

More on LL Grammars + Interpreters

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CSE 401/M501 – Section 5
Autumn 2020

Announcements

- Parser & AST due *tonight!*
- Homework 3 (LL grammars) due next Thursday
 - *Please start early – the programming portion may take a while!*

Agenda

- **What LL (Top-Down) Parsing Looks Like**
- LL Grammar Issues
 - FIRST Conflict
 - FIRST FOLLOW Conflict
 - Recursion
 - Indirect Left Recursion
- Interpreters vs Compilers

L L (1)



Left-to-Right
Only takes one pass,
performed from the left

Leftmost
At each point, finds the
derivation for the leftmost
handle (**top-down**)

1 Terminal Lookahead
Must determine derivation
from the next unparsed
terminal in the string

Top-Down Derivation of "a z x"

0. $S ::= a B$

1. $B ::= C x \mid y$

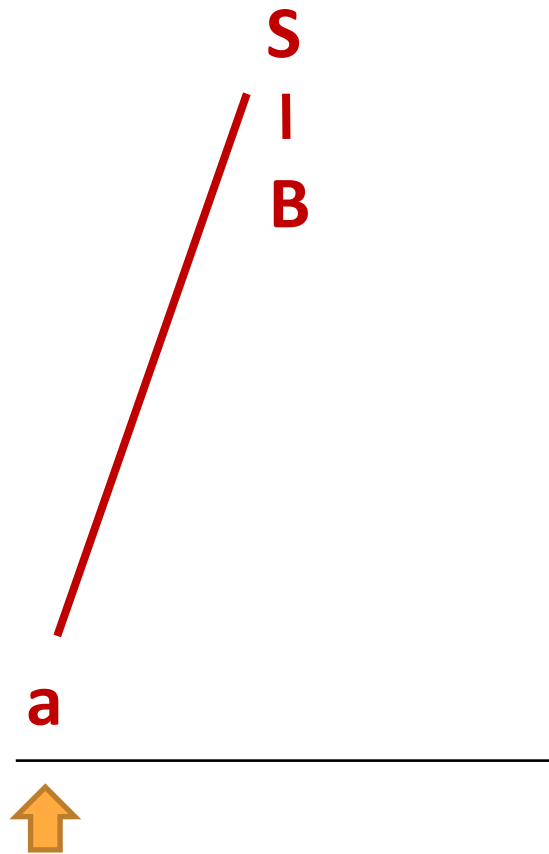
2. $C ::= \epsilon \mid z$

Lookahead Remaining

a

z x

Top-Down Derivation of "a z x"



0. $S ::= a B$

1. $B ::= C x \mid y$

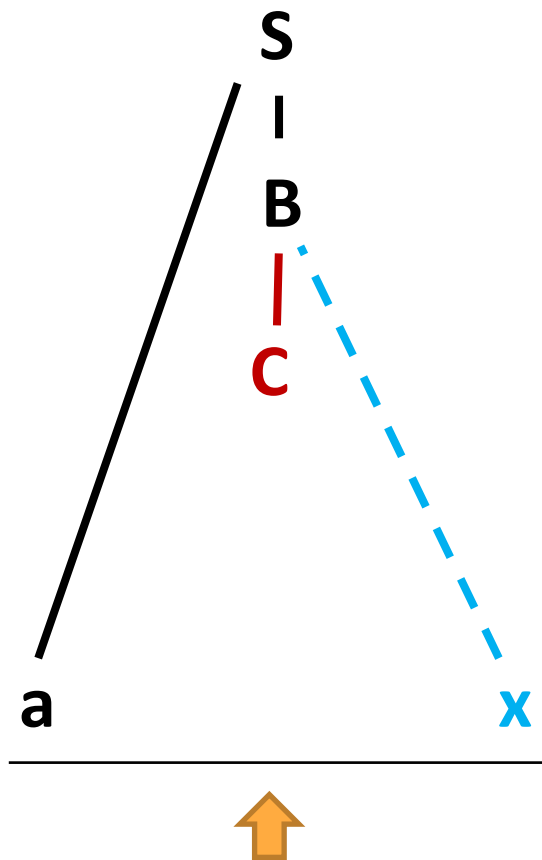
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Lookahead Remaining

a

z x

Top-Down Derivation of "a z x"



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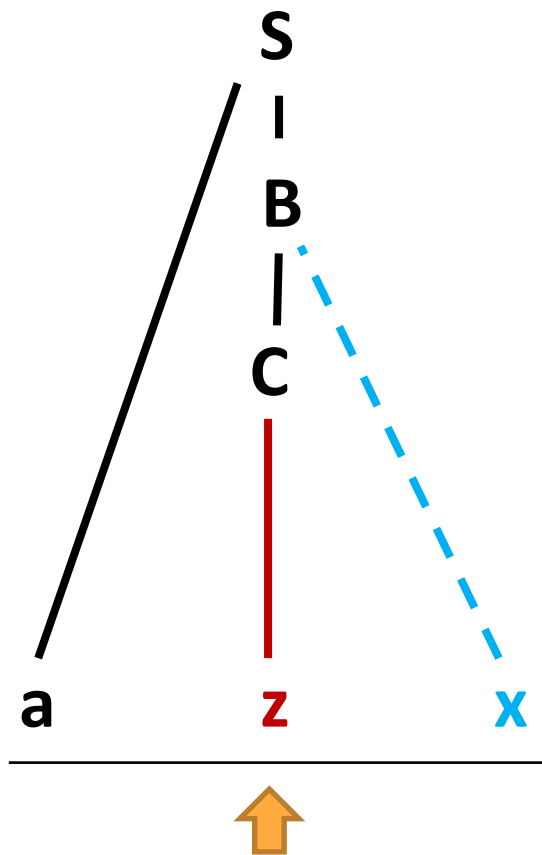
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Lookahead Remaining

z

x

Top-Down Derivation of "a z x"



0. $S ::= a B$

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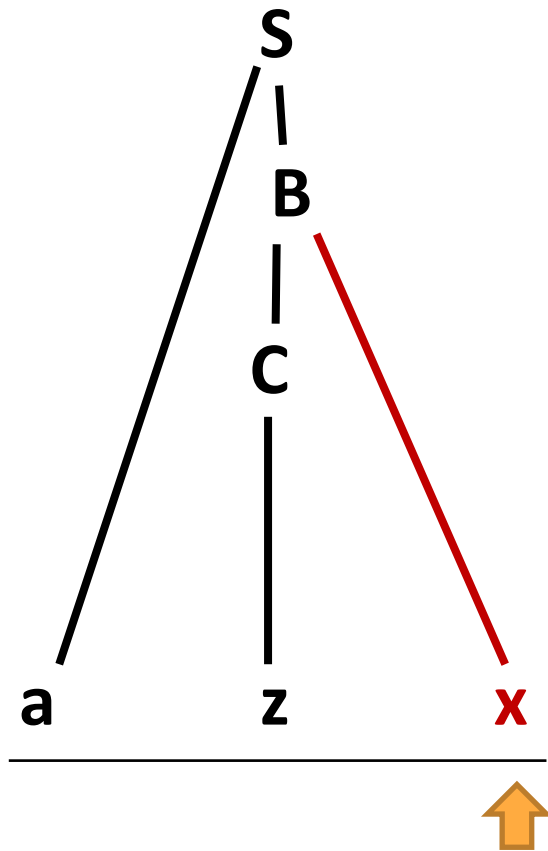
2. $C ::= \epsilon \mid z$

Lookahead Remaining

z

x

Top-Down Derivation of "a z x"



0. $S ::= a B$

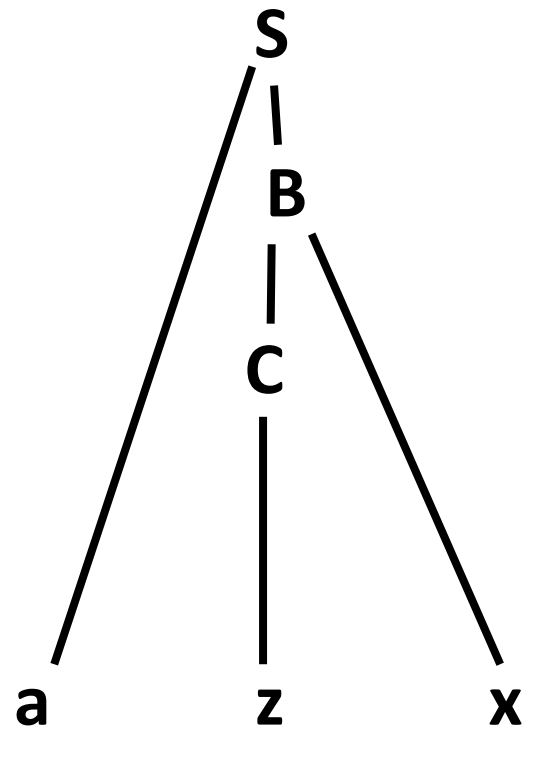
1. $B ::= C x \mid y$

2. $C ::= \epsilon \mid z$

Lookahead Remaining

x

Top-Down Derivation of "a z x"



0. $S ::= a B$

1. $B ::= C x \mid y$

2. $C ::= \epsilon \mid z$

Successful parse!

