

# Section 3: LR Parsing

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Adapted from Autumn 2019

# Announcements

- Scanner is due tonight
  - Be sure to test, push, and tag!
- Every person has 4 late days
  - Up to 2 can be used per assignment
  - Each late day gives an extra 24 hour chunk (including weekend days)
  - We recommend not using your late days this early if possible!
  - **Submitting project components a day late uses a late day from each partner!**

# Get Your LR Jargon On

- Frontier
  - The upper “layer” of the current parse tree (held in the stack)

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# Get Your LR Jargon On

- Frontier
  - The upper “layer” of the current parse tree (held in the stack)
- Sentential Form
  - A string that can be generated at any point in a derivation (can be reached using any number of productions from the start symbol)
- Handle
  - An occurrence of the right side of a production in the frontier that is used in the rightmost derivation to arrive at the current string
  - Given the derivation ...  $\Rightarrow$  **aAbcde**  $\Rightarrow$  **abbcde**, using the production **A ::= b**:
    - The production ‘**A ::= b**’ at index 1 would be a handle of **abbcde**

# A Little Bit More Jargon

- Viable Prefix
  - The prefixes of a right sentential form that do not extend beyond the end of its handle
  - Perhaps less confusing -> the set of prefixes that can appear on the stack of a shift-reduce parser

# A Little Bit More Jargon

- Viable Prefix
  - The prefixes of a right sentential form that do not extend beyond the end of its handle
  - Perhaps less confusing -> the set of prefixes that can appear on the stack of a shift-reduce parser
- Item
  - A marked production (a production with a '.' in it)
    - $[A ::= .XY], [A ::= X.Y], [A ::= XY.]$

