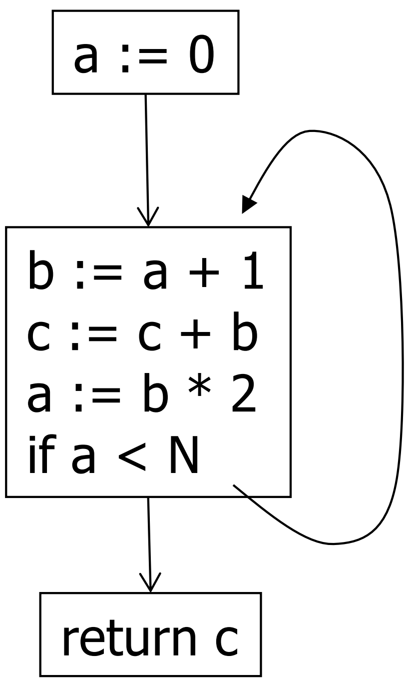


How Does Φ "Know" What to Pick?

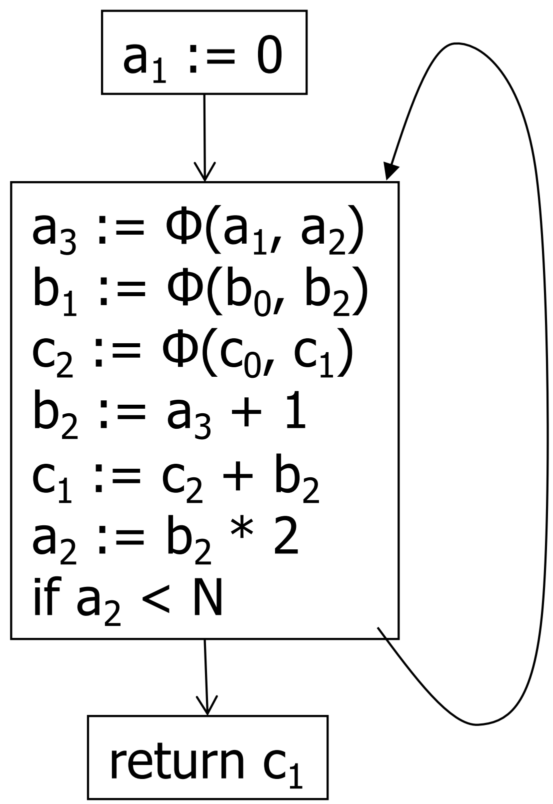
- It doesn't !
- Φ-functions don't actually exist at runtime
  - When we're done using the SSA IR, we translate back out of SSA form, removing all Φ-functions
    - Basically by adding code to copy all SSA x_i values to the single, non-SSA variable x
  - For analysis, all we typically need to know is the connection of uses to definitions – no need to "execute" anything
    - So Φ-functons are (only) compile-time bookkeeping

Example With a Loop

Original



SSA



Notes:

- Loop-back edges are also merge points, so require Φ-functions
- Convention: a_0, b_0, c_0 are initial values of a, b, c on entry to initial block
- b_1 is dead – can delete later
- c is live on entry – either input parameter or uninitialized

What does SSA “buy” us?

- No need for DU or UD chains – implicit in SSA
- Compact representation
- SSA is “recent” (i.e., 80s)
- Prevalent in real compilers for { } languages

Converting To SSA Form

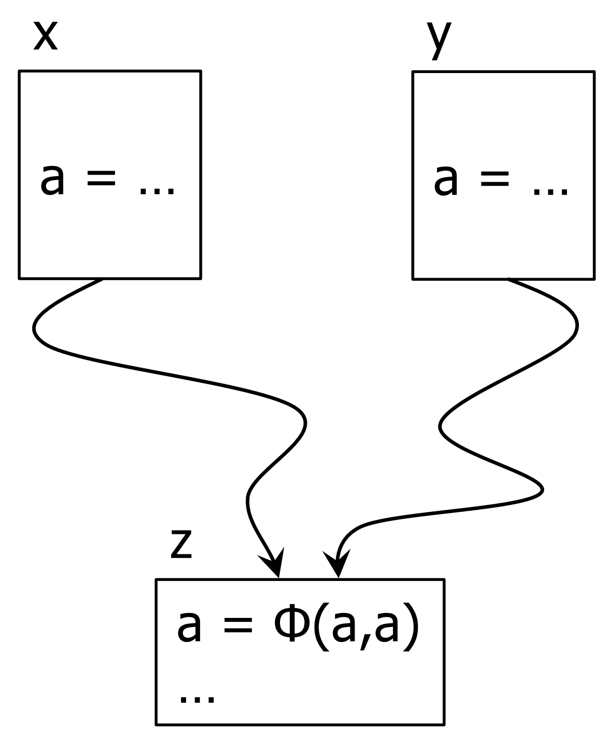
- Basic idea
  - First, add Φ-functions
  - Then, rename all definitions and uses of variables by adding subscripts

Inserting Φ-Functions

- Could simply add Φ-functions for every variable at every join point(!)
- Called “maximal SSA”
- But
  - Wastes *way* too much space and time
  - Not needed in many cases

Path-convergence criterion

- Insert a Φ-function for variable *a* at point z when:
  - There are blocks x and y, both containing definitions of *a*, and $x \neq y$
  - There are nonempty paths from x to z and from y to z
  - These paths have no common nodes other than z



Details

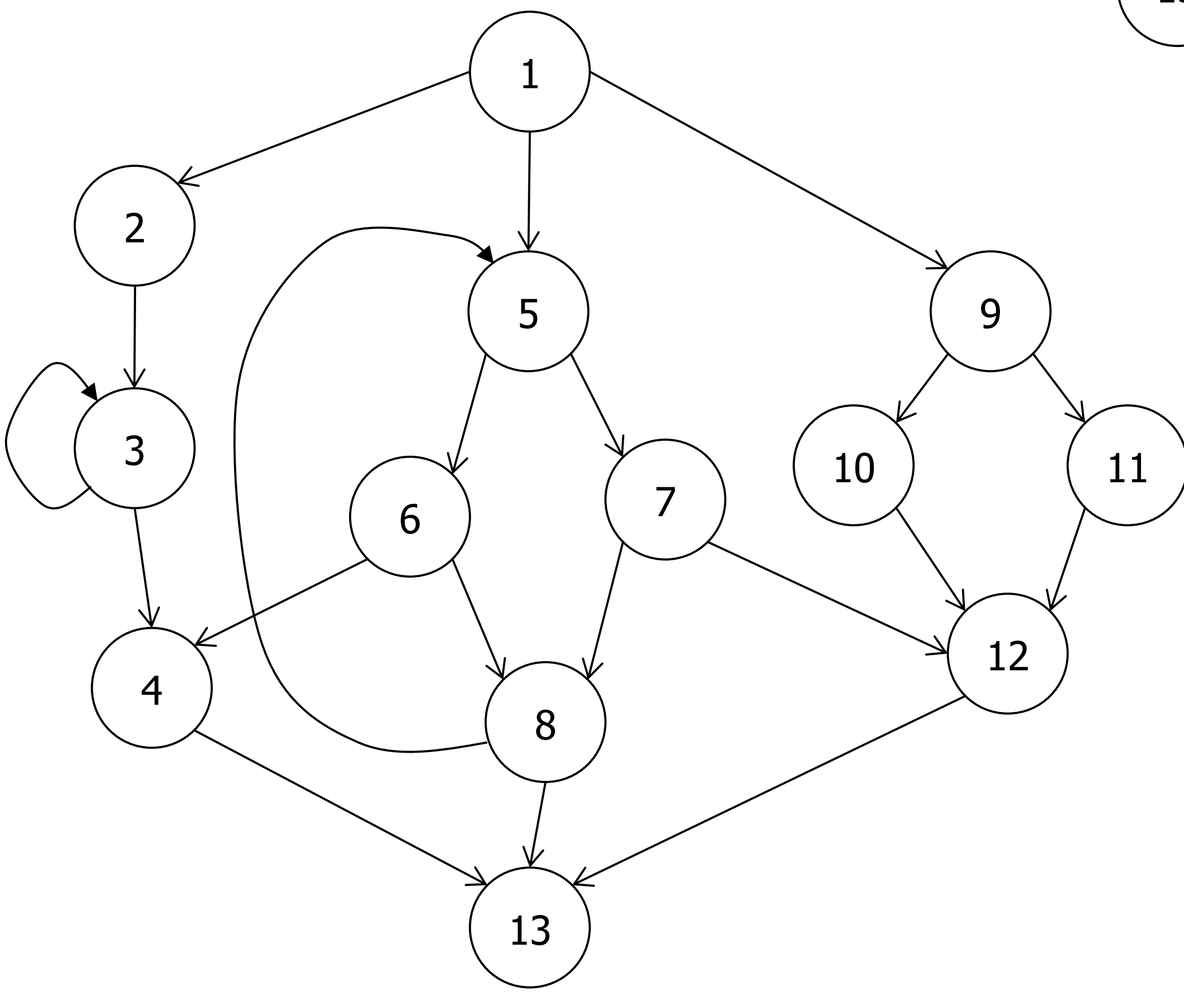
- The start node of the flow graph is considered to define every variable (even if "undefined")
- Each Φ-function itself defines a variable, which may create the need for a new Φ-function
  - So we need to keep adding Φ-functions until things converge
- How can we do this efficiently?
  Use a new concept: dominance frontiers