Agenda

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Canonical FIRST Conflict

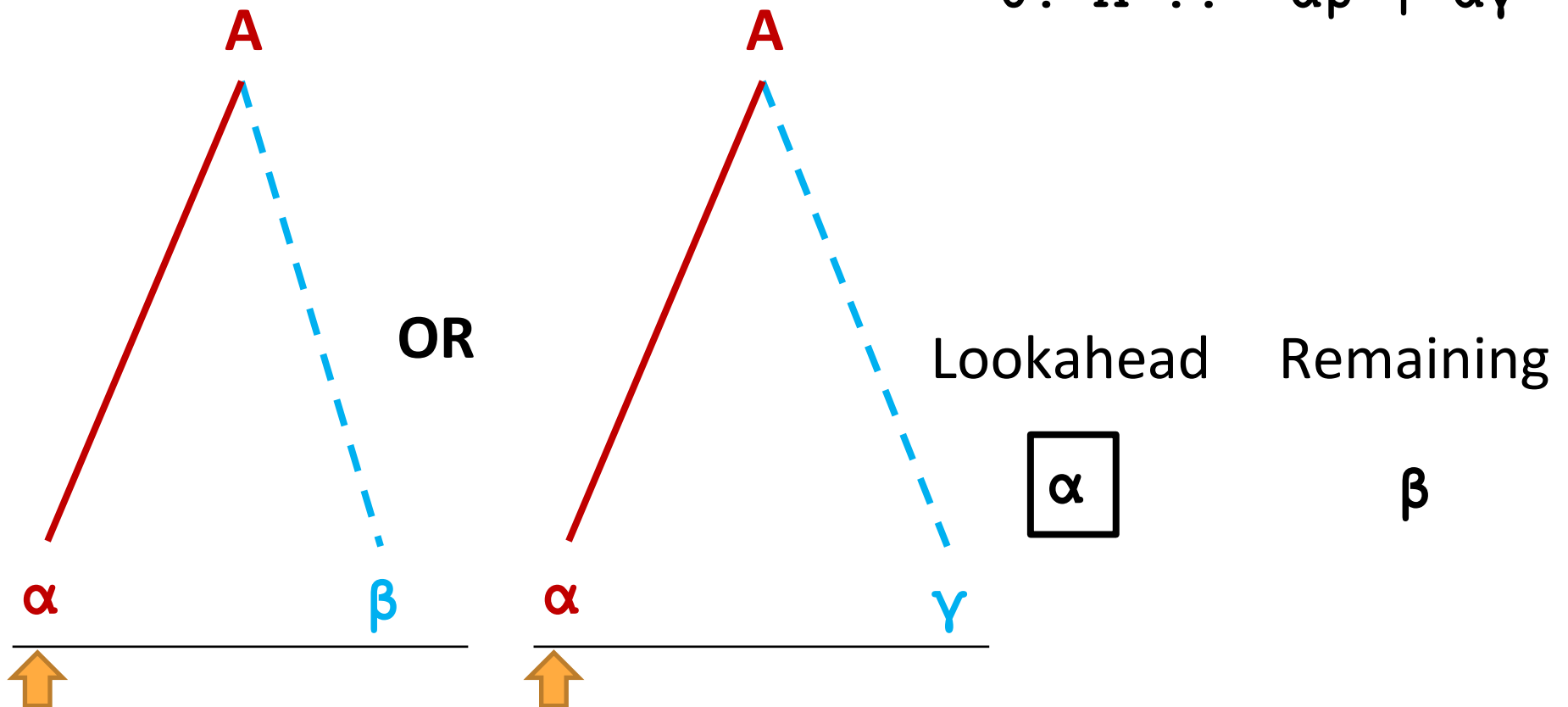
Problem

$$0. \ A ::= \alpha\beta \mid \alpha\gamma$$

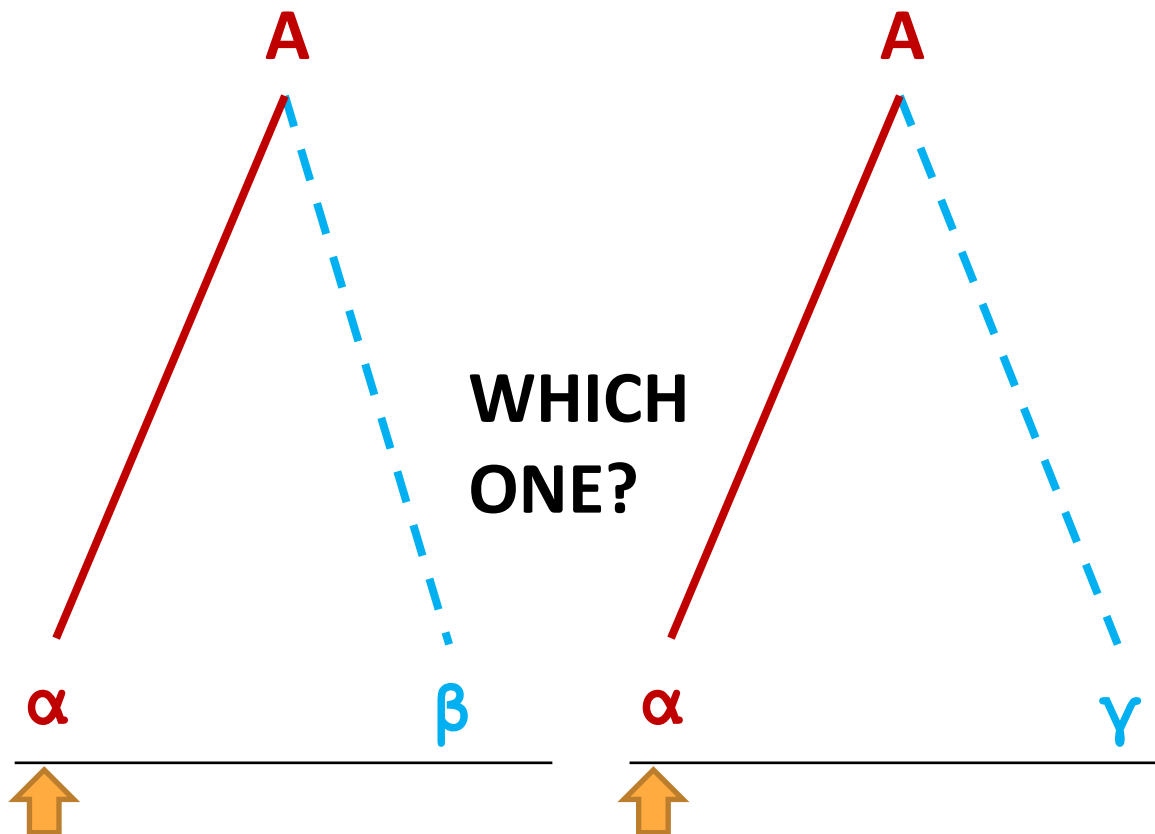
The FIRST sets of the right-hand sides for the **SAME NON-TERMINAL** must be disjoint!

Let's try a top-down derivation of $\alpha\beta$

0. $A ::= \alpha\beta \mid \alpha\gamma$



Let's try a top-down derivation of $\alpha\beta$



0. $A ::= \alpha\beta \mid \alpha\gamma$

We don't know!

We are using an $LL(1)$ parser, we can't see more than α !

Canonical FIRST Conflict Solution

Solution

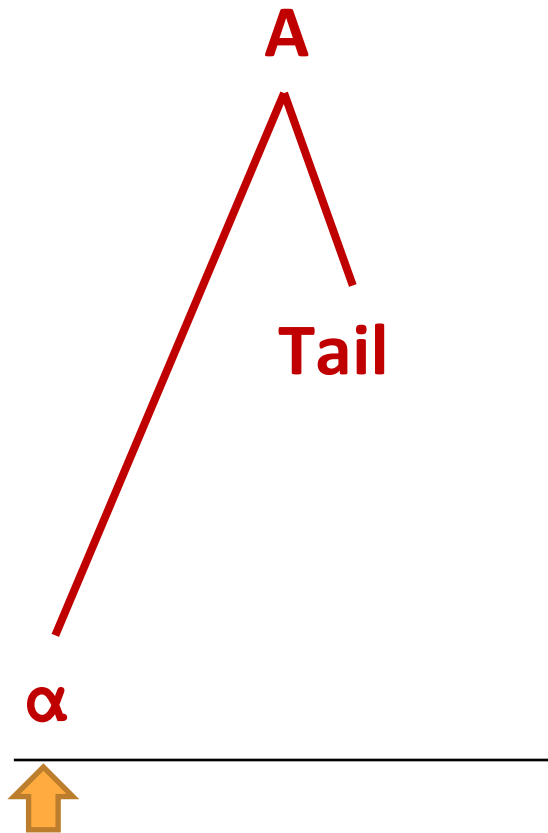
0. $A ::= \alpha\beta \mid \alpha\gamma$

0. $A ::= \alpha \text{ Tail}$

1. $\text{Tail} ::= \beta \mid \gamma$

Factor out the
common prefix

Top-Down Derivation of “ $\alpha\beta$ ”



0. $A ::= \alpha \text{ Tail}$

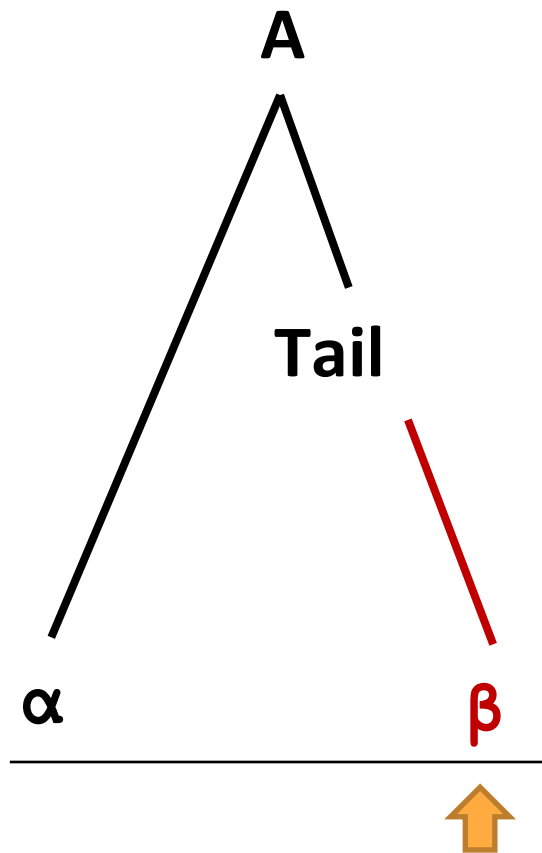
1. $\text{Tail} ::= \beta \mid \gamma$

Lookahead Remaining

α

β

Top-Down Derivation of “ $\alpha\beta$ ”



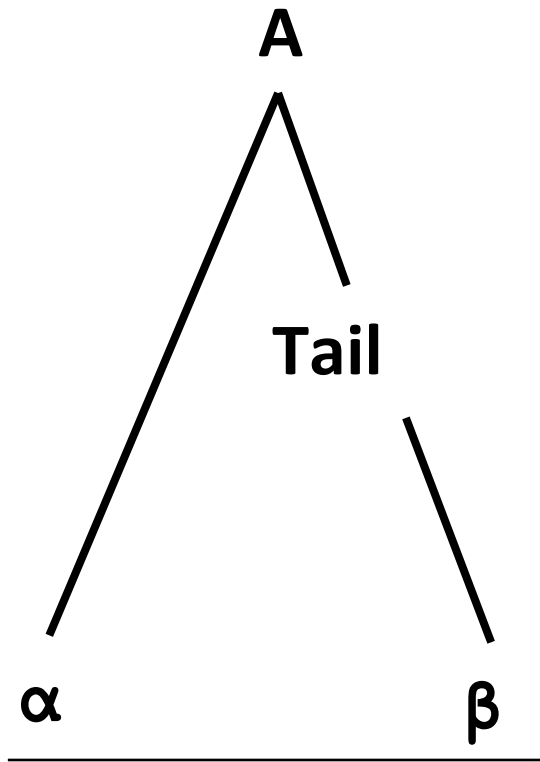
0. $A ::= \alpha \text{ Tail}$

1. $\text{Tail} ::= \beta \mid \gamma$

Lookahead Remaining

β

Top-Down Derivation of “ $\alpha\beta$ ”



0. $A ::= \alpha \text{ Tail}$

1. $\text{Tail} ::= \beta \mid \gamma$

Successful parse!

Changing original grammar a little (Grammar 1)

0. $S ::= a B \mid a w$

1. $B ::= C x \mid y$

2. $C ::= \varepsilon \mid z$

Lookahead

Remaining

a

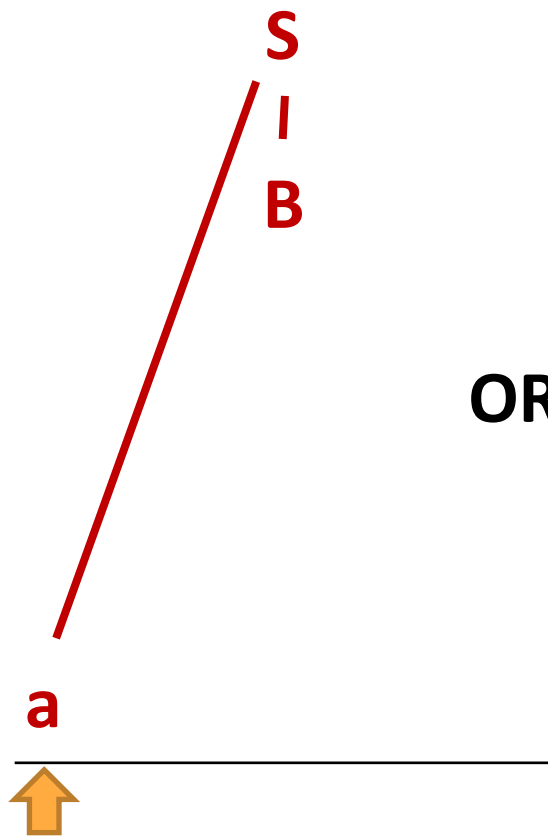
z x

What's the issue?

0. $S ::= \mathbf{a} B \mid \mathbf{a} w$
1. $B ::= C x \mid y$
2. $C ::= \varepsilon \mid z$

There's a FIRST Conflict!

Top-Down Derivation of "a z x": LL(1) can't parse



OR



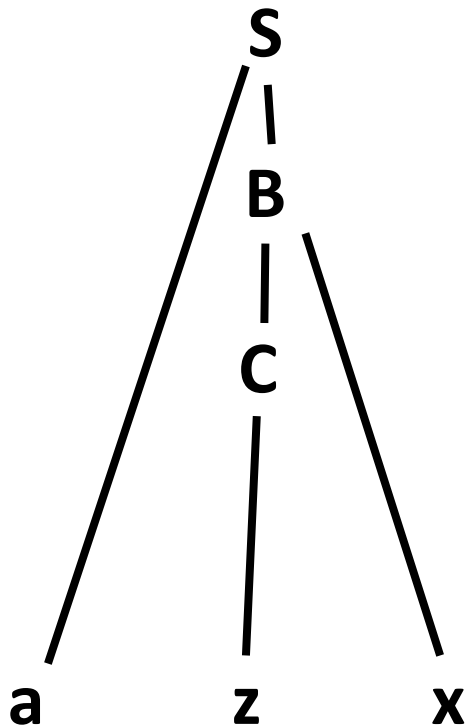
- 0. **S** ::= **a B** | **a w**
- 1. **B** ::= **C x** | **y**
- 2. **C** ::= ϵ | **z**

Lookahead Remaining

a

z x

Parse Tree without changing Grammar



0. $S ::= a B \mid a w$

1. $B ::= C x \mid y$

2. $C ::= \varepsilon \mid z$

Applying the Fix: Factor out the Common Prefix

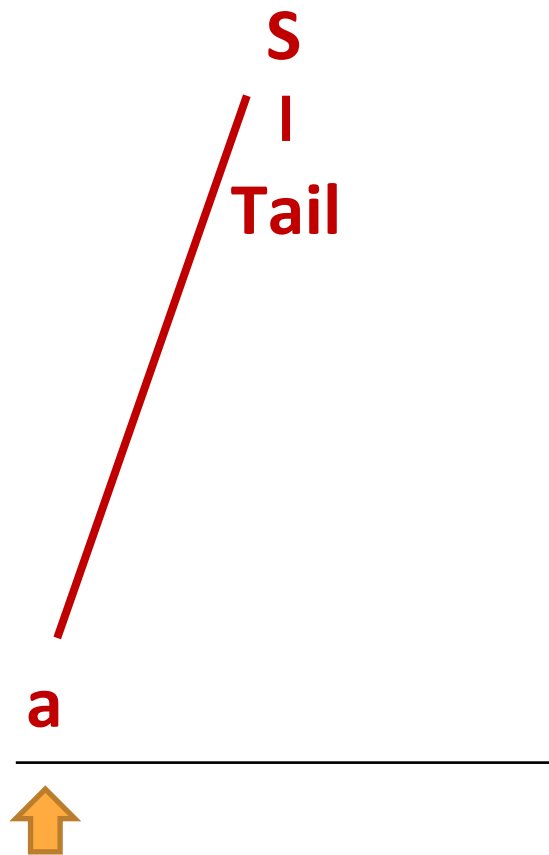
0. **S ::= a Tail**

1. **Tail ::= B | w**

2. B ::= C x | y

3. C ::= ε | z

Top-Down Derivation of "a z x"



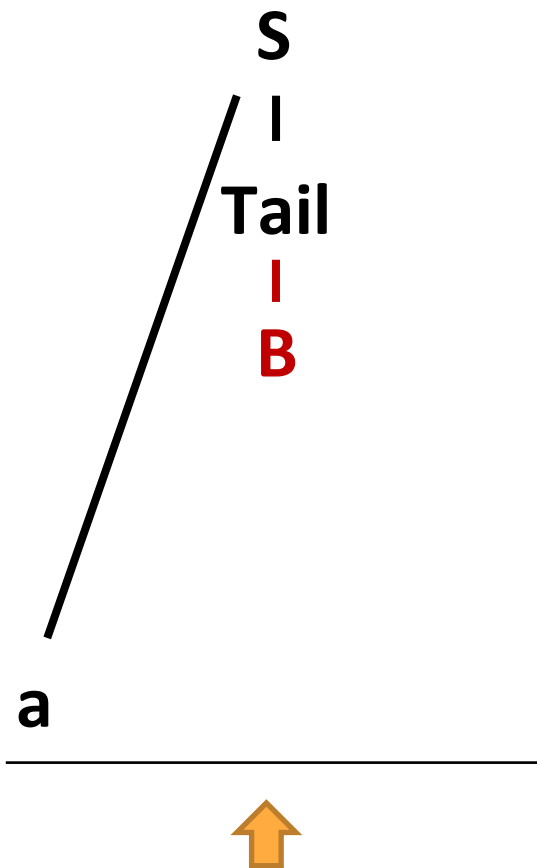
0. **S ::= a Tail**
1. **Tail ::= B | w**
2. **B ::= C x | y**
3. **C ::= ε | z**