# LR (0)

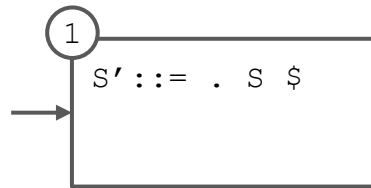


## Problem 1 (On Worksheet)

0.  $S' ::= S \$$
1.  $S ::= a Z$
2.  $S ::= b$
3.  $Z ::= a$
4.  $Z ::= b S$

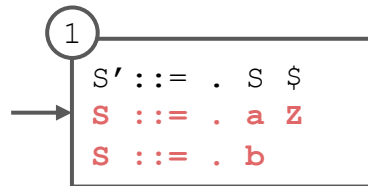
# State Diagram Construction

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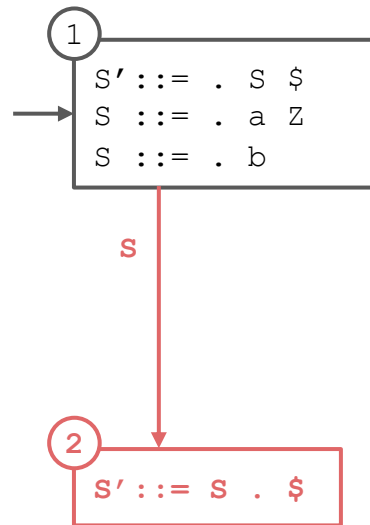
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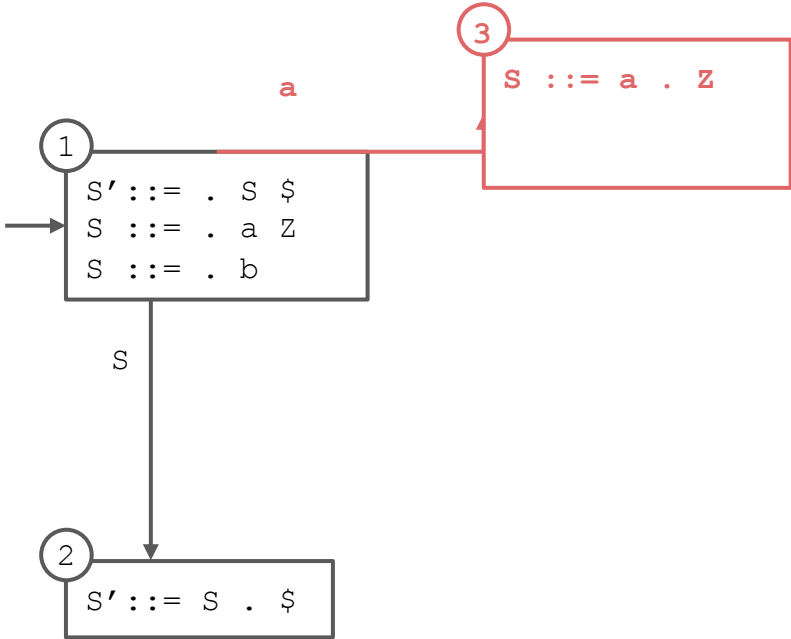
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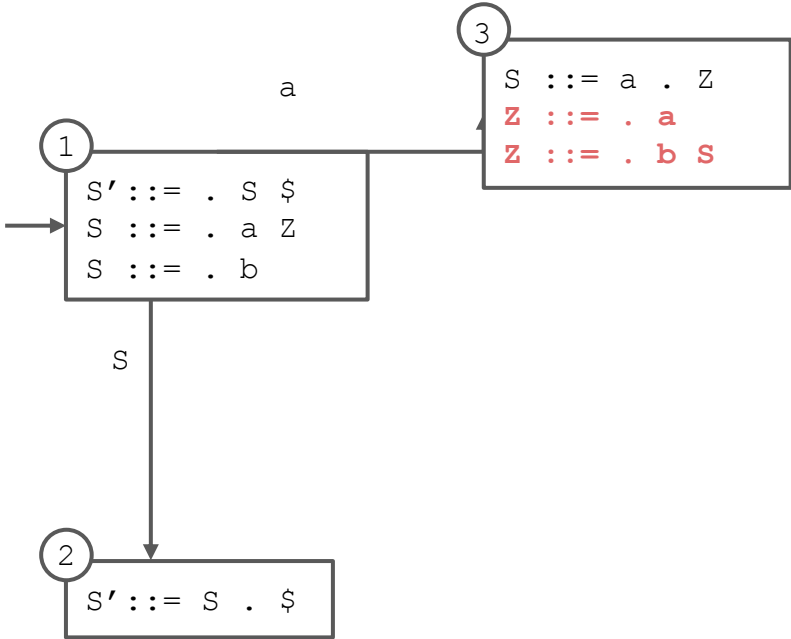
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