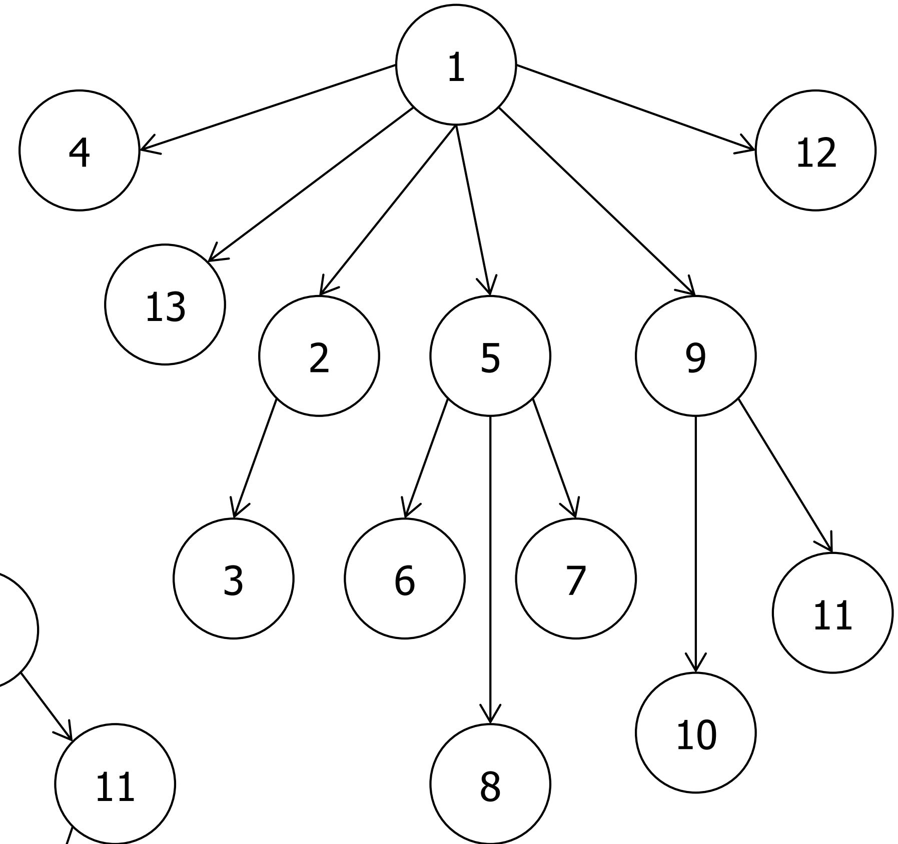
Dominators

- Definition: a block x *dominates* a block y iff every path from the entry of the control-flow graph to y includes x
- So, by definition, x dominates x
- We can associate a Dom(inator) set with each CFG node x – set of all blocks dominated by x

  $| \text{Dom}(x) | \geq 1$
- Properties:
  - Transitive: if a dom b and b dom c, then a dom c
  - There are no cycles, thus can represent the dominator relationship as a tree

Example





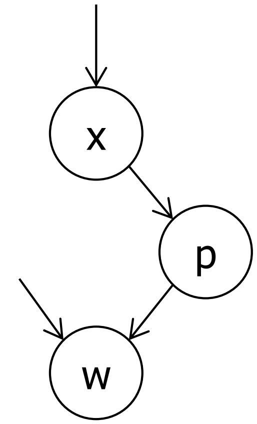
Dominators and SSA

- One property of SSA is that definitions dominate uses; more specifically:
  - If $x := \Phi(\ldots,x_i,\ldots)$ is in block b, then the definition of x_i dominates the i^{th} predecessor of b
  - If x is used in a non-Φ statement in block b, then the definition of x dominates block b

Dominance Frontier (1)

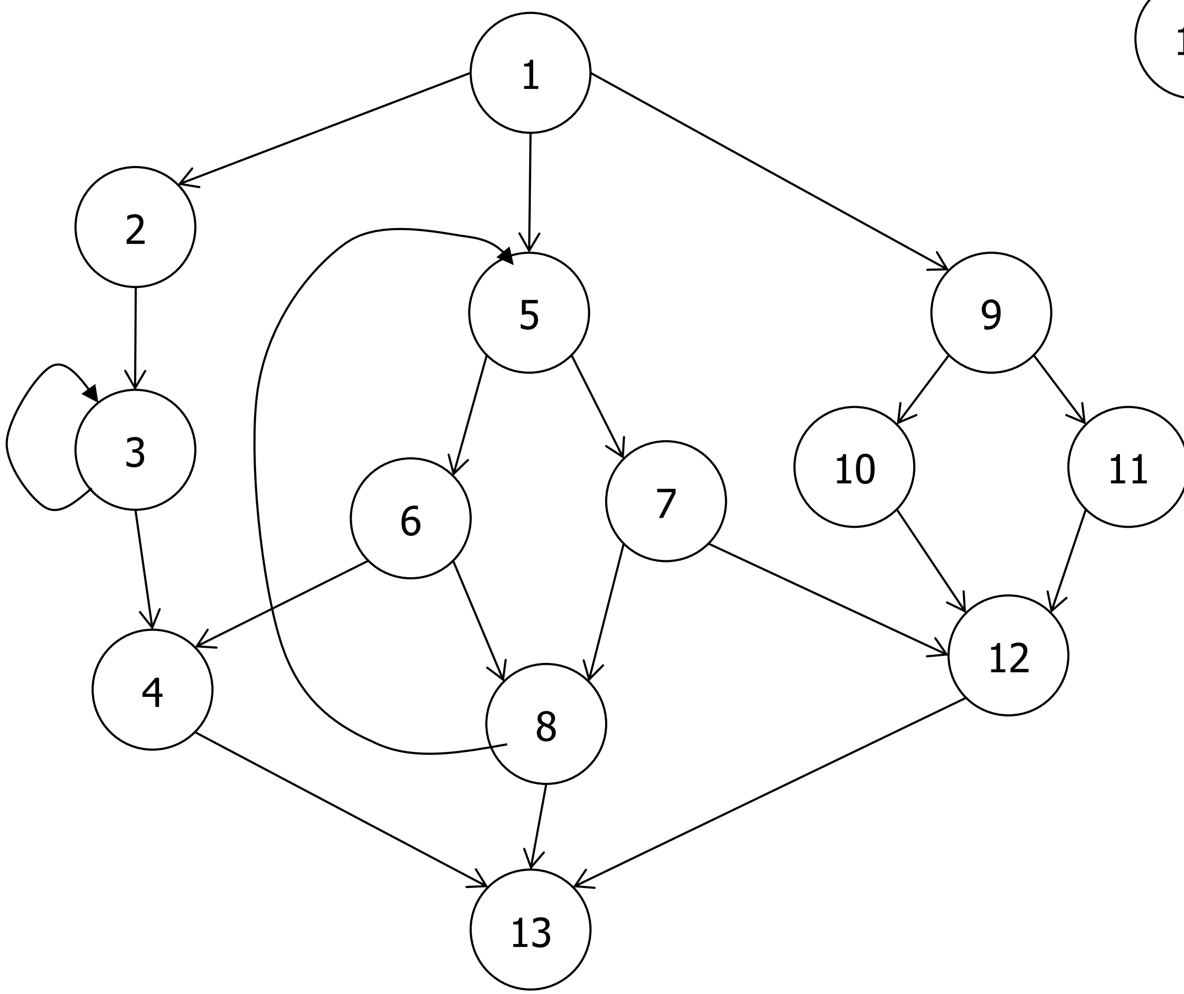
- To get a practical algorithm for placing Φ-functions, we need to avoid looking at all combinations of nodes leading from x to y
- Instead, use the dominator tree in the flow graph

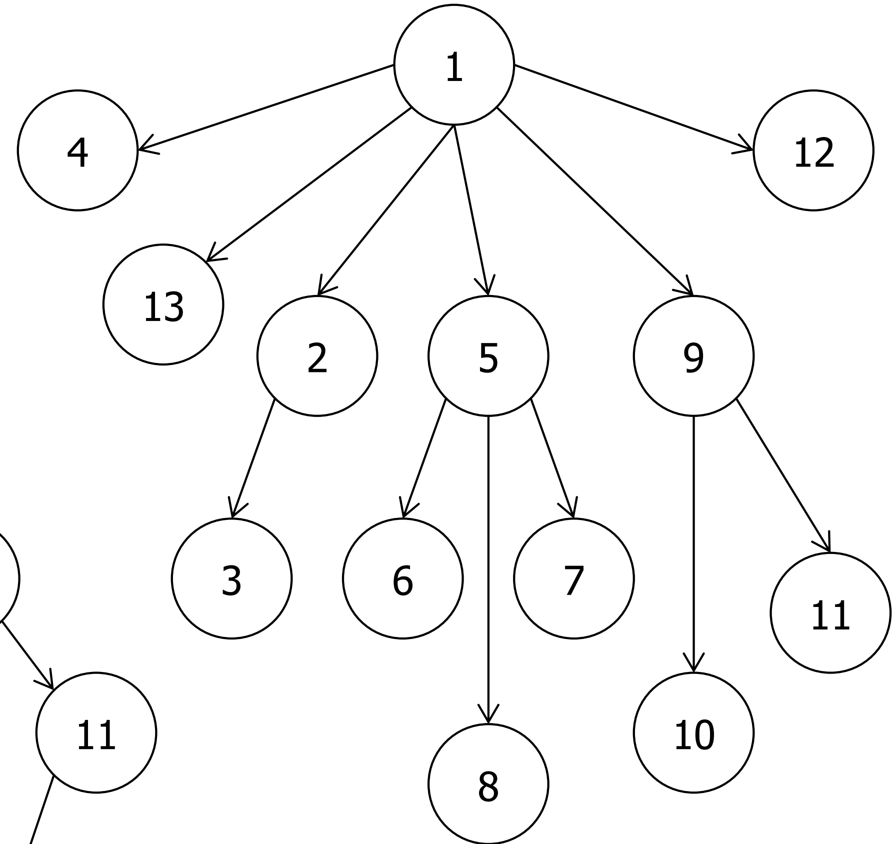
Dominance Frontier (2)



- Definitions
  - x *strictly dominates* y if x dominates y and $x \neq y$
  - The *dominance frontier* of a node x is the set of all nodes w such that x dominates a predecessor p of w, but x does not strictly dominate w
    - ∴ if x and w are different, then x does not dominate w – there is some other path to w that does not go through x
    - But x can be in *it's own* dominance frontier! That can happen if there is a back edge to x from some node that x dominates (i.e., x is the head of a loop)
- Essentially, the dominance frontier is the border between dominated and undominated nodes