Lookahead Remaining

a

z x

Top-Down Derivation of "a z x"



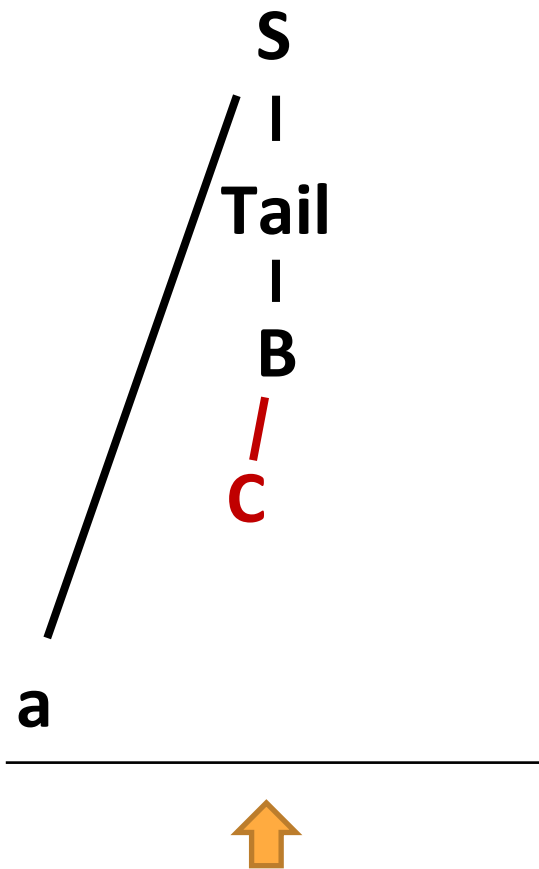
0. **S ::= a Tail**
1. **Tail ::= B | w**
2. **B ::= C x | y**
3. **C ::= ε | z**

Lookahead Remaining

z

x

Top-Down Derivation of "a z x"



0. $S ::= a \text{ Tail}$

1. $\text{Tail} ::= B \mid w$

2. $B ::= C \ x \mid y$

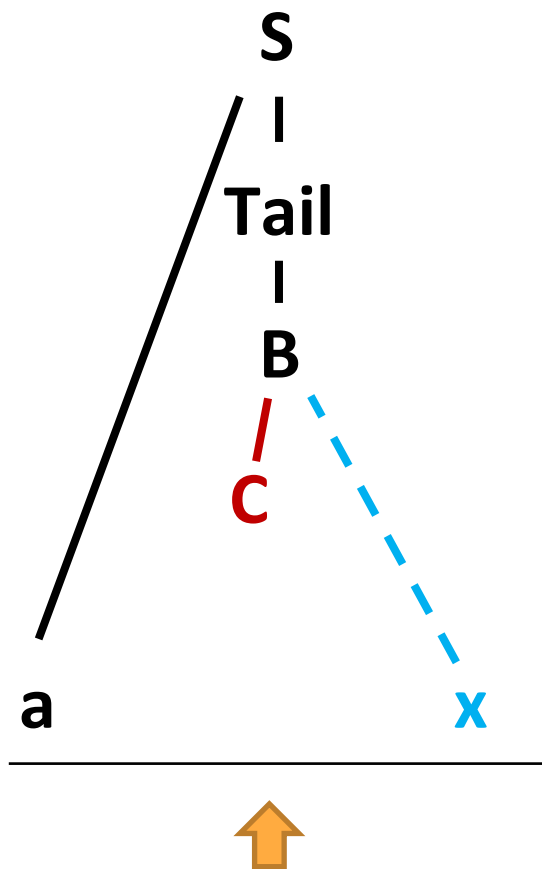
3. $C ::= \epsilon \mid z$

Lookahead Remaining

z

x

Top-Down Derivation of "a z x"



0. **S ::= a Tail**

1. **Tail ::= B | w**

2. **B ::= C x | y**

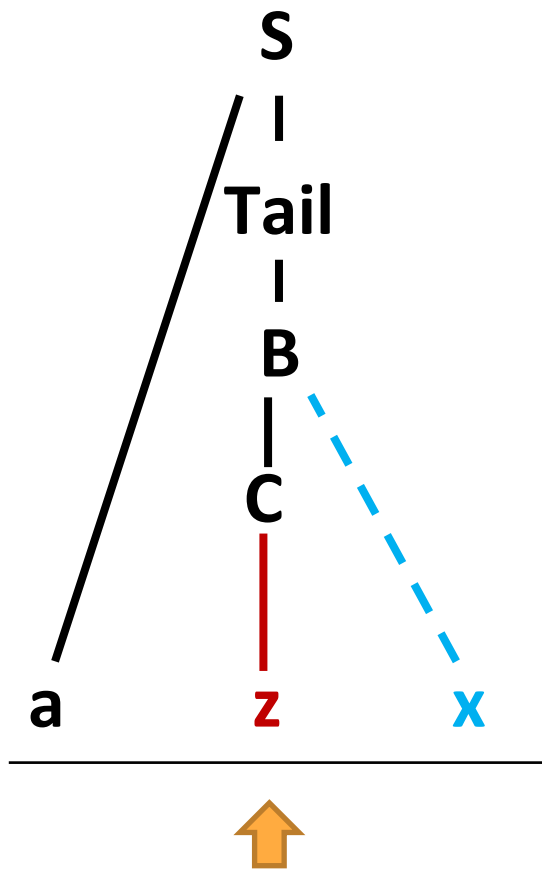
3. **C ::= ε | z**

Lookahead Remaining

z

x

Top-Down Derivation of "a z x"



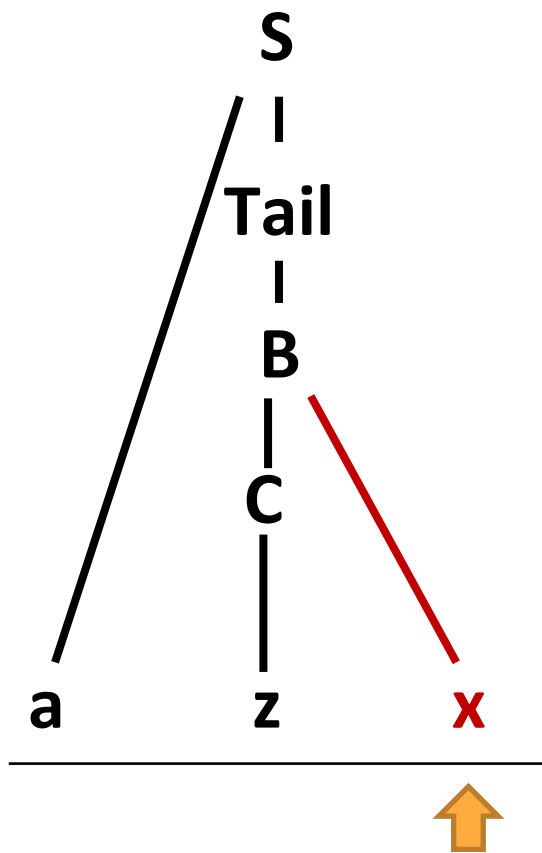
0. **S ::= a Tail**
1. **Tail ::= B | w**
2. **B ::= C x | y**
3. **C ::= ε | z**

Lookahead Remaining

z

x

Top-Down Derivation of "a z x"



0. $S ::= a \text{ Tail}$

1. $\text{Tail} ::= B \mid w$

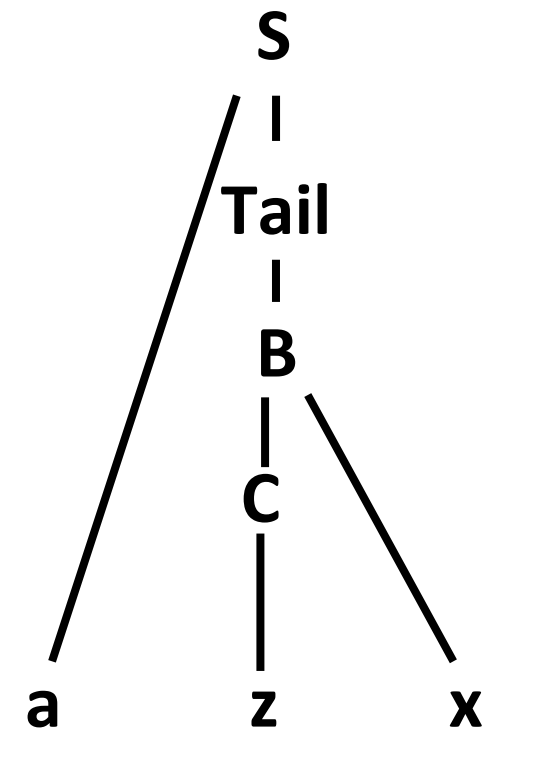
2. $B ::= C \ x \mid y$

3. $C ::= \epsilon \mid z$

Lookahead Remaining

x

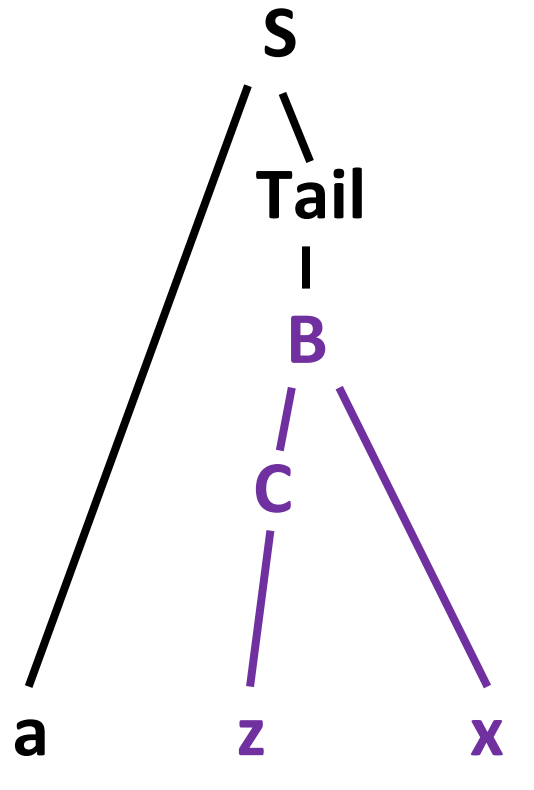
Top-Down Derivation of "a z x"



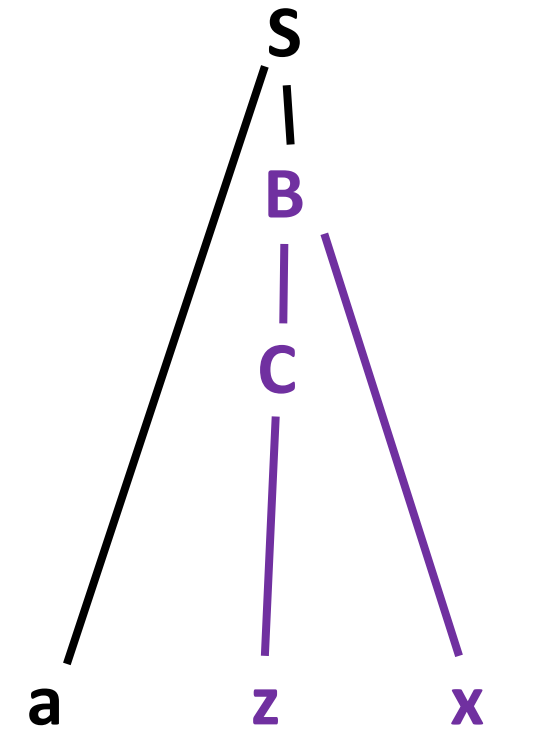
0. **S ::= a Tail**
1. **Tail ::= B | w**
2. **B ::= C x | y**
3. **C ::= ε | z**

Success!

Comparing Parse Trees



Purple trees
are the same!