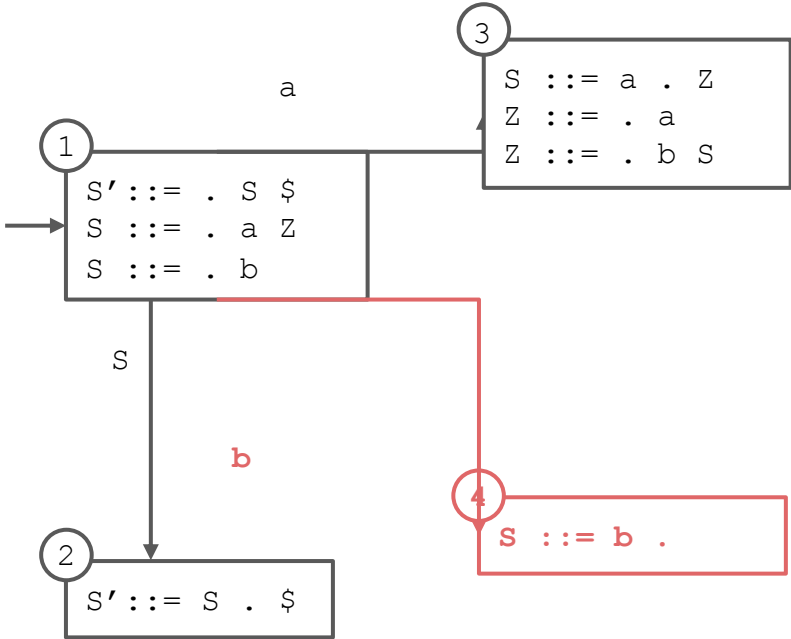
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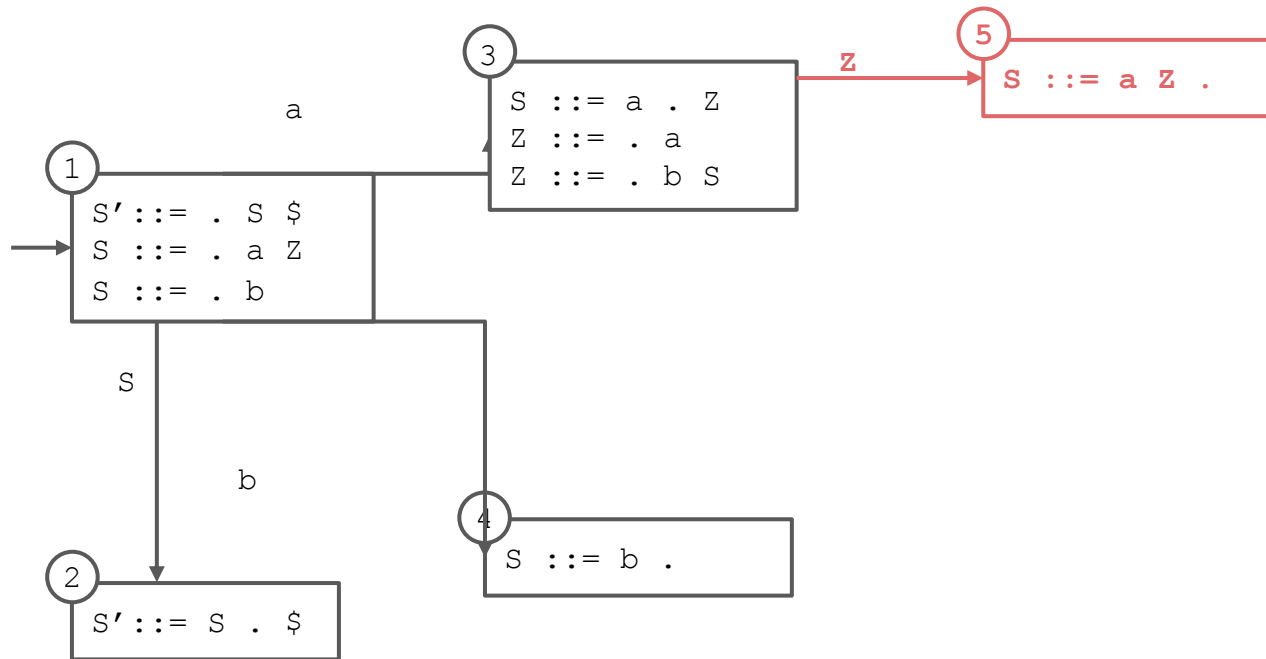
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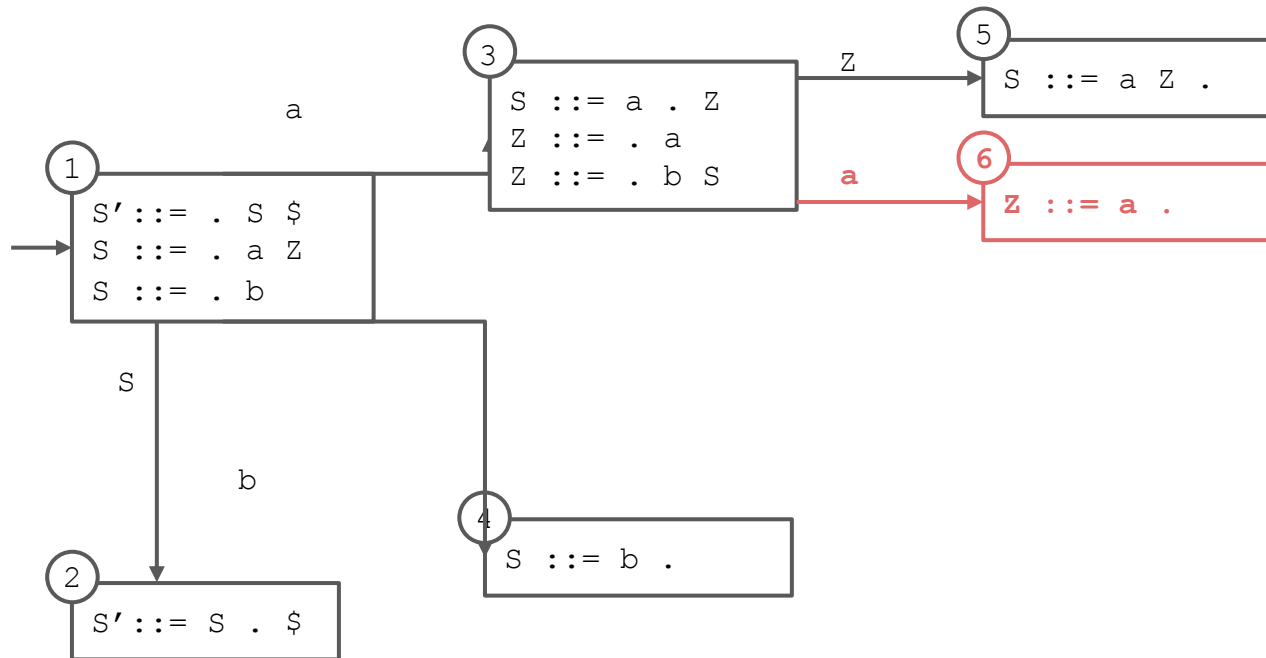
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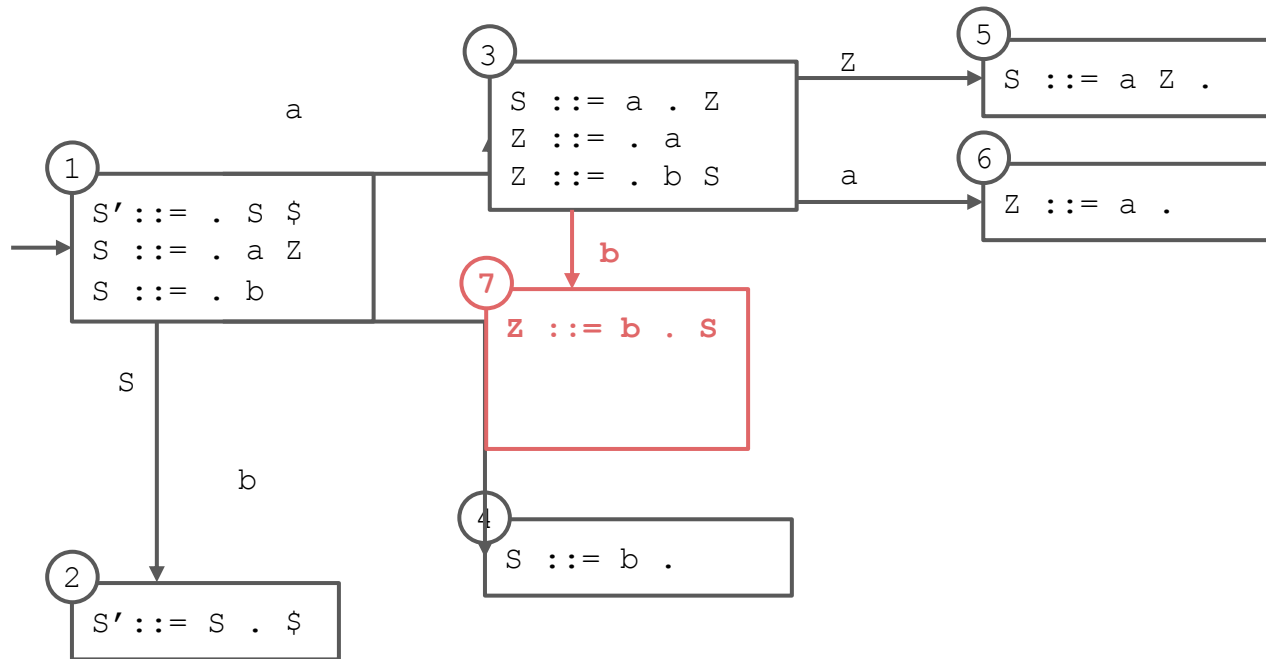