Example



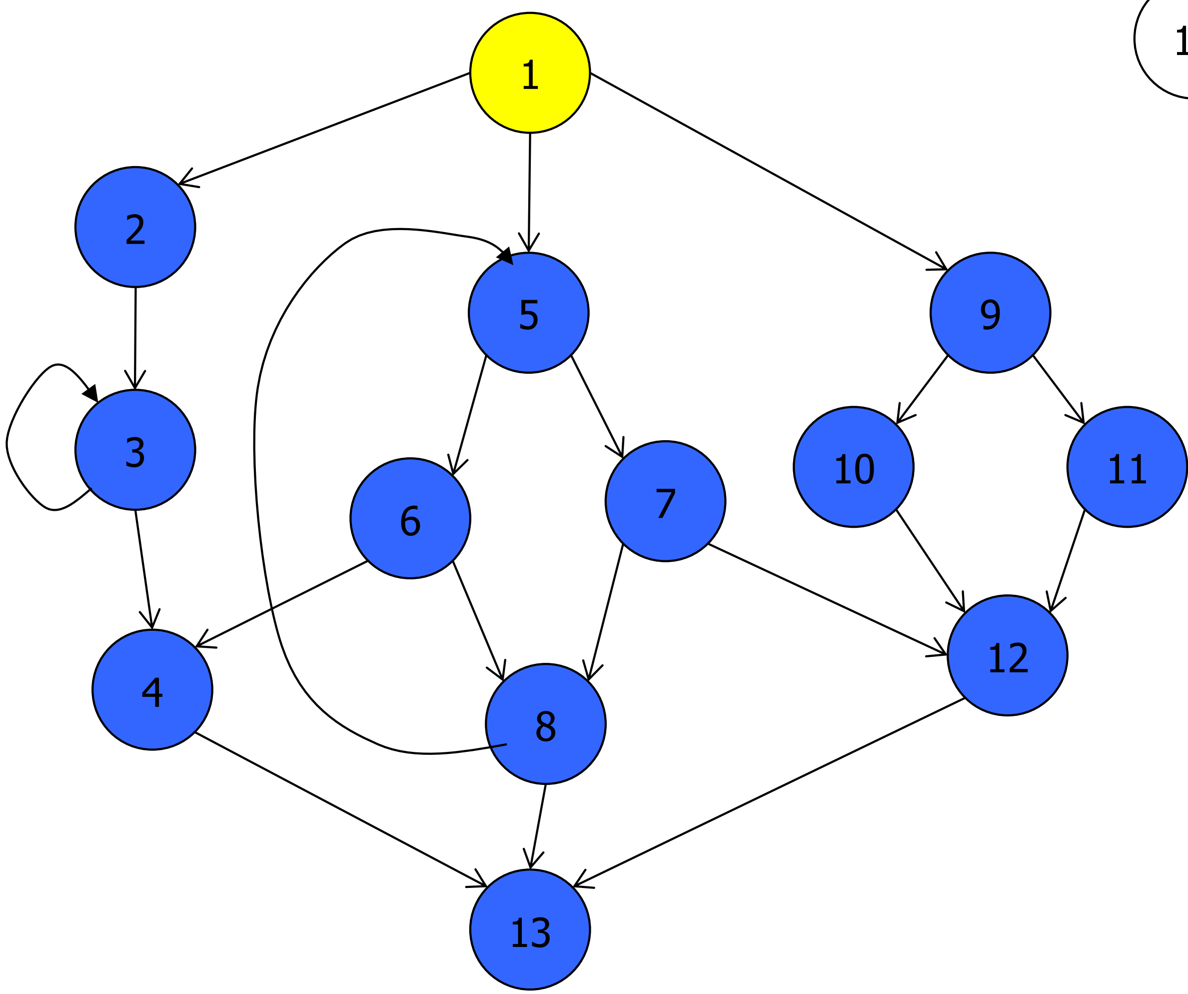


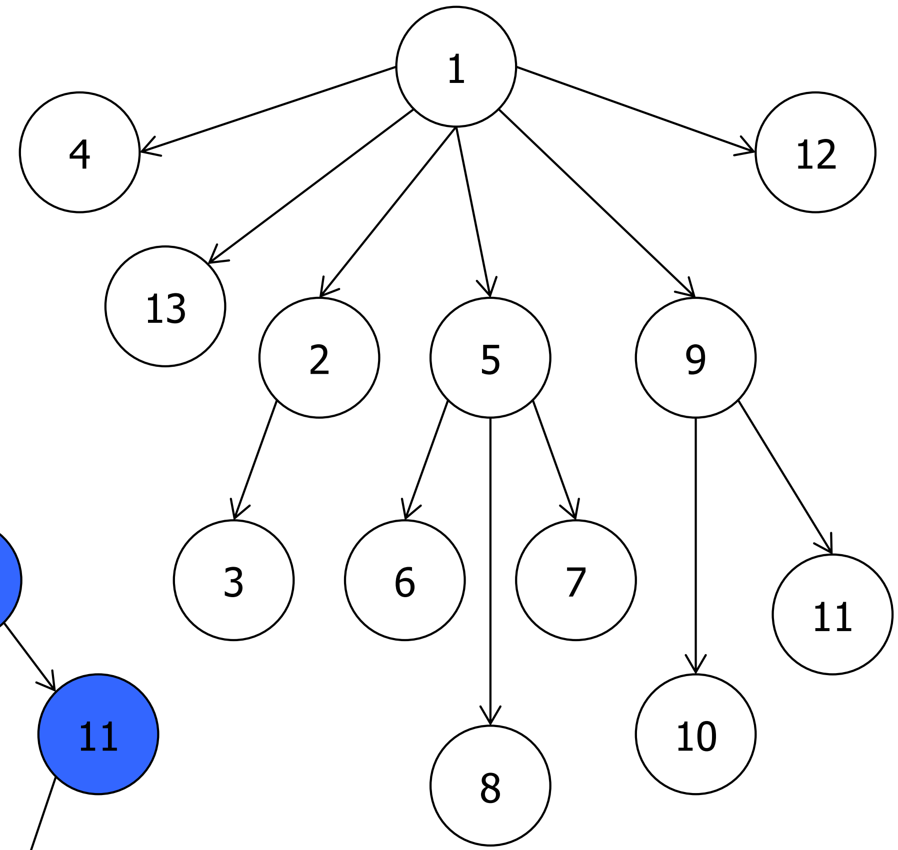
= x

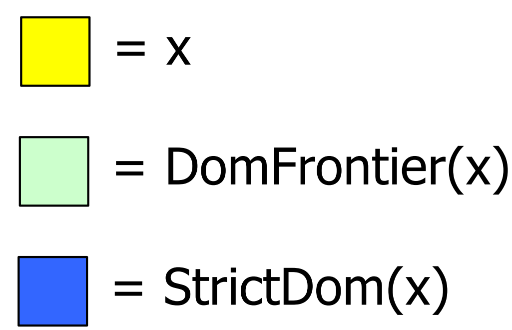
= DomFrontier(x)

= StrictDom(x)