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Canonical FIRST FOLLOW Conflict

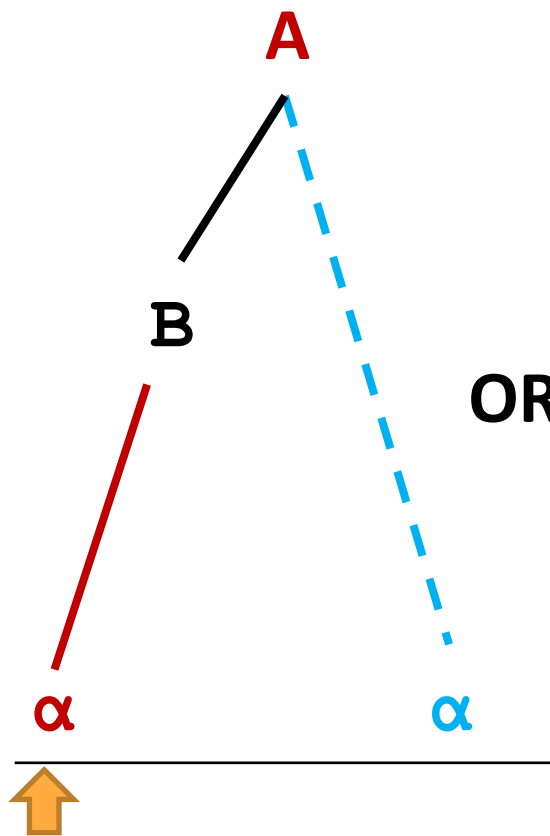
Problem

0. $A ::= B \alpha$

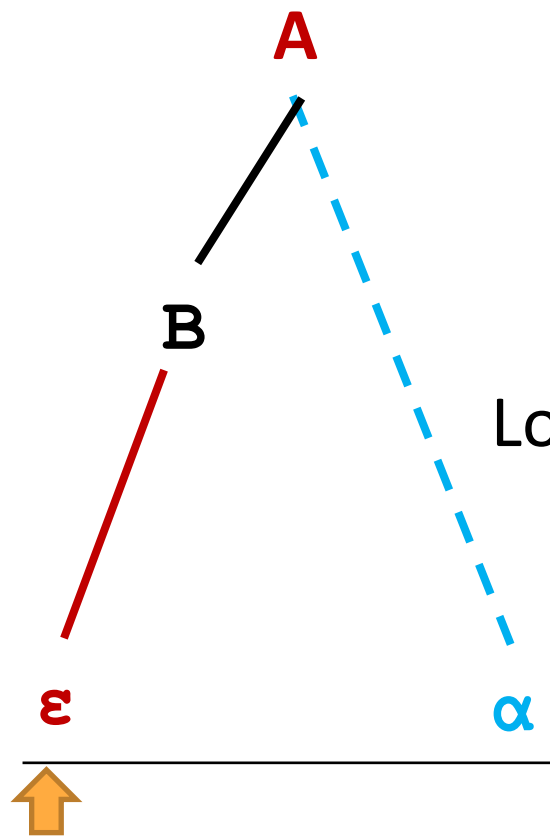
1. $B ::= \alpha \mid \epsilon$

Because B is nullable, its FOLLOW set must be disjoint from the FIRST sets of its right-hand sides!

Let's try a top-down derivation of " α "



OR



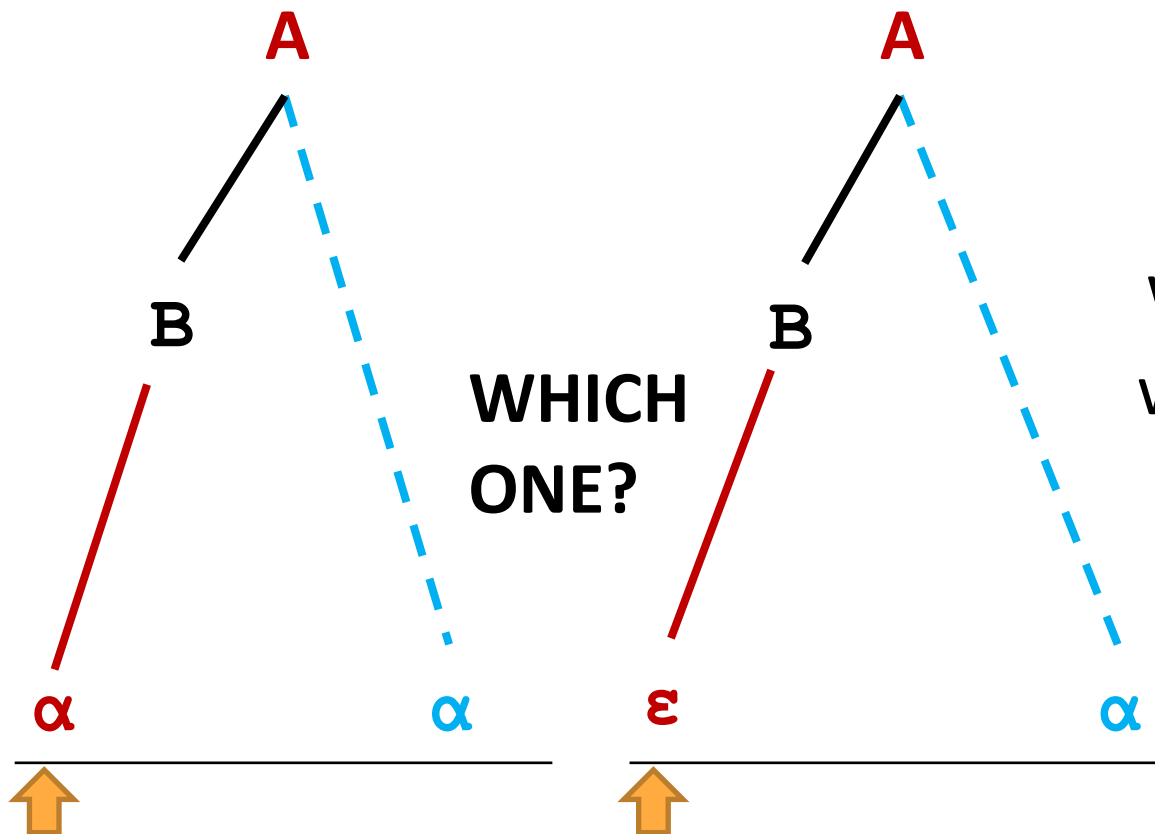
0. $A ::= B \alpha$

1. $B ::= \alpha \mid \epsilon$

Lookahead Remaining

α

Let's try a top-down derivation of " α "



0. $A ::= B \alpha$

1. $B ::= \alpha \mid \epsilon$

We don't know! Again,
we can't see more than
 α !

Canonical FIRST FOLLOW Conflict Solution

Solution

0. $A ::= B \alpha$

1. $B ::= \alpha \mid \varepsilon$

Substitute the
common prefix

0. $A ::= \alpha\alpha \mid \alpha$

0. $A ::= \alpha \text{ Tail}$

Factor out the
tail

1. $\text{Tail} ::= \alpha \mid \varepsilon$

Watch out for Nullability! (Grammar 2)

Changing the grammar again...

0. $S ::= a B$

1. $B ::= C x \mid y$

2. $C ::= \epsilon \mid x$

Lookahead

Remaining

a

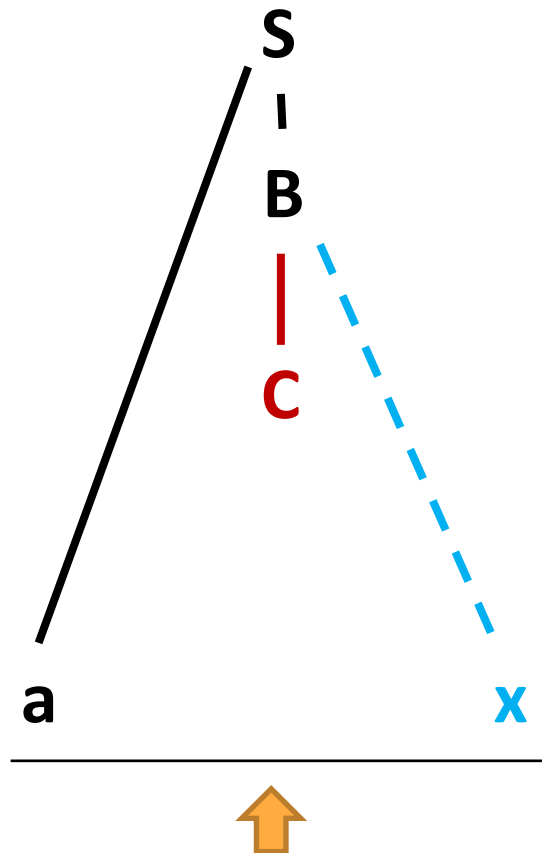
x

What's the issue?

0. $S ::= a B$
1. $B ::= C \mathbf{x} \mid y$
2. $C ::= \varepsilon \mid \mathbf{x}$

FIRST FOLLOW Conflict

Top down derivation of "ax"



0. $S ::= a B$

1. $B ::= C x \mid y$

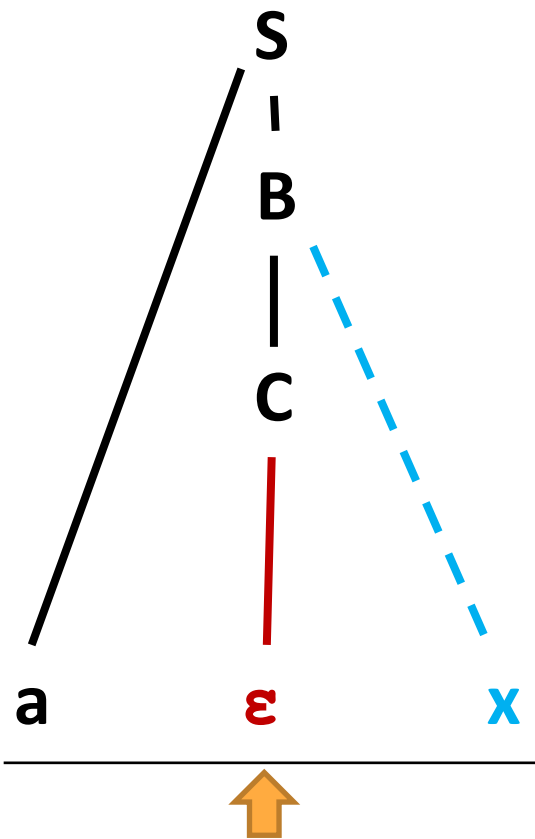
2. $C ::= \epsilon \mid \mathbf{x}$

Lookahead Remaining

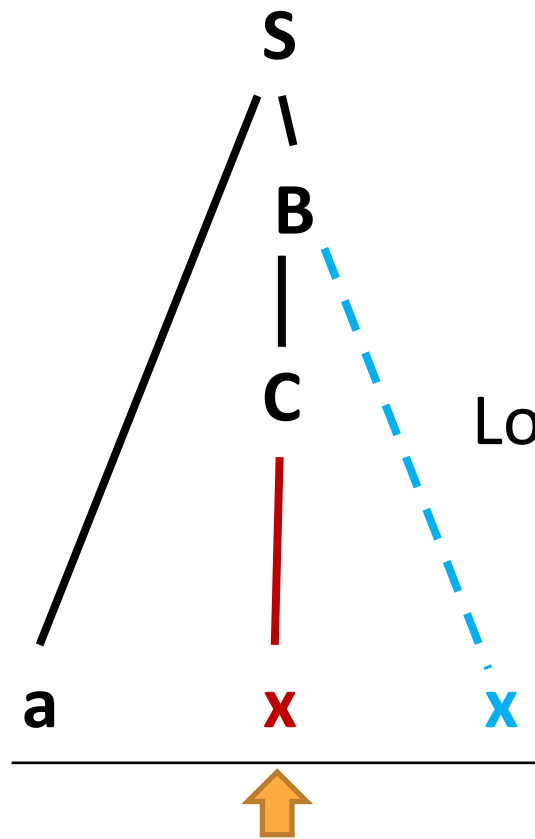
x

Top down derivation of "ax"

- 0. $S ::= a B$
- 1. $B ::= C x \mid y$
- 2. $C ::= \epsilon \mid x$



OR



Lookahead Remaining

x

Applying the Fix: Substitute the Common Prefix, then Factor

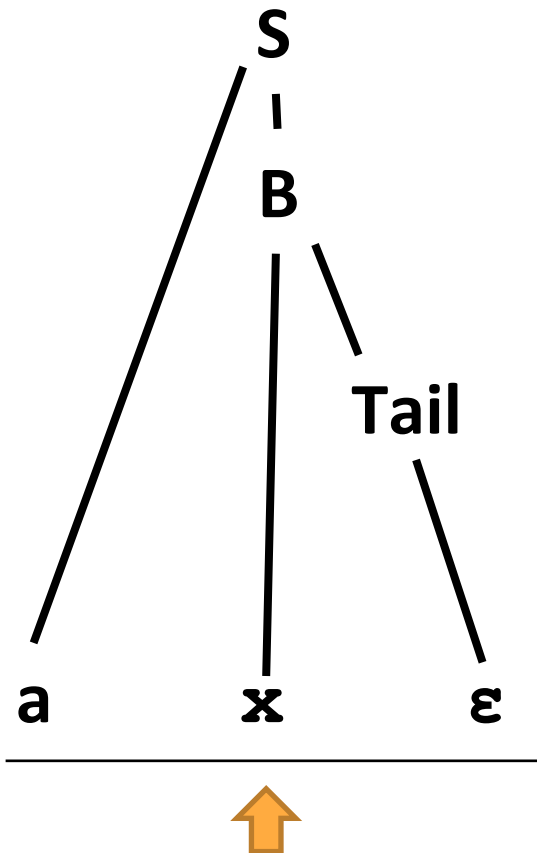
1

0. $S ::= a B$
1. $B ::= x \mid xx \mid y$
~~2. $C ::= \epsilon \mid x$~~

2

0. $S ::= a B$
1. $B ::= x Tail \mid y$
2. $Tail ::= x \mid \epsilon$

Top down derivation of "ax"



0. $S ::= a B$

1. $B ::= \mathbf{x Tail} \mid \mathbf{y}$

2. $Tail ::= \mathbf{x} \mid \mathbf{\epsilon}$

Lookahead Remaining

x

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Canonical Left Recursion Problem