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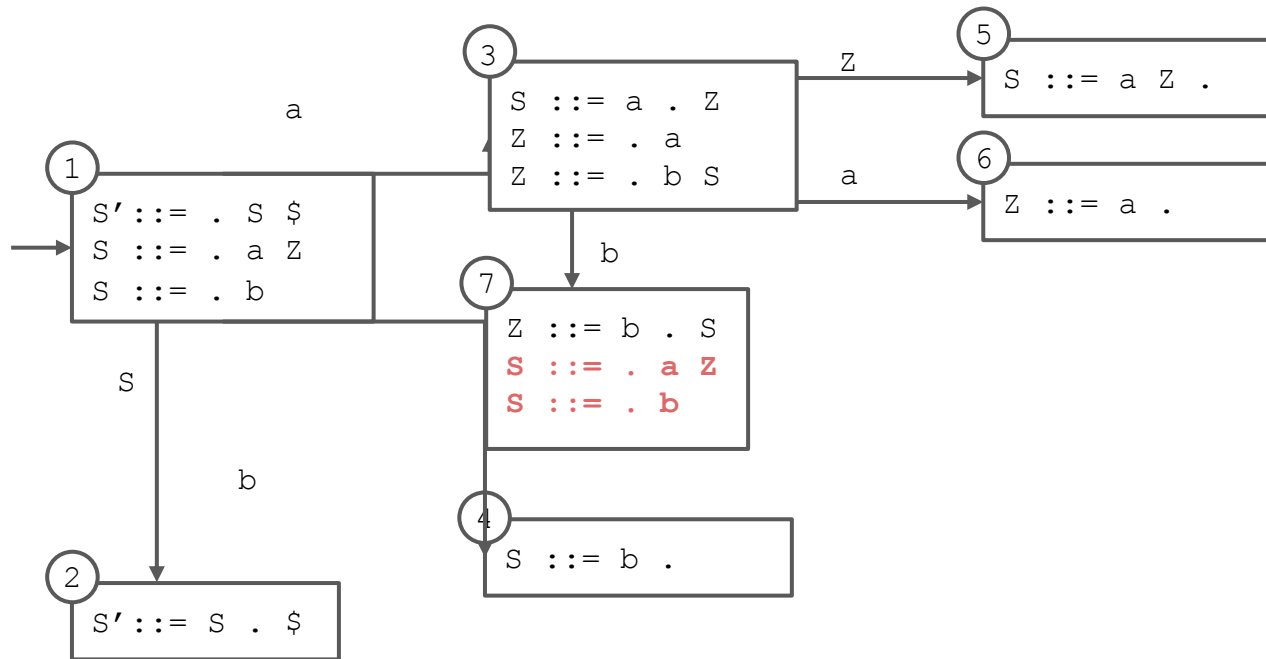
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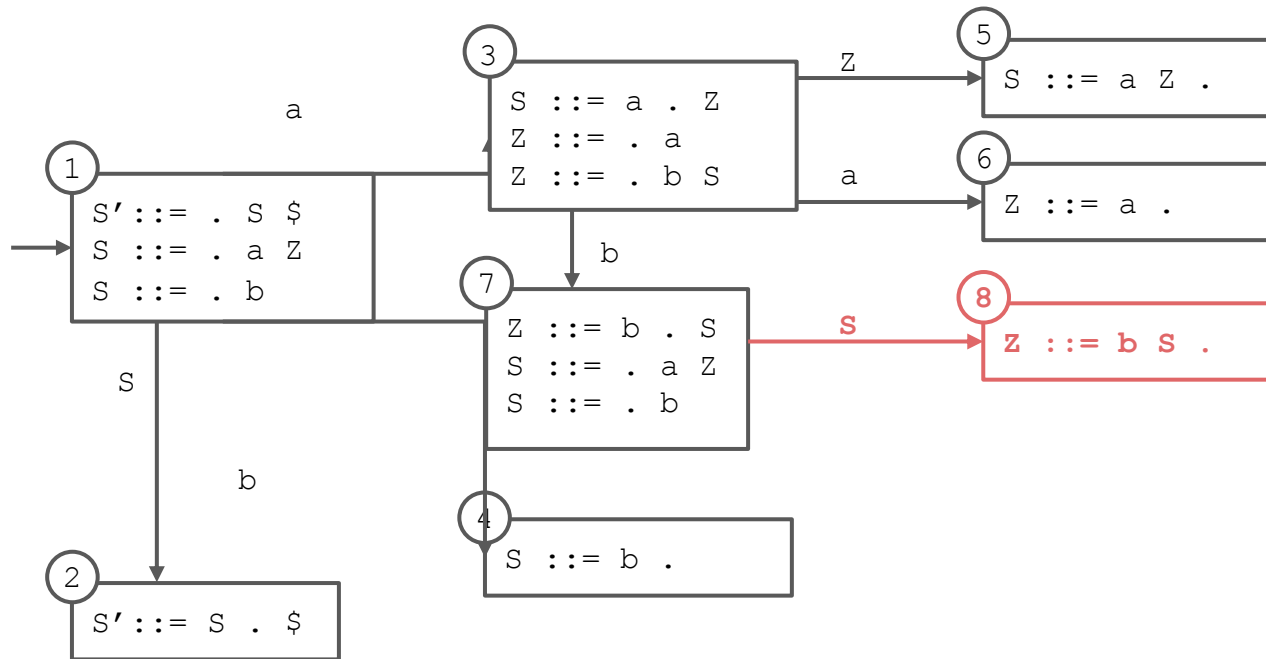
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