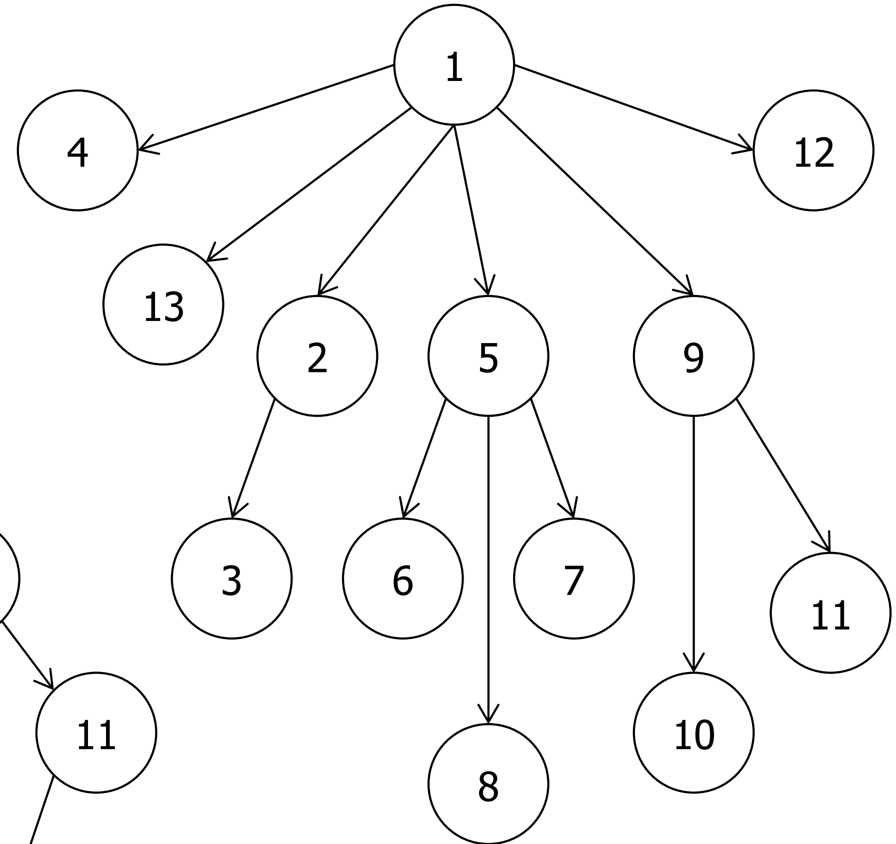
Example

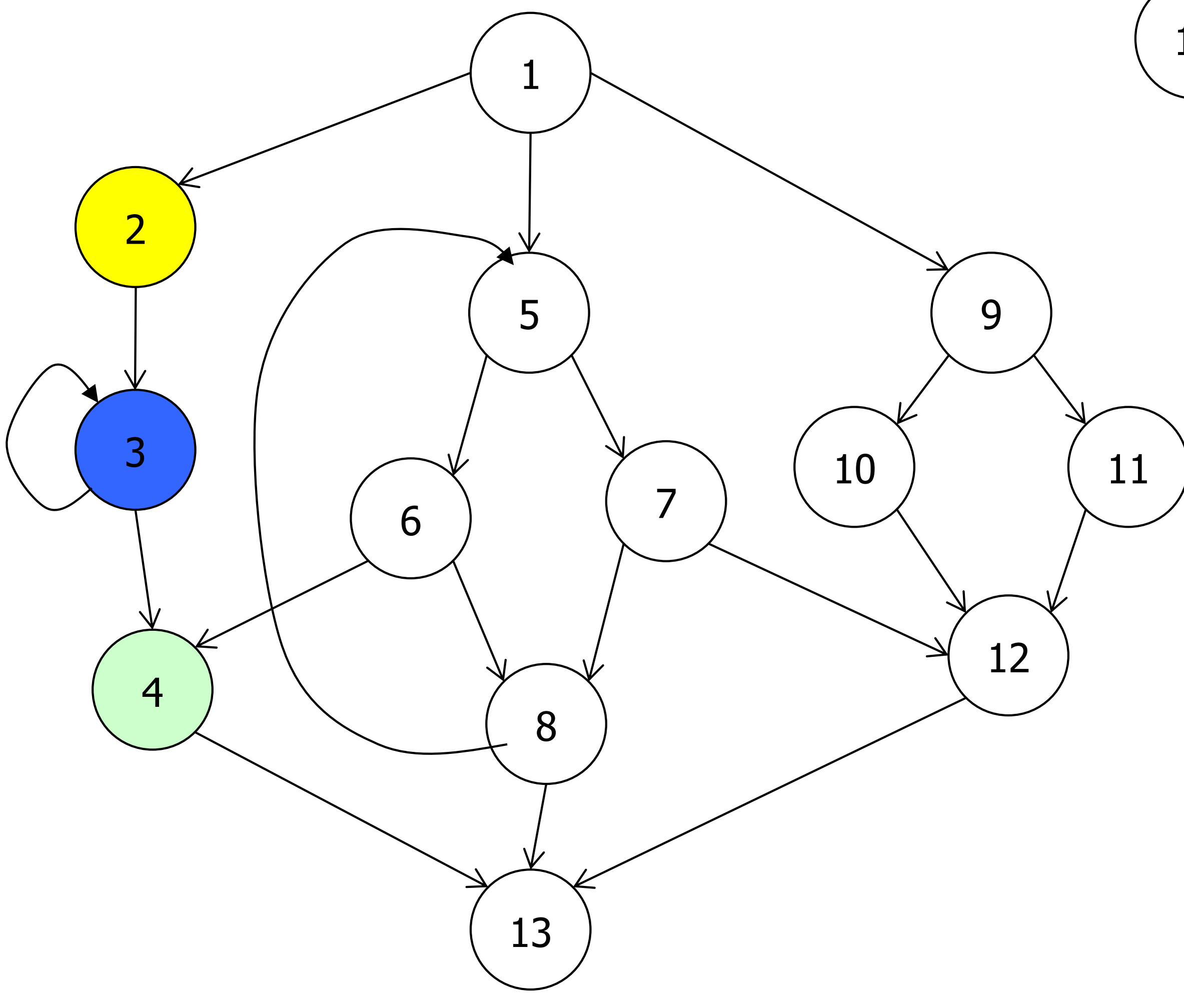

1
2
3
4
5
6
7
8
9
10
11
12
13

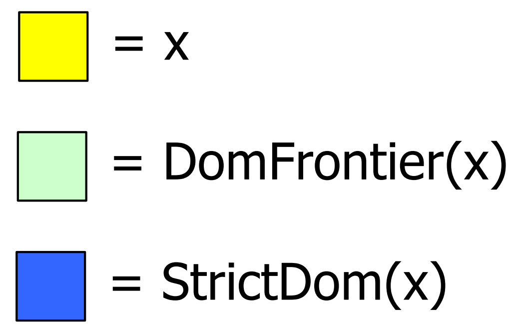

1
4
12
13
2
5
9
3
6
7
11
8
10


= x
= DomFrontier(x)
= StrictDom(x)