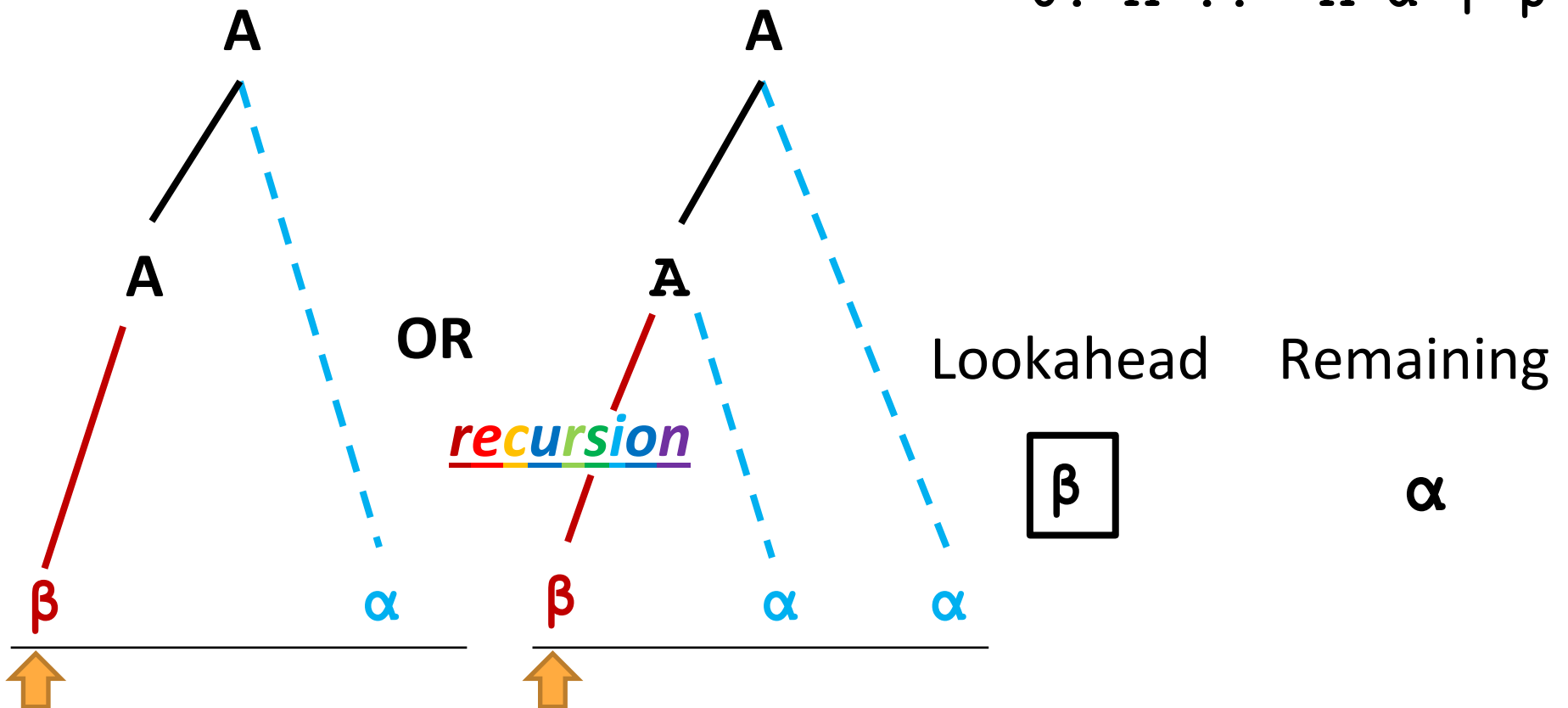
Problem

$$0. \mathbf{A} ::= \mathbf{A} \alpha \mid \beta$$

Left recursion can't be parsed by LL(1) parsers!

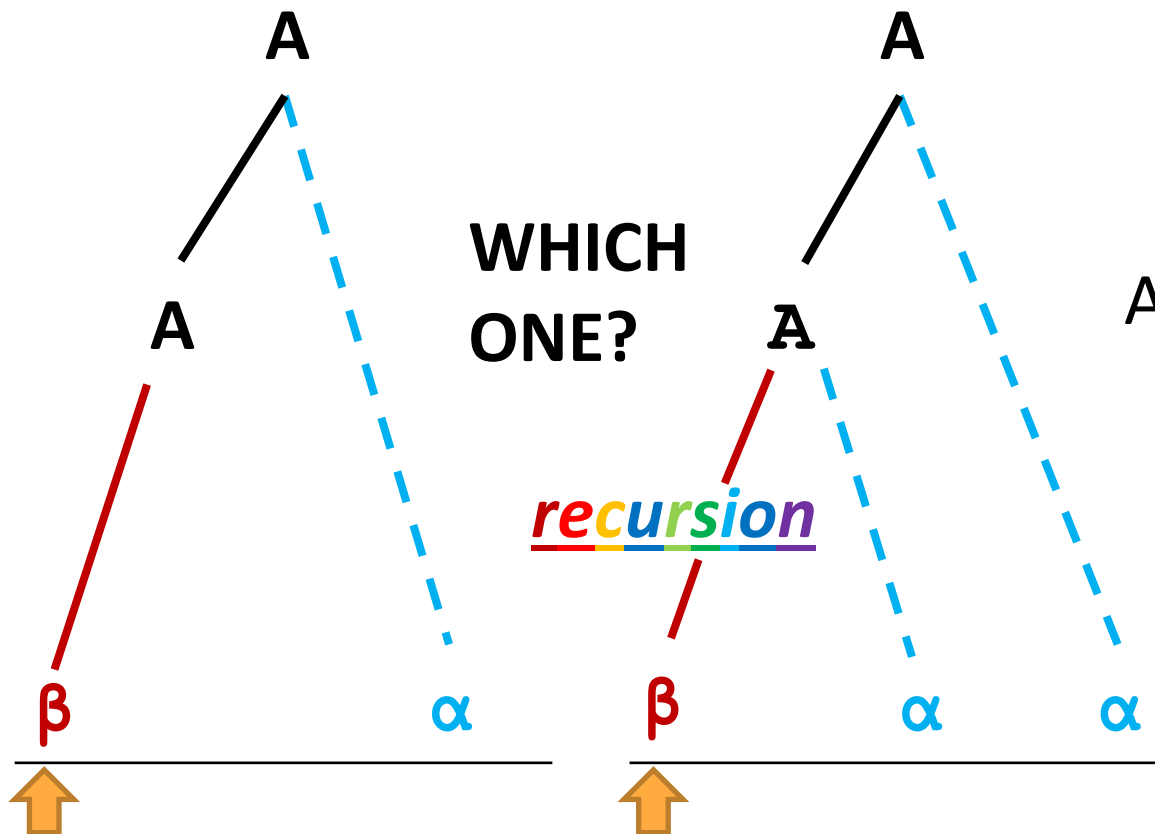
Let's try a top-down derivation of " $\beta\alpha$ "

0. $A ::= A \alpha \mid \beta$



Let's try a top-down derivation of " $\beta\alpha$ "

0. $A ::= A \alpha \mid \beta$



All we know right now is that it starts with β !

Canonical Left Recursion Solution

Solution

0. $A ::= A \alpha \mid \beta$

0. $A ::= \beta \text{ Tail}$

1. $\text{Tail} ::= \alpha \text{ Tail} \mid \epsilon$

1. Turn suffix of all A's recursive rhs into a tail non-terminal.
 2. Append the tail to every non-recursive rhs.
3. Add the empty string (ϵ) as a rhs for the tail production.

Let's change the grammar again! (Grammar 3)

0. $S ::= SB \mid a \mid w$

1. $B ::= Cx \mid y$

2. $C ::= \varepsilon \mid z$

Lookahead

Remaining

a

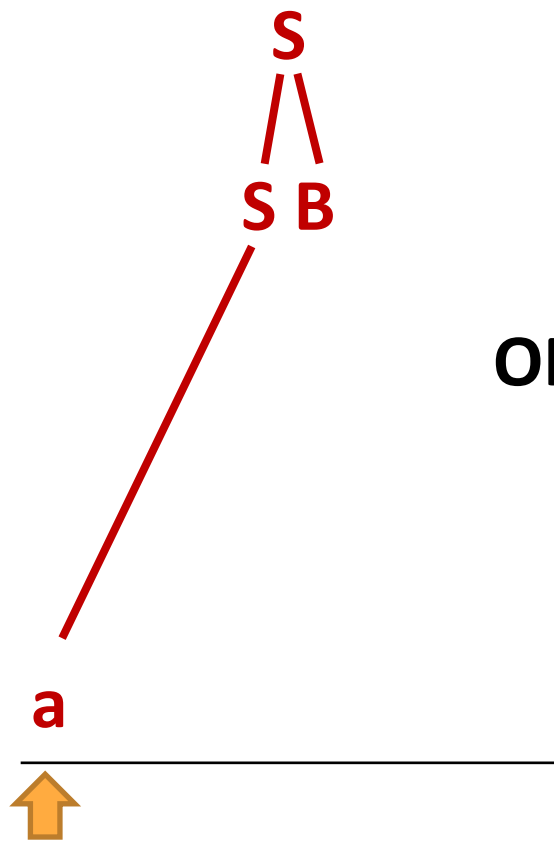
zx

What's the issue?

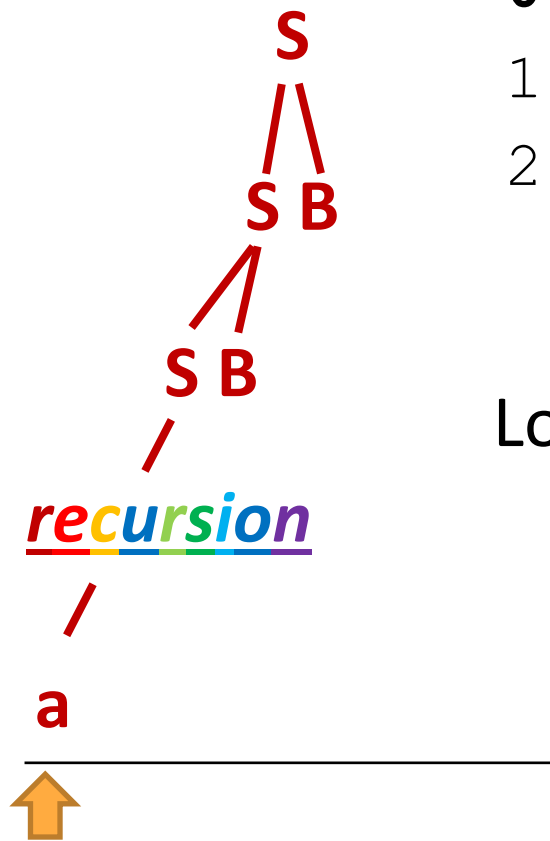
0. $S ::= \mathbf{S} B \mid a \mid w$
1. $B ::= C x \mid y$
2. $C ::= \varepsilon \mid z$

Left recursion can't be parsed by LL(1) parsers!

Top-Down Derivation of "a z x"



OR



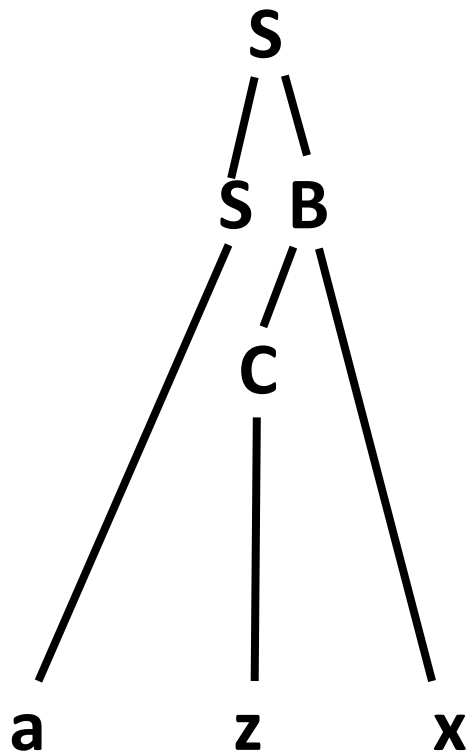
- 0. **S** ::= **S B** | **a** | **w**
- 1. **B** ::= **C x** | **y**
- 2. **C** ::= ϵ | **z**

Lookahead Remaining

a

z x

Parse Tree without changing Grammar



0. $S ::= S B \mid a \mid w$
1. $B ::= C x \mid y$
2. $C ::= \varepsilon \mid z$

Applying the fix:

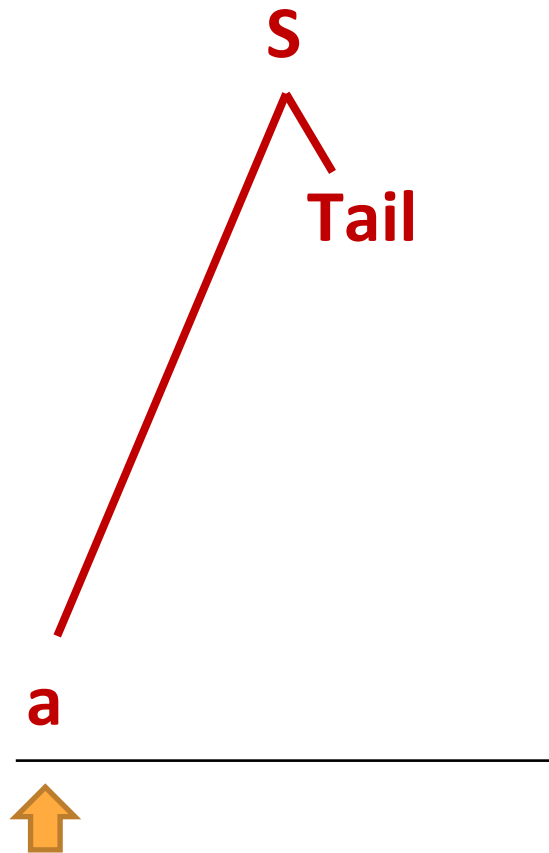
0. $S ::= a \text{ Tail} \mid w \text{ Tail}$

1. $\text{Tail} ::= B \text{ Tail} \mid \varepsilon$

2. $B ::= C x \mid y$

3. $C ::= \varepsilon \mid z$

Top-Down Derivation of "a z x"