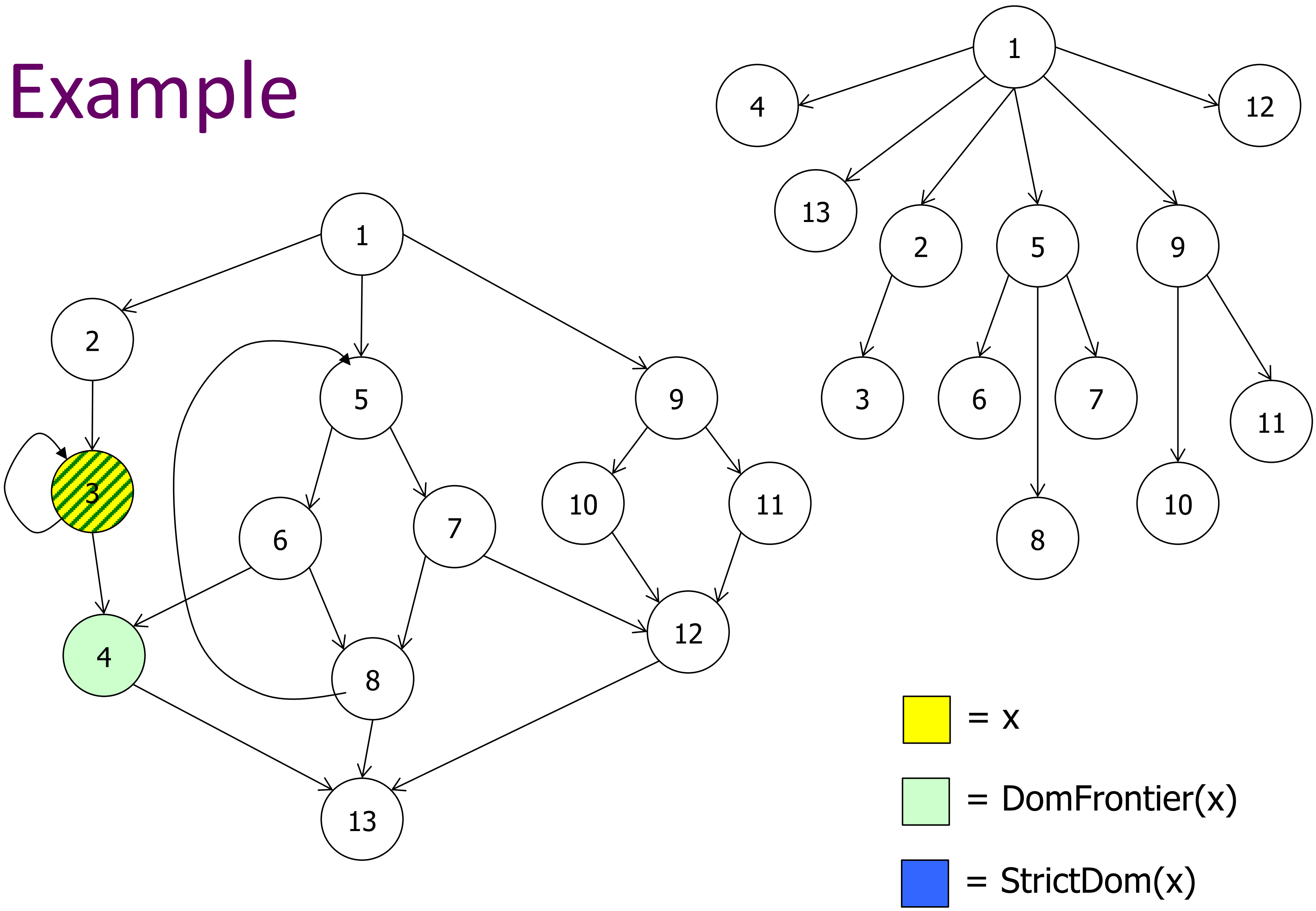
Example


1
4
12
13
2
5
9
3
6
7
11
8
10

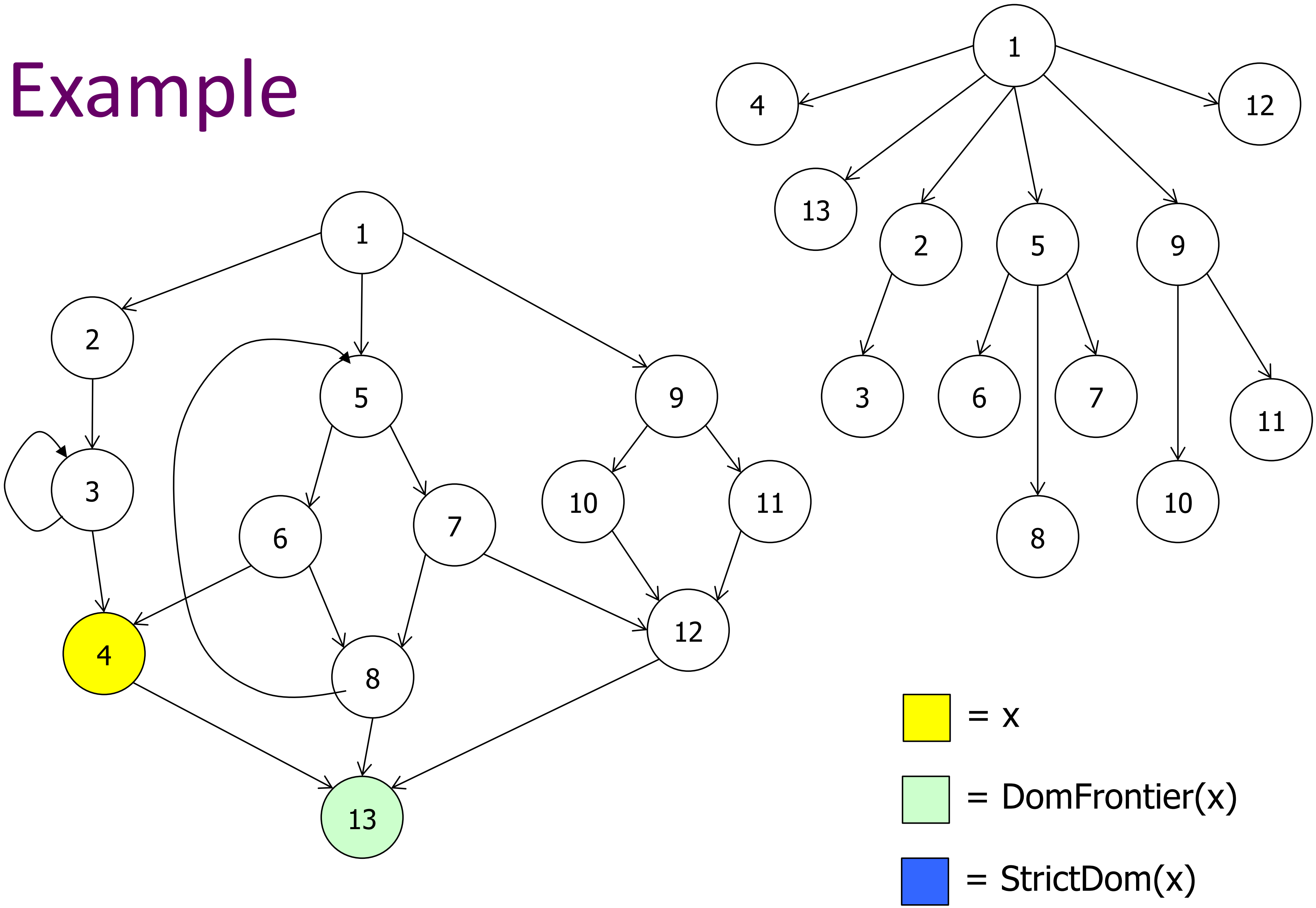

1
2
5
9
3
6
7
10
11
4
8
12
13


= x
= DomFrontier(x)
= StrictDom(x)

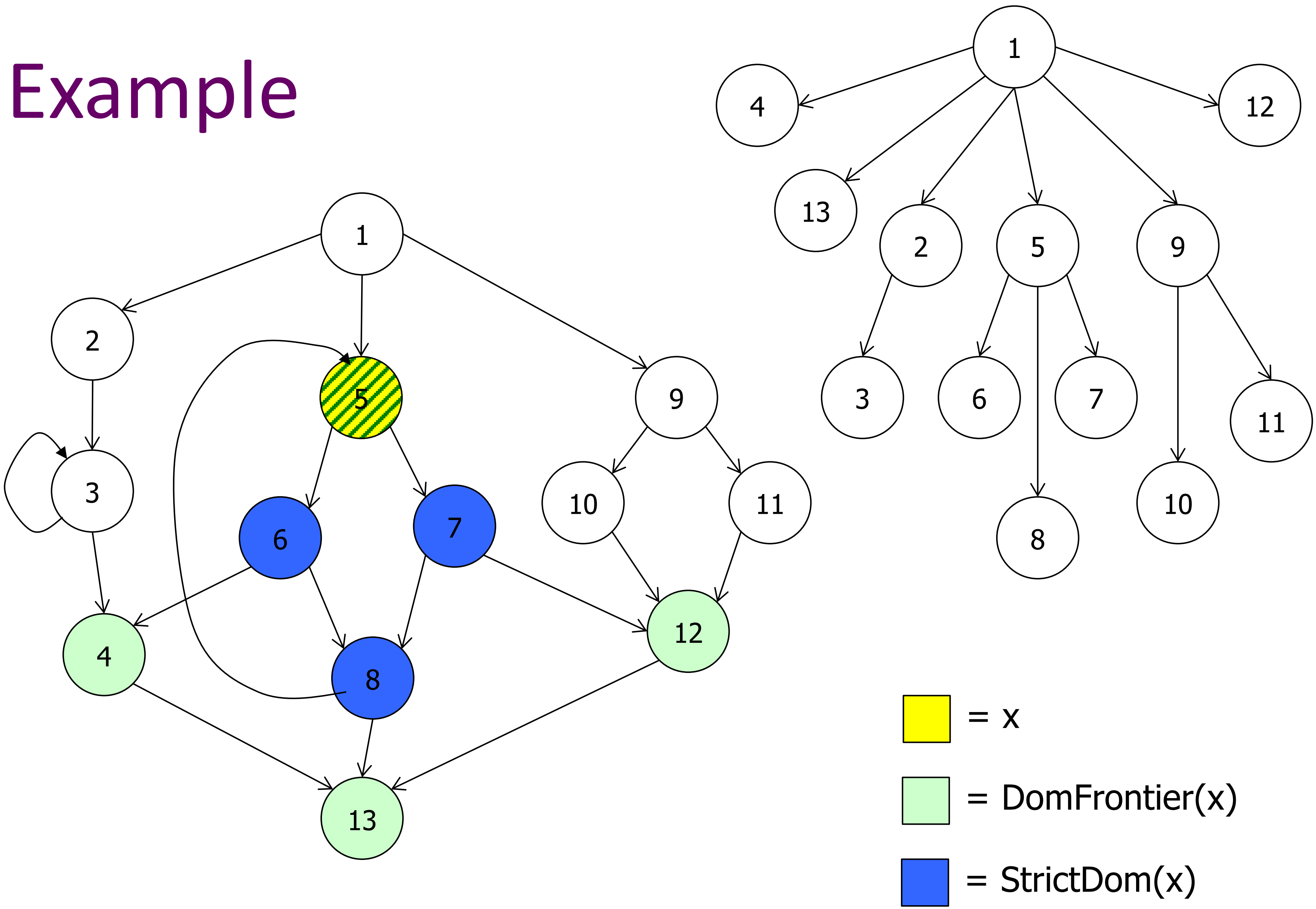
Example


1
2
5
9
3
6
7
10
11
4
8
12
13
1
4
12
13
2
5
9
3
6
7
11
10
8
= x
= DomFrontier(x)
= StrictDom(x)