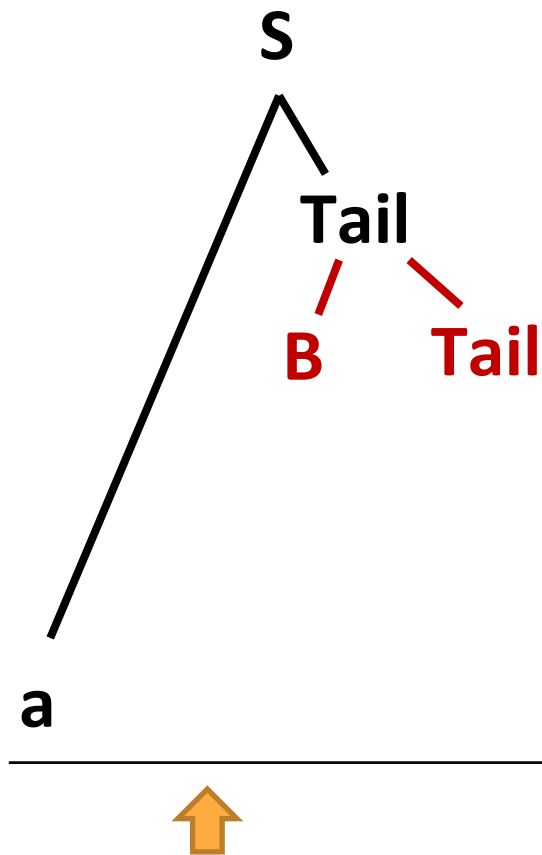
0. $S ::= a \text{ Tail} \mid w \text{ Tail}$
1. $\text{Tail} ::= B \text{ Tail} \mid \varepsilon$
2. $B ::= C x \mid y$
3. $C ::= \varepsilon \mid z$

Lookahead Remaining

a

z x

Top-Down Derivation of "a z x"



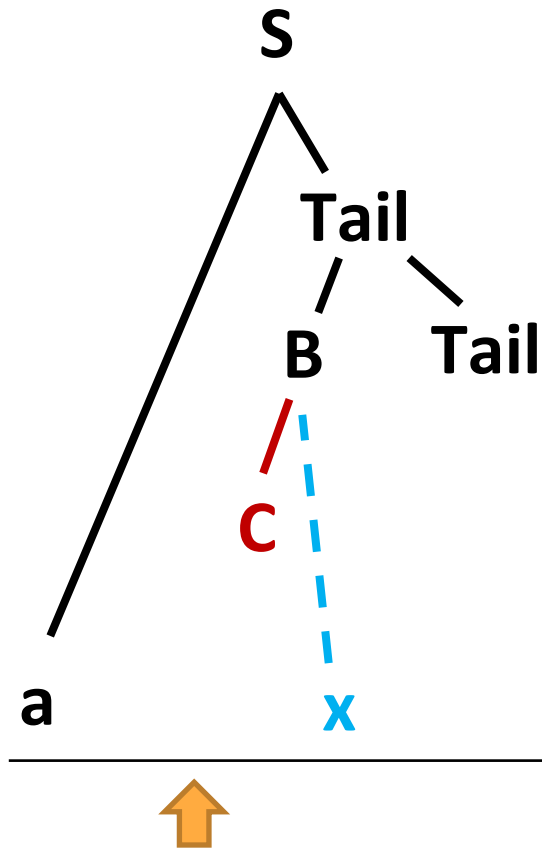
0. $S ::= a \text{ Tail} \mid w \text{ Tail}$
1. $\text{Tail} ::= B \text{ Tail} \mid \varepsilon$
2. $B ::= C x \mid y$
3. $C ::= \varepsilon \mid z$

Lookahead Remaining

z

x

Top-Down Derivation of "a z x"



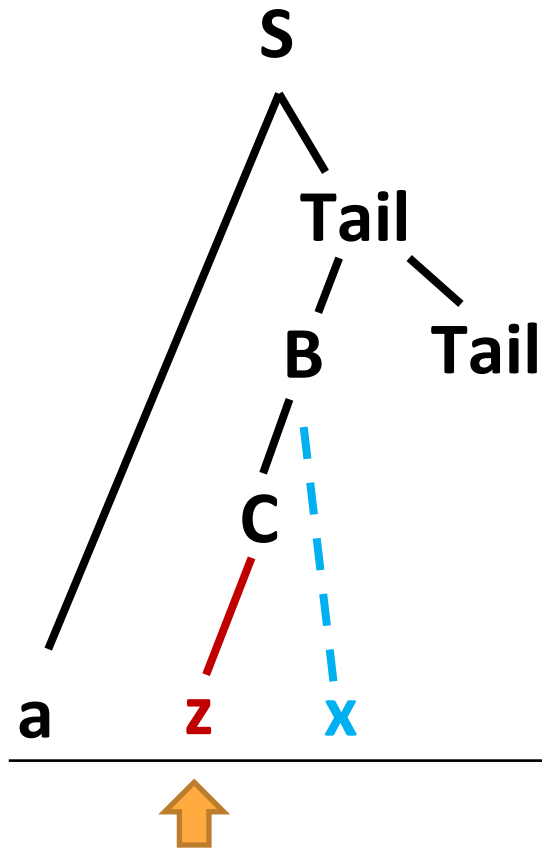
0. $S ::= a \text{ Tail} \mid w \text{ Tail}$
1. $\text{Tail} ::= B \text{ Tail} \mid \epsilon$
2. $B ::= C x \mid y$
3. $C ::= \epsilon \mid z$

Lookahead Remaining

z

x

Top-Down Derivation of "a z x"



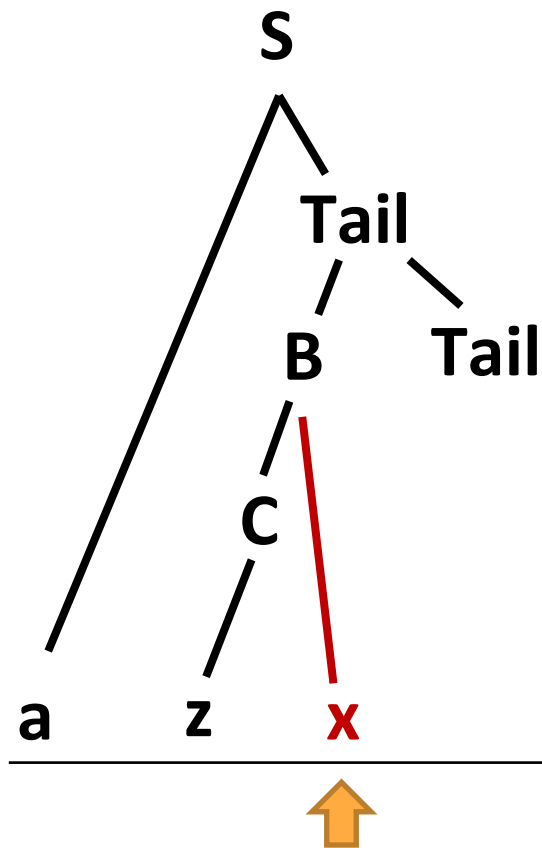
0. $S ::= a \text{ Tail} \mid w \text{ Tail}$
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3. $C ::= \epsilon \mid z$

Lookahead Remaining

z

x

Top-Down Derivation of "a z x"

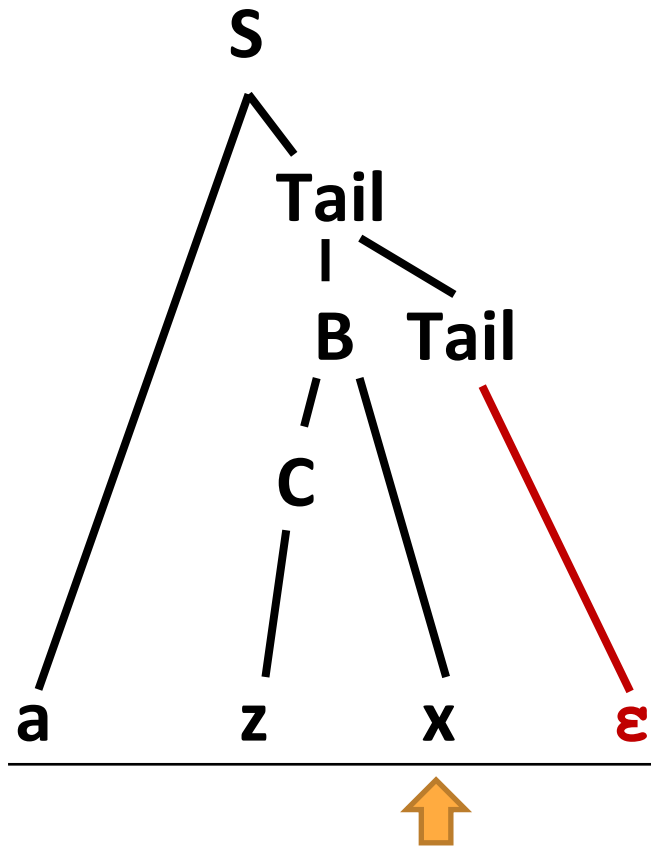


0. $S ::= a \text{ Tail} \mid w \text{ Tail}$
1. $\text{Tail} ::= B \text{ Tail} \mid \varepsilon$
2. $B ::= C x \mid y$
3. $C ::= \varepsilon \mid z$

Lookahead Remaining

x

Top-Down Derivation of "a z x"

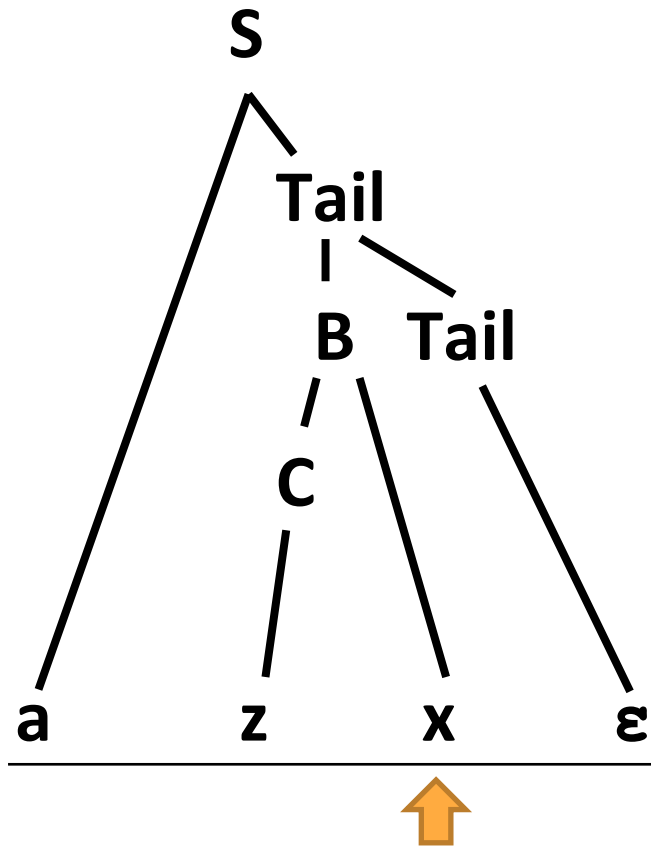


0. $S ::= a \text{ Tail} \mid w \text{ Tail}$
1. $\text{Tail} ::= B \text{ Tail} \mid \epsilon$
2. $B ::= C x \mid y$
3. $C ::= \epsilon \mid z$

Lookahead Remaining



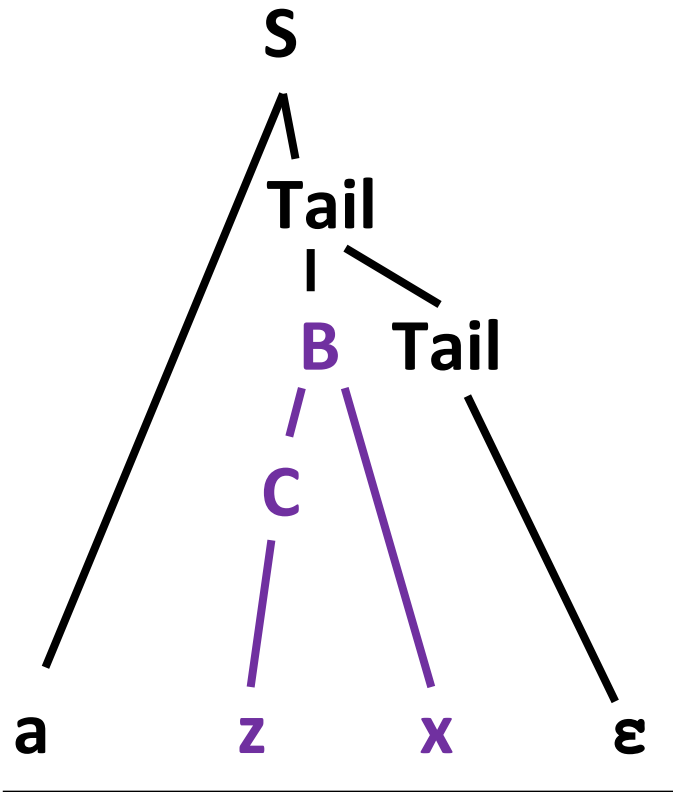
Top-Down Derivation of "a z x"



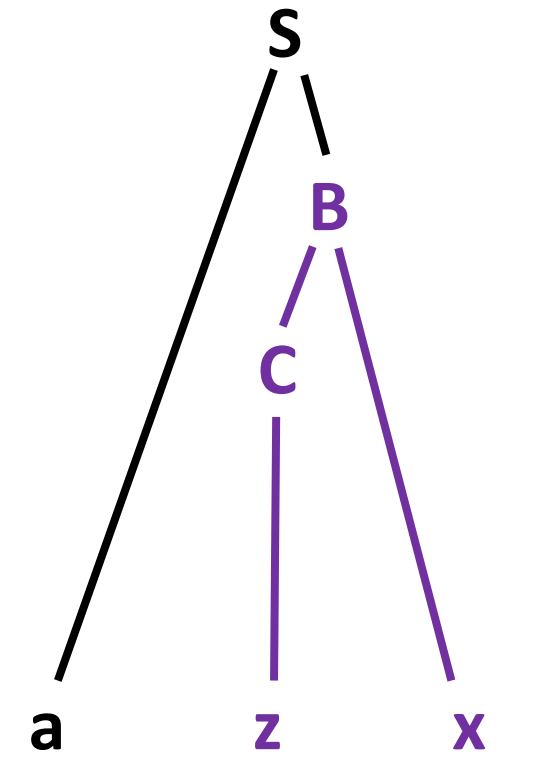
0. $S ::= a \text{ Tail} \mid w \text{ Tail}$
1. $\text{Tail} ::= B \text{ Tail} \mid \epsilon$
2. $B ::= C x \mid y$
3. $C ::= \epsilon \mid z$

Success!

Top-Down Derivation of "a z x"



Purple trees are the same again!



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 - **Indirect Left Recursion**
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Canonical Indirect Left Recursion Problem

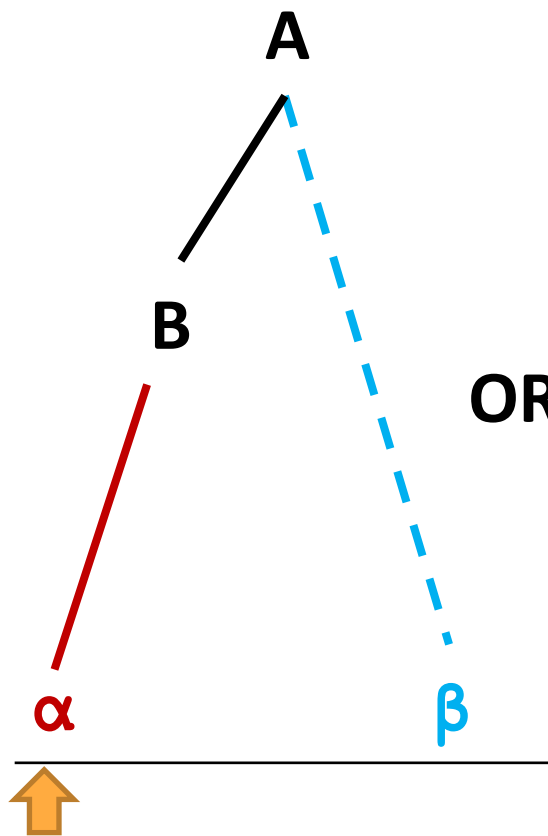
Problem

0. $A ::= B \beta$

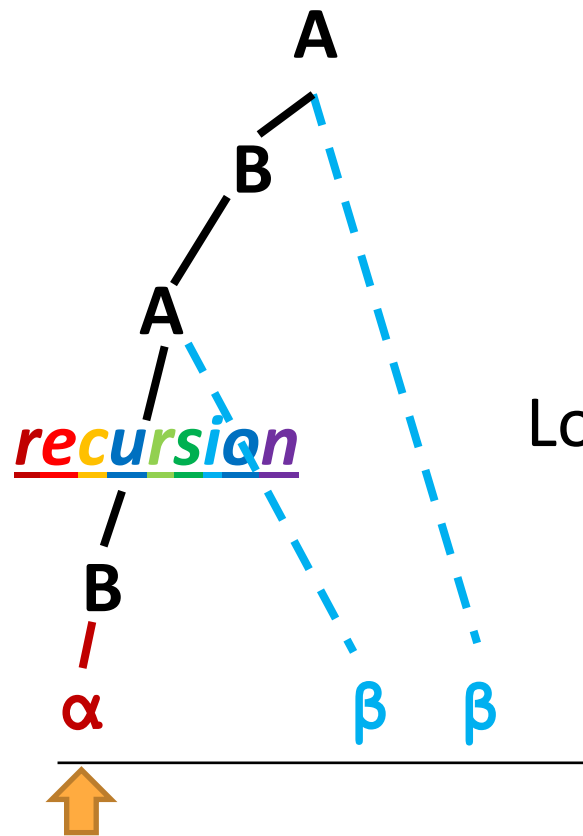
1. $B ::= A \mid \alpha$

Indirect Left recursion can't be parsed by LL(1)
parsers either

Let's try a top-down derivation of " $\alpha\beta$ "



OR



0. $A ::= B \beta$

1. $B ::= A \mid \alpha$

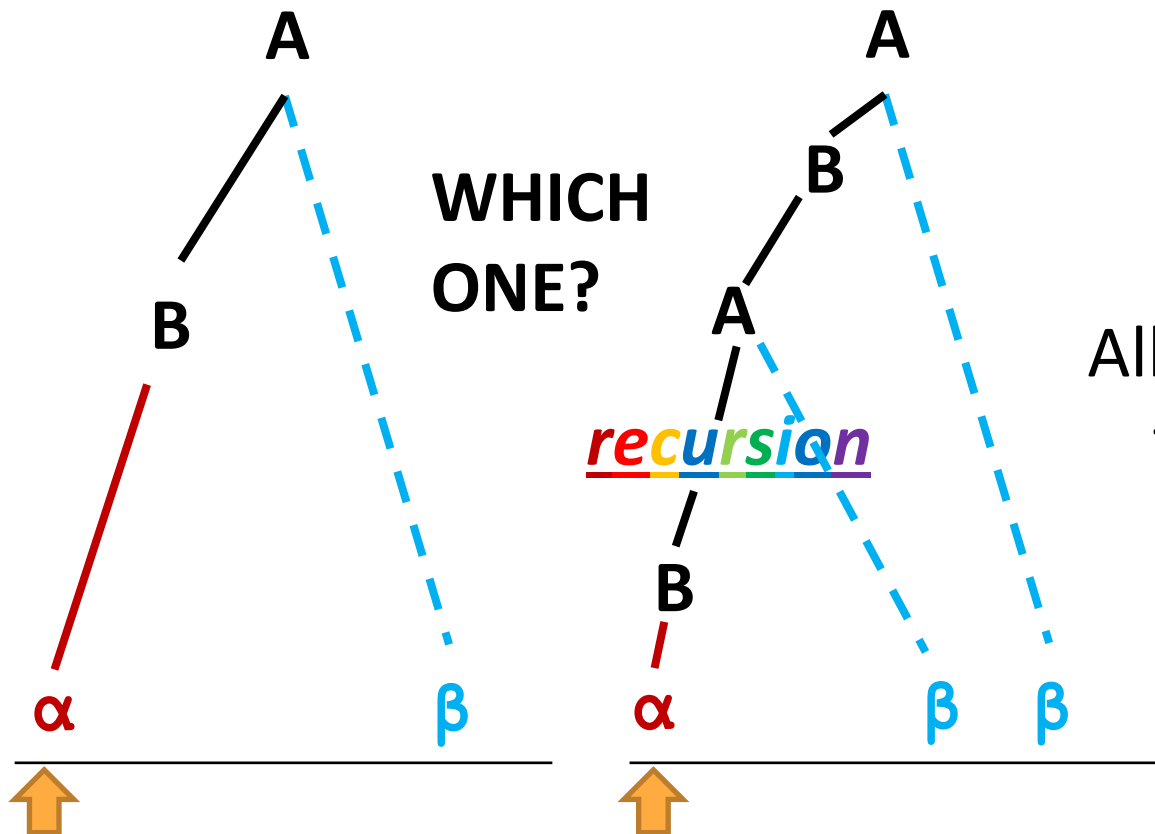
Lookahead

Remaining

α

β

Let's try a top-down derivation of " $\alpha\beta$ "



0. $A ::= B \beta$
1. $B ::= A \mid \alpha$

All we know right now is that it starts with α !

Canonical Indirect Left Recursion Solution

Solution

0. $A ::= B \beta$

1. $B ::= A \mid \alpha$

Substitute B into A

0. $A ::= A \beta \mid \alpha \beta$

Fix via typical Left
Recursion Solution

0. $A ::= \alpha \beta \text{Tail}$

1. $\text{Tail} ::= \beta \text{Tail} \mid \epsilon$

Note: **NEVER** remove the starting non-terminal!
The start symbol of the new grammar should be the same

Indirect Left Recursion

Changing the grammar for the last time...

0. $S ::= B w \mid a B$

1. $B ::= S \mid z x$

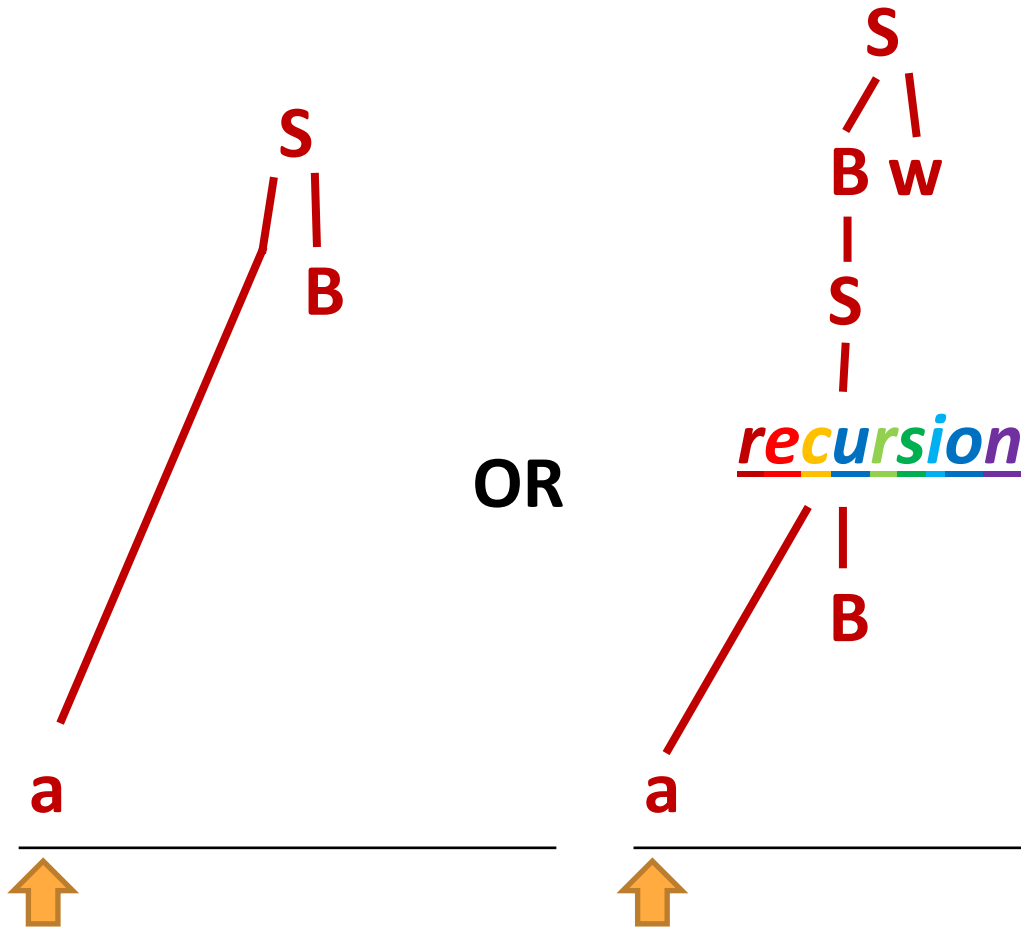
Lookahead

a

Remaining

z x

Indirect Left Recursion



OR

- 0. $S ::= B w \mid a B$
- 1. $B ::= S \mid z x$

Lookahead	Remaining
a	z x

Applying the fix: Substitute, then eliminate left recursion!

1 0. $S ::= S w \mid z x w \mid a S \mid a z x$
 ~~1. $B ::= S \mid z x$~~

2 0. $S ::= z x w T \mid a S T \mid a z x T$
 1. $T ::= w T \mid \epsilon$

3 0. $S ::= z x w T \mid a aT$
 1. $aT ::= S T \mid z x T$
 2. $T ::= w T \mid \epsilon$

Something else is wrong!
What is it?

Eliminating FIRST CONFLICT

4: sub S into
the S in aT

$$0. S ::= z x w T \mid a aT$$

$$1. aT ::= z x w T T \mid a aT T \mid z x T$$

$$2. T ::= w T \mid \epsilon$$

5: Add a tail
to zx as it
appears twice
in (4)

$$0. S ::= z x w T \mid a aT$$

$$1. aT ::= z x zT \mid a aT T$$

$$2. zT ::= w T T \mid T$$

$$3. T ::= w T \mid \epsilon$$

Eliminating FIRST CONFLICT

6: Sub rule
3 into the T
in rule 2.

0. $S ::= z x w T \mid a aT$
1. $aT ::= z x zT \mid a aT T$
2. $zT ::= w T T \mid w T \mid \epsilon$
3. $T ::= w T \mid \epsilon$

7: Notice in
rule 2, $w T T$ is
the same as $w T$,
since it's
recursively
appending w 's on
the right. Get rid
of $w T T$.

0. $S ::= z x w T \mid a aT$
1. $aT ::= z x zT \mid a aT T$
2. $zT ::= w T \mid \epsilon$
3. $T ::= w T \mid \epsilon$

Eliminating FIRST CONFLICT

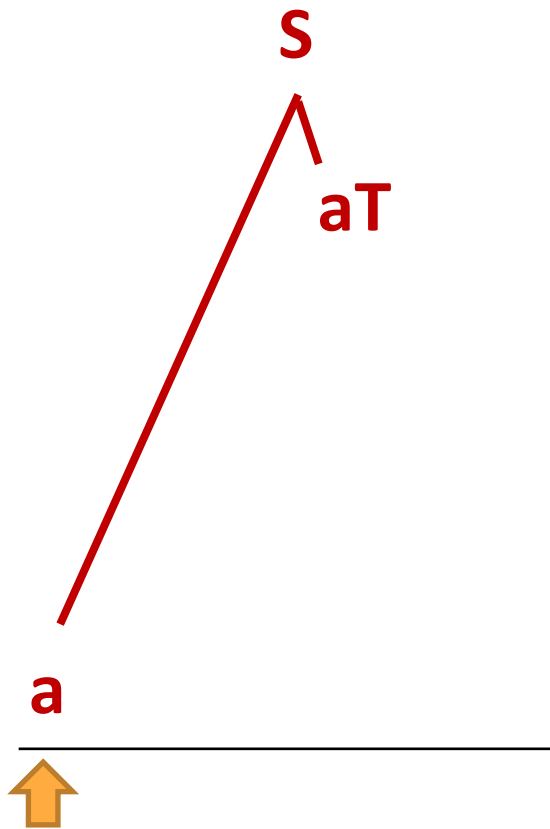
8: zT has been deleted because it was the same production as T . zT was replaced by T .