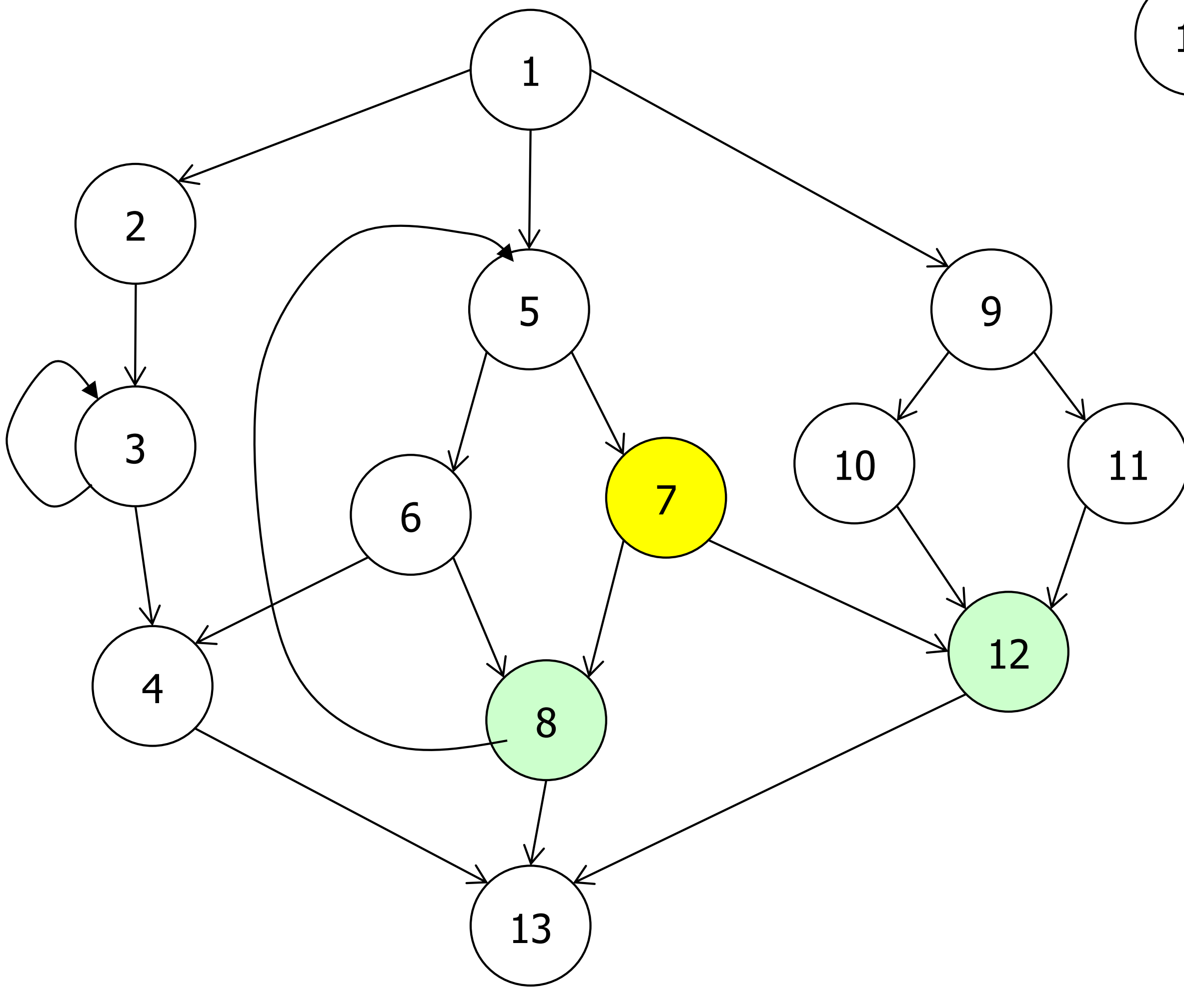
Example

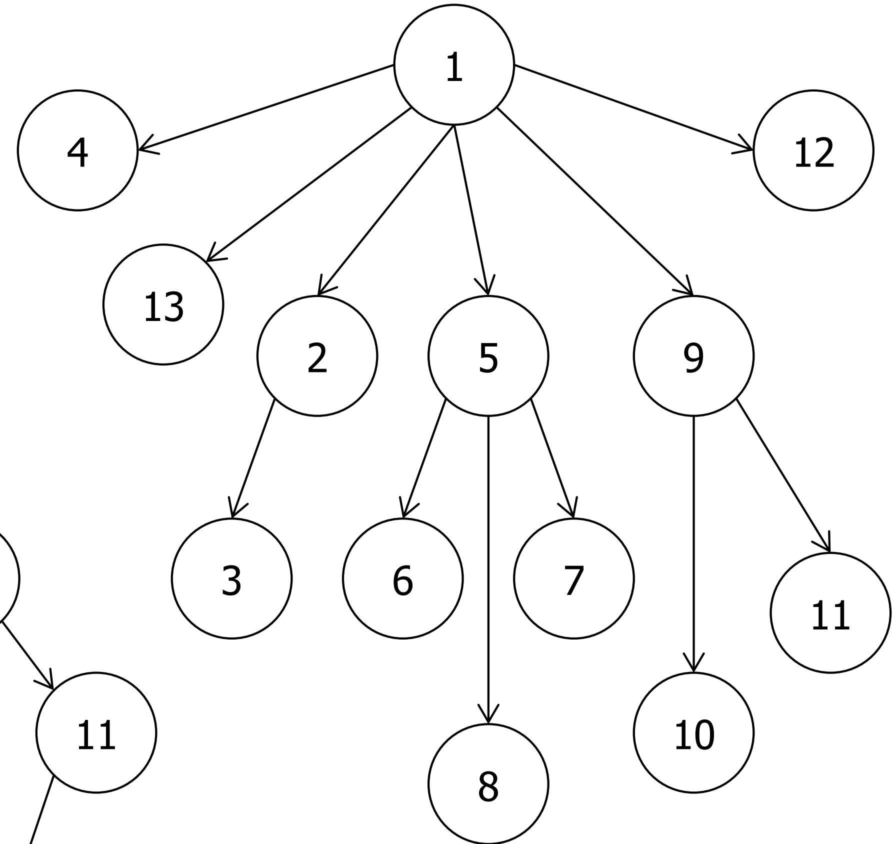


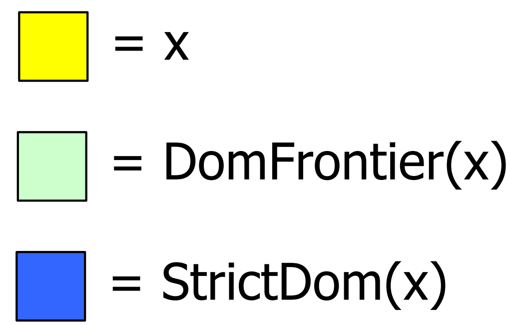
Example



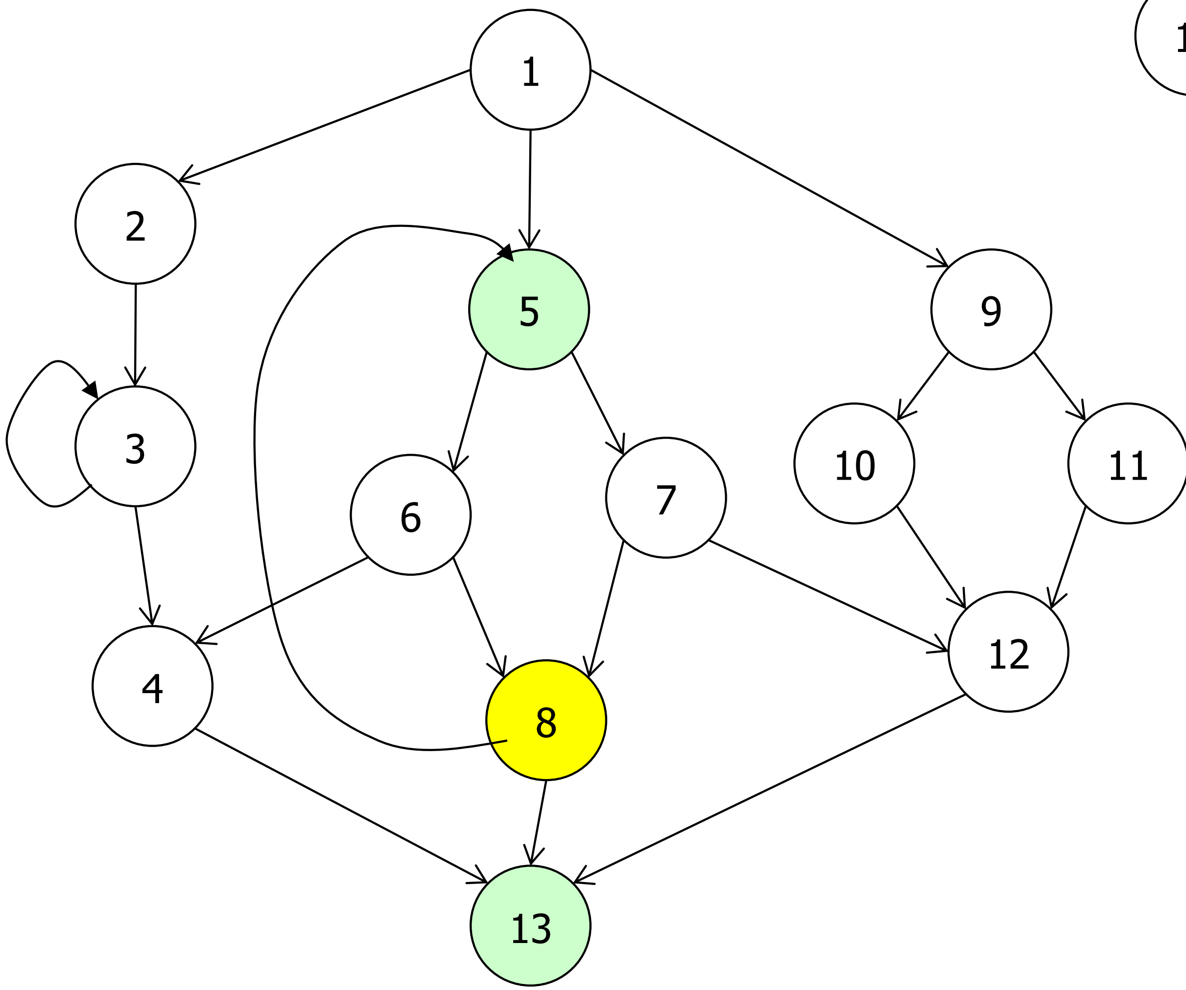
Example

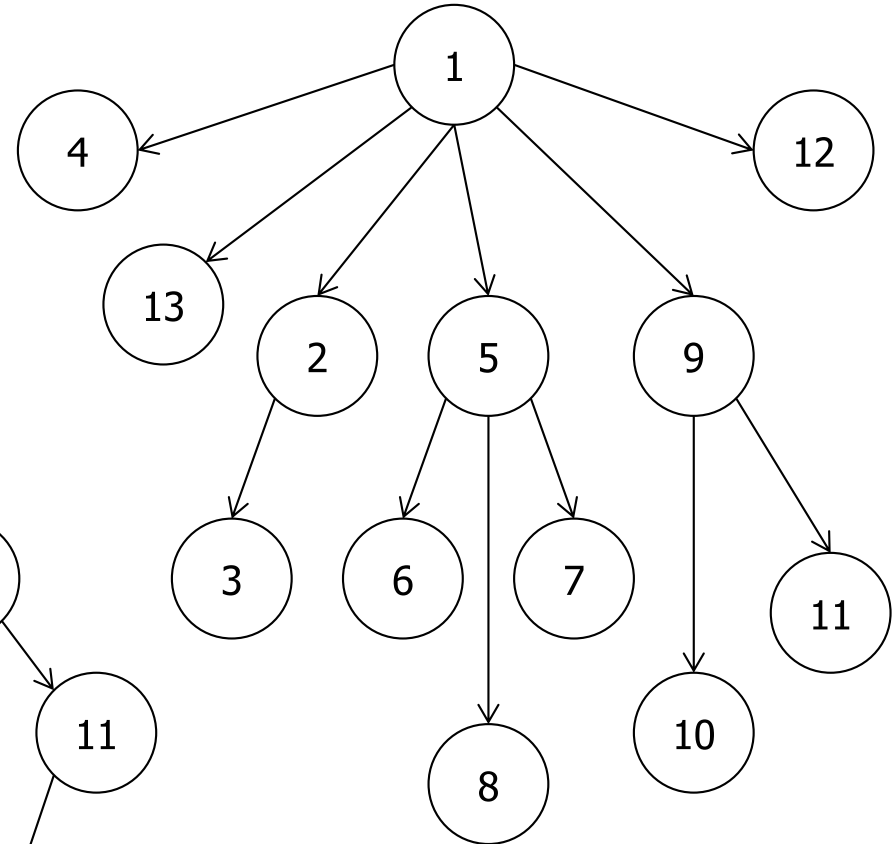

1
2
3
4
5
6
7
8
9
10
11
12
13


1
4
12
13
2
5
9
3
6
7
11
8
10


= x
= DomFrontier(x)
= StrictDom(x)

Example



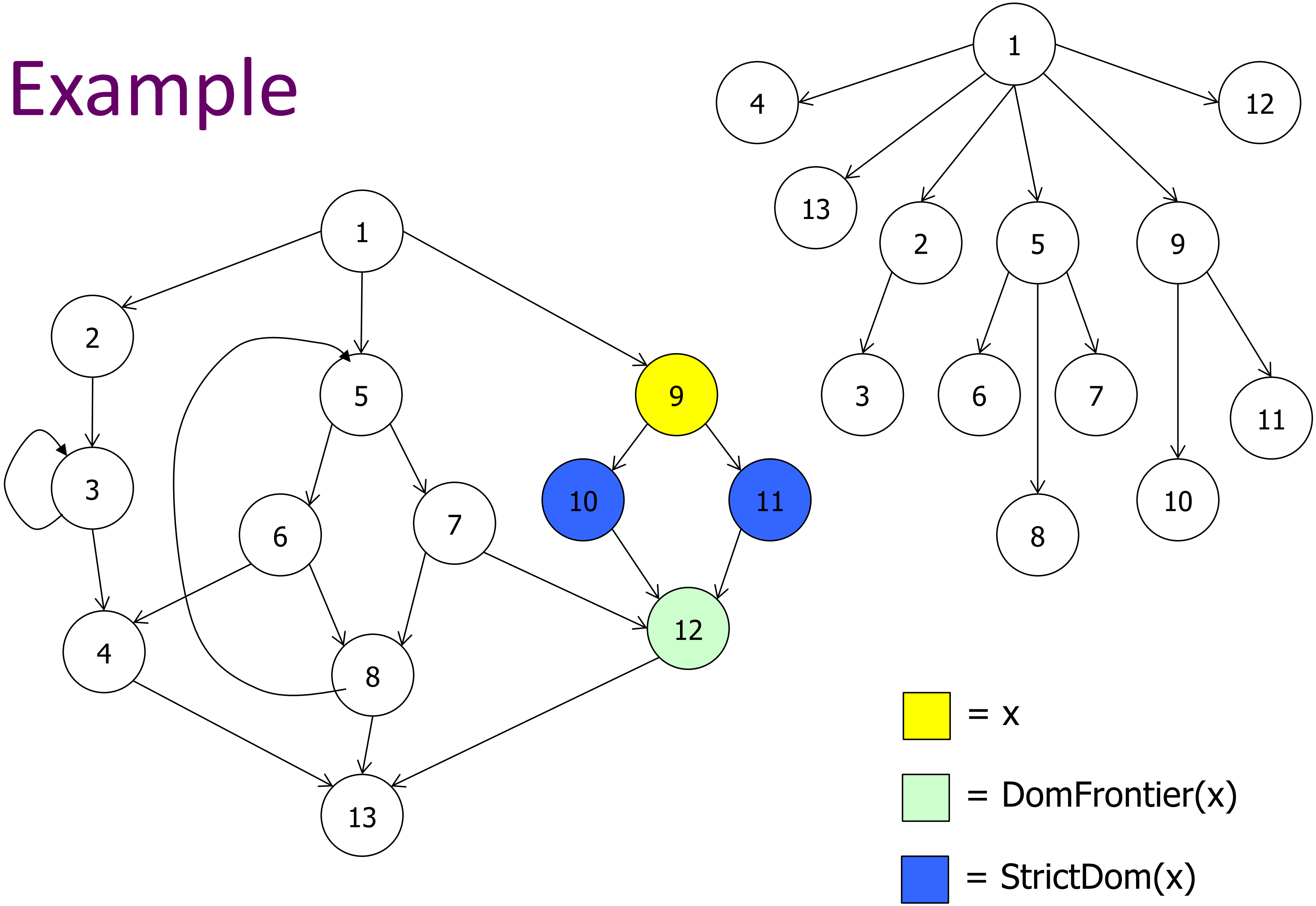


= x

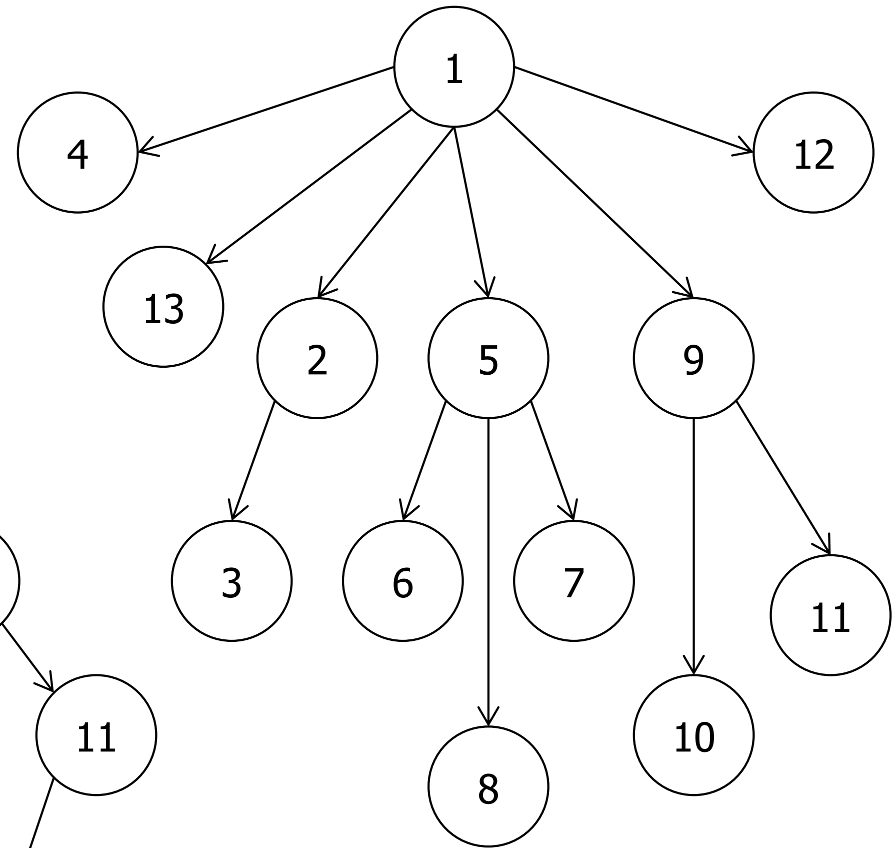
= DomFrontier(x)

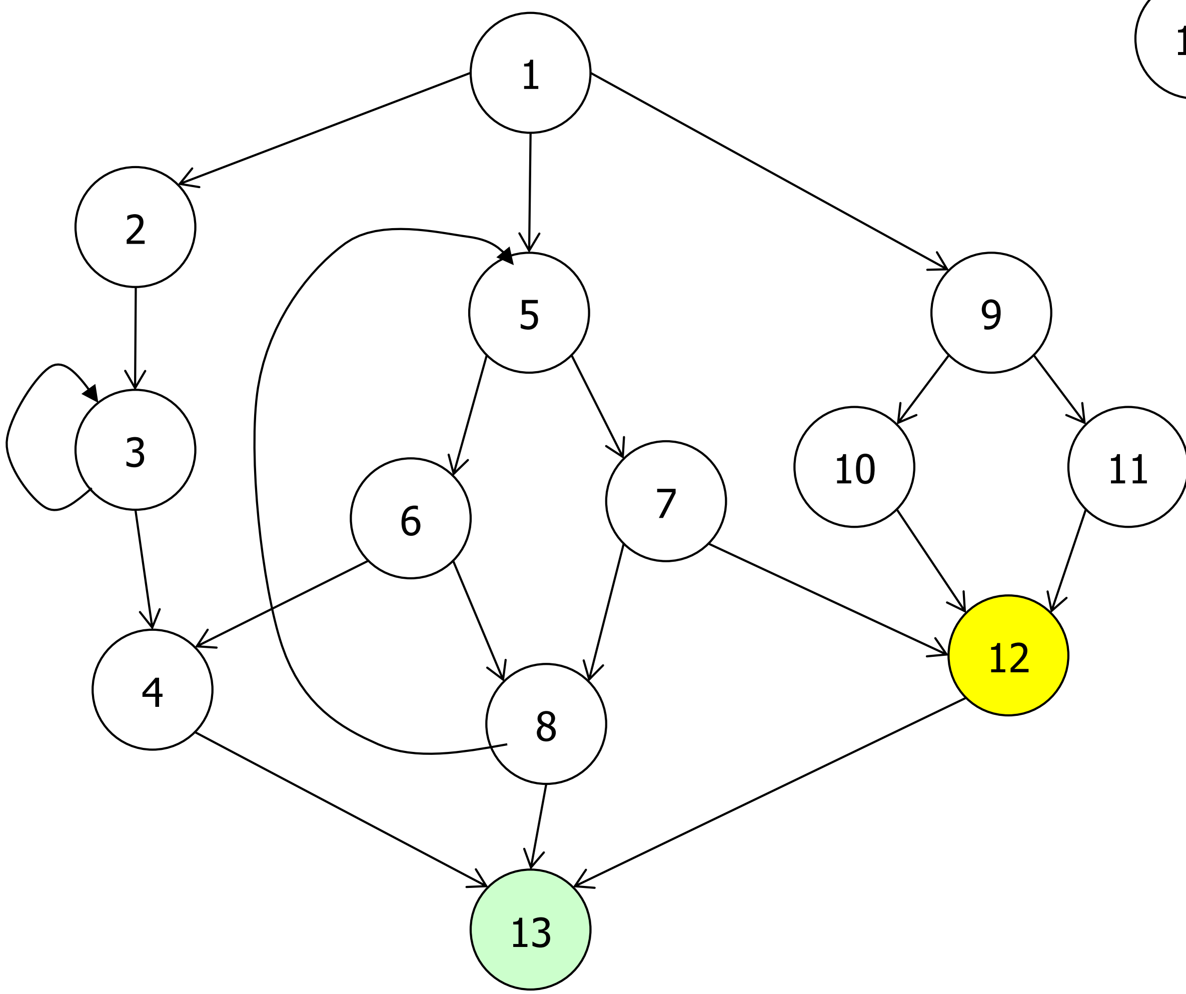
= StrictDom(x)

Example



Example



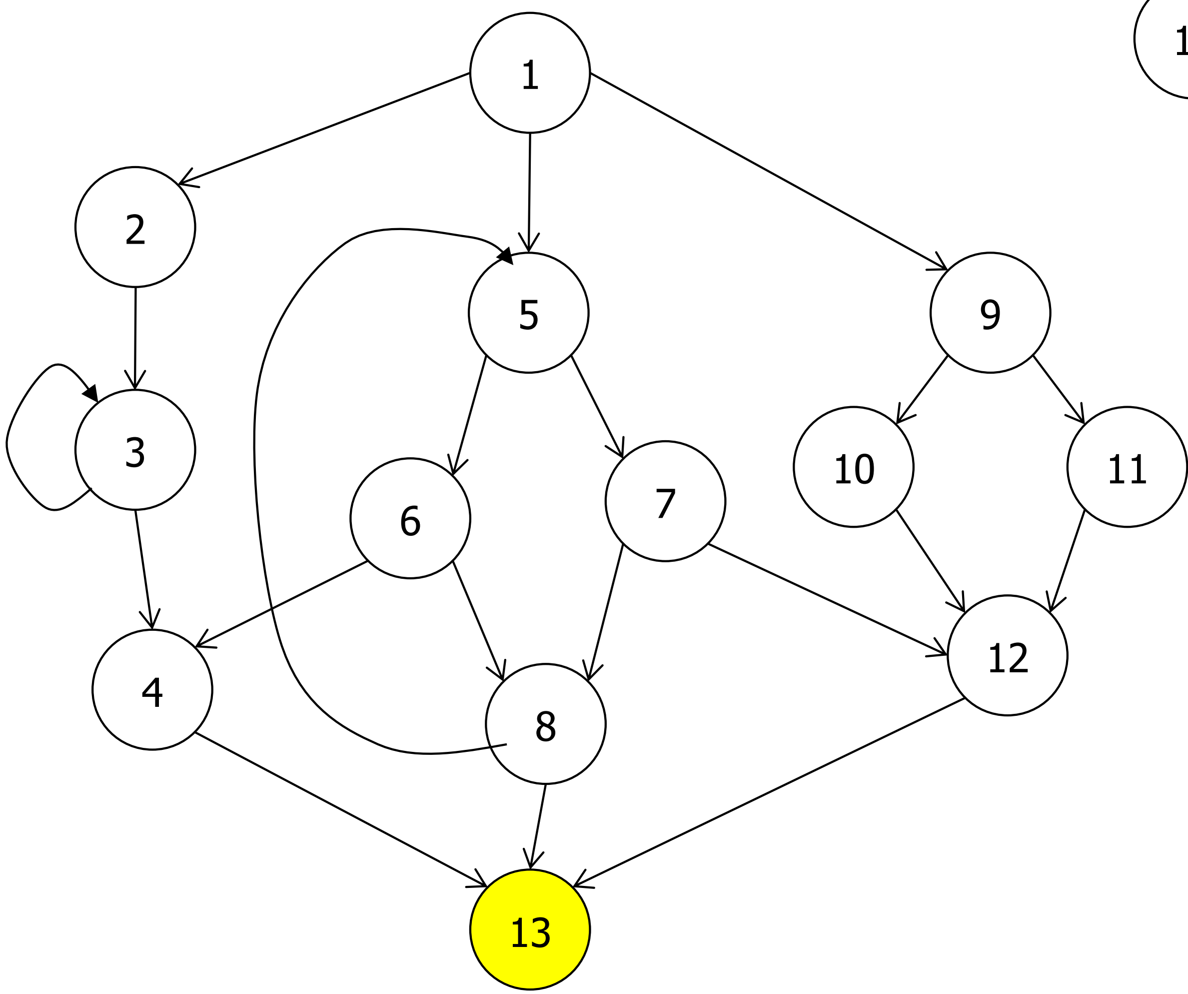


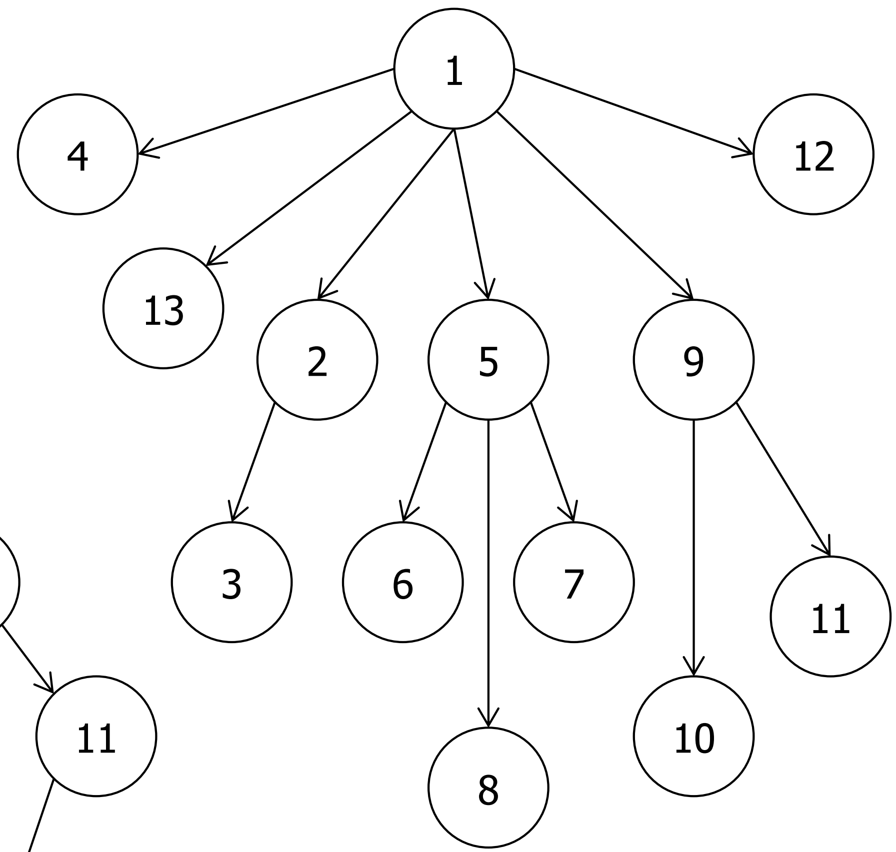
= x

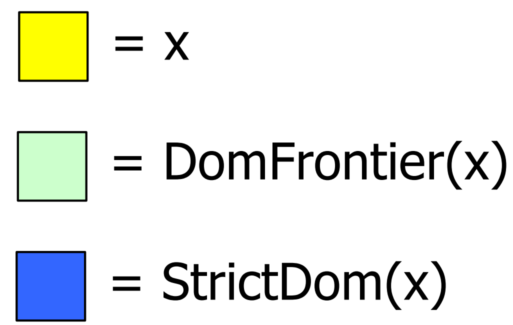
= DomFrontier(x)

= StrictDom(x)

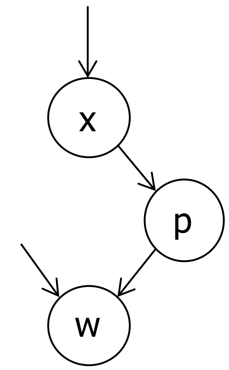
Example


1
2
5
9
3
6
7
10
11
4
8
12
13


1
4
12
13
2
5
9
3
6
7
11
10
8


= x
= DomFrontier(x)
= StrictDom(x)

Dominance Frontier Criterion for Placing Φ-Functions



- If a node x contains the definition of variable *a*, then every node in the dominance frontier of x needs a Φ-function for *a*
  - Idea: Everything dominated by x will see x's definition of *a*. The dominance frontier represents the first nodes we could have reached via an alternative path, which *will* have an alternate reaching definition of *a* (recall the convention that the entry node defines all variables with version 0 - a_0)
    - Why is this right for loops? Hint: strict dominance…
  - Since the Φ-function itself is a definition, this placement rule needs to be iterated until it reaches a fixed-point
- Theorem: this algorithm places exactly the same set of Φ-functions as the path convergence criterion (above)

Placing Φ-Functions: Details

- See the book for the full construction, but the basic steps are:
  1. Compute the dominance frontiers for each node in the flowgraph
  2. Insert just enough Φ-functions to satisfy the criterion. Use a worklist algorithm to avoid reexamining nodes unnecessarily
  3. Walk the dominator tree and rename the different definitions of each variable *a* to be a_1, a_2, a_3, …

SSA Optimizations

- Why go to the trouble of translating to SSA?
- Because SSA makes many optimizations and analyses simpler and more efficient
  - We'll give a couple of examples
- But first, what do we know? (i.e., what information is stored in the compiler SSA graph data structures?)

SSA Data Structures

For each …

- Statement: links to containing block, next and previous statements, variables defined, variables used
- Variable: link to its (single) definition and (possibly multiple) use sites
- Block: List of contained statements, ordered list of predecessor(s) & successor(s) blocks

Dead-Code Elimination

- A variable is live ⇔ its list of uses is not empty(!)
  - That's it! Nothing further to compute
- Algorithm to delete dead code:

```
while there is some variable v with no uses
    if the statement that defines v has no
        other side effects, then delete it
```

  - Need to remove this statement from the list of uses for its operand variables – which may cause those variables to become dead

Simple Constant Propagation

- If c is a constant in *v* := c, any use of *v* can be replaced by c
  - So update every use of v to use constant c
- If the c_i's in *v* := $\Phi(c_1, c_2, \ldots, c_n)$ are all the same constant c, we can replace this with *v* := c
- Incorporate copy propagation, constant folding, and others in the same worklist algorithm

Simple Constant Propagation

W := list of all statements in SSA program
while W is not empty
  remove some statement S from W
  if S is *v*:=Φ(c, c, …, c), replace S with *v*:=c
  if S is *v*:=c
    delete S from the program
    for each statement T that uses *v*
      substitute c for *v* in T
      add T to W

Converting Back from SSA

- Unfortunately, real machines do not include a Φ instruction
- So after analysis, optimization, and transformation, need to convert back to a "Φ-less" form for execution
  - (Also sometimes needed for different kinds of analysis or transformation. A production optimizer might convert the IR into and out of SSA form multiple times)

Translating Φ-functions

- The meaning of $x := \Phi(x_1, x_2, \ldots, x_n)$ is "set $x = x_1$ if arriving on predecessor block edge 1, set $x = x_2$ if arriving on edge 2, etc."
- So, for each i, insert $x = x_i$ at the end of predecessor block i
- Rely on copy propagation and coalescing in register allocation to eliminate redundant copy instructions

SSA Wrapup

- More details needed to fully and efficiently implement SSA, but these are the main ideas
  – See recent compiler books (but not the Dragon book!)
- Allows efficient implementation of many optimizations
- SSA is used in most modern optimizing compilers (llvm is based on it) and has been retrofitted into many older ones (gcc is a well-known example)
- Not a silver bullet – some optimizations still need non-SSA forms – but very effective for many