0. $S ::= z x w T \mid a aT$

1. $aT ::= z x T \mid a aT T$

2. $T ::= w T \mid \epsilon$

Top-down derivation of "azx"



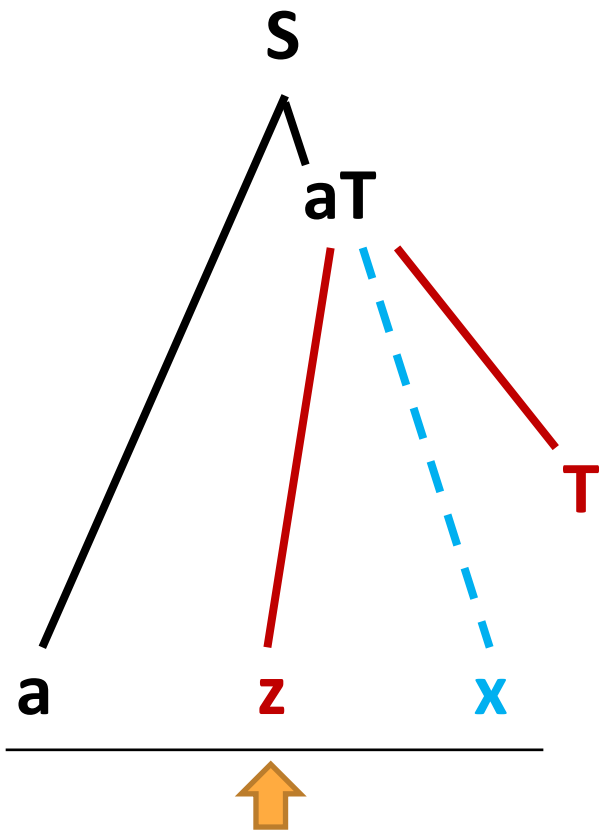
0. $S ::= z x w T \mid a aT$
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Lookahead Remaining

a

z x

Top-down derivation of "azx"



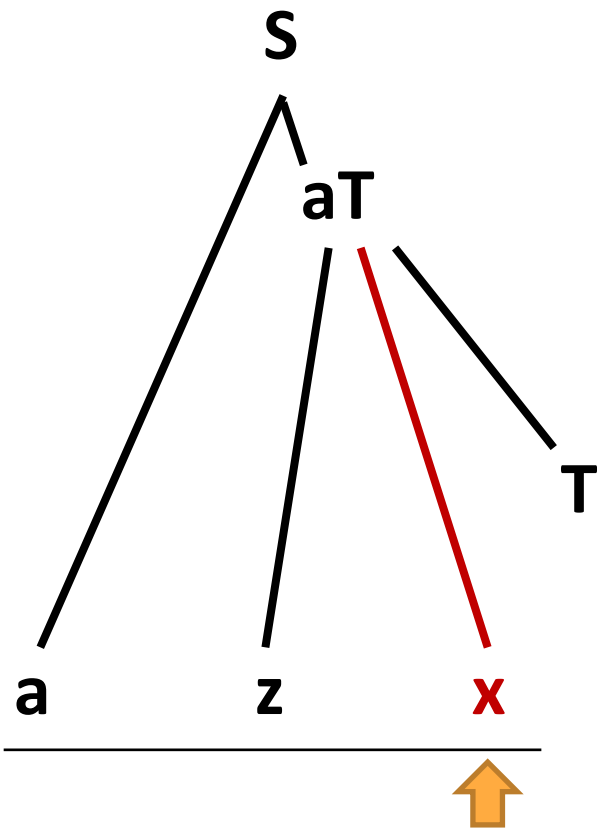
0. $S ::= z x w T \mid a aT$
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Lookahead Remaining

z

x

Top-down derivation of "azx"

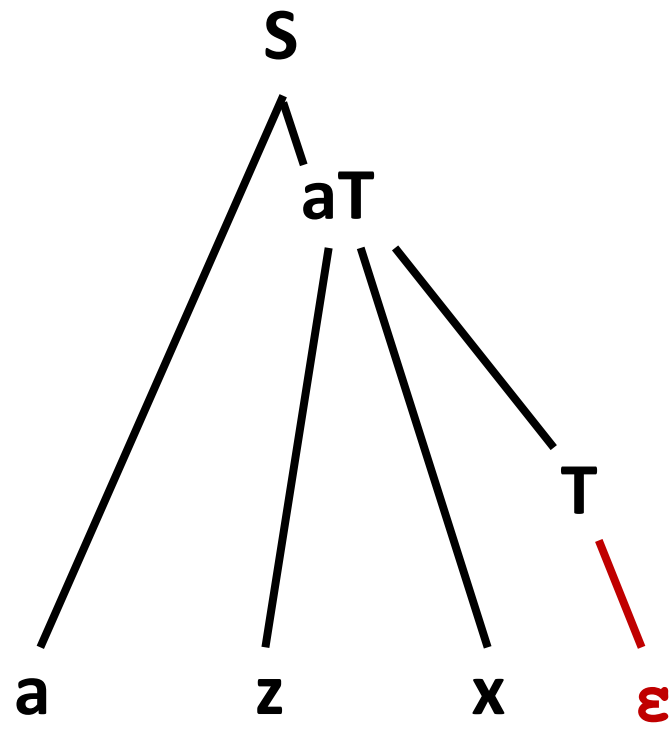


0. $S ::= z x w T \mid a aT$
1. $aT ::= z x T \mid a aT T$
2. $T ::= w T \mid \epsilon$

Lookahead Remaining

x

Top-down derivation of "azx"



0. $S ::= z x w T \mid a aT$
1. $aT ::= z x T \mid a aT T$
2. $T ::= w T \mid \epsilon$

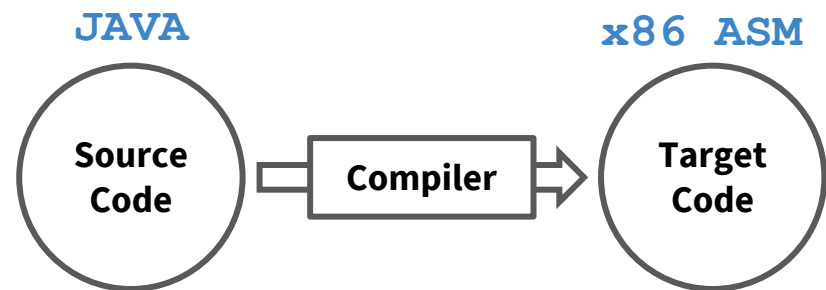
Success!

Agenda

- What LL (Top-Down) Parsing Looks Like
- LL Grammar Issues
 - FIRST Conflict
 - FIRST FOLLOW Conflict
 - Left Recursion
 - Indirect Left Recursion
- **Interpreters vs Compilers**

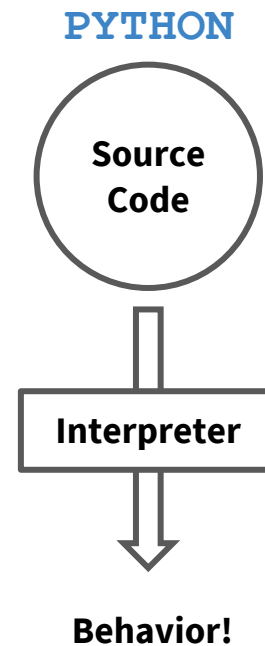
Interpreters vs. Compilers

- Compilers
 - Translate between different languages
 - e.g. MiniJava \Rightarrow x86 ASM
 - e.g. Java \Rightarrow Java Byte Code



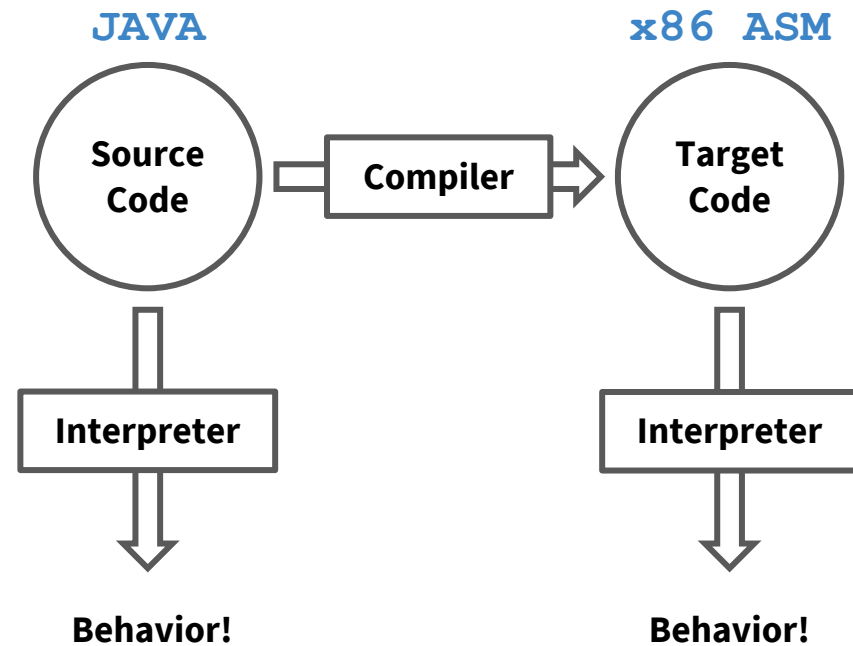
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- Interpreters
 - Take action upon a piece of code as it is read



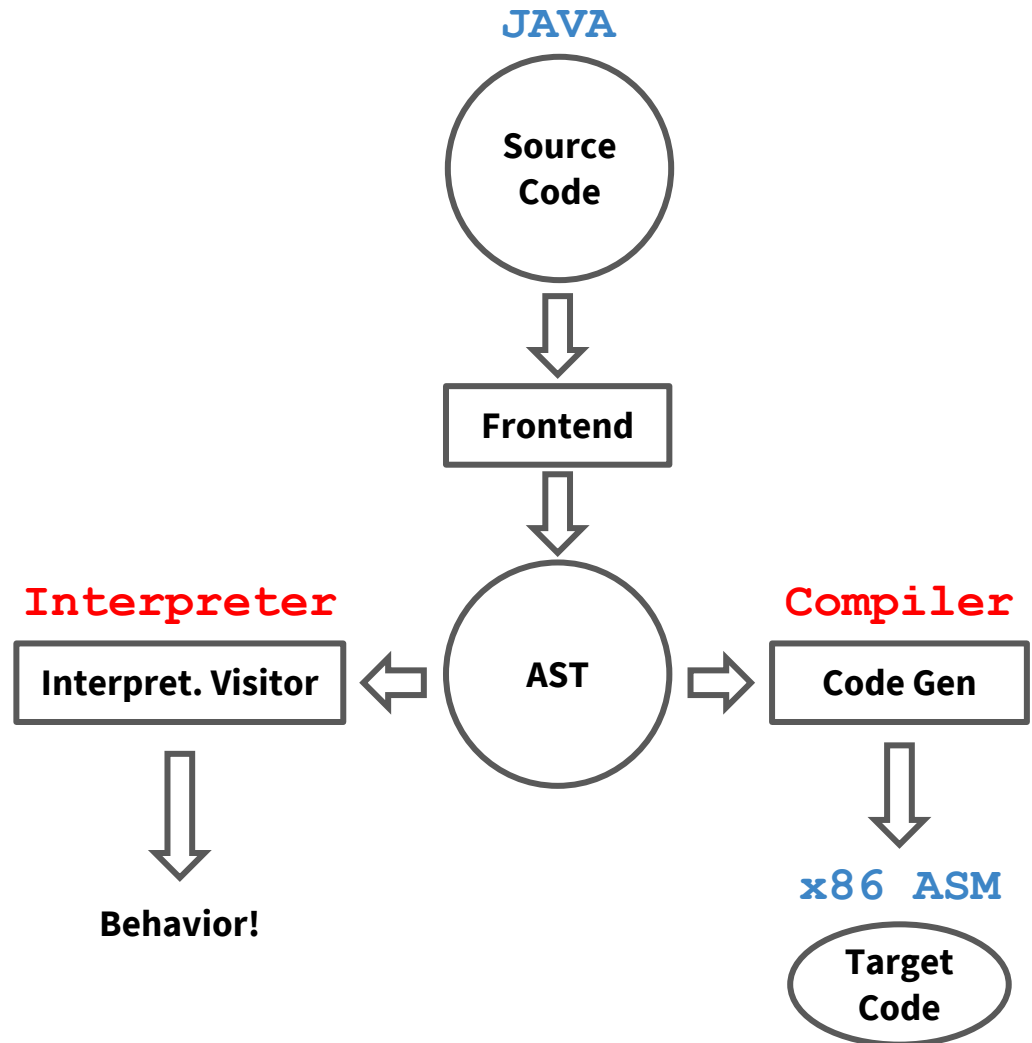
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Implementation

- Frontend is the same!
- Interpreters
 - Execute the AST
- Compilers
 - Translate the AST



Interpreter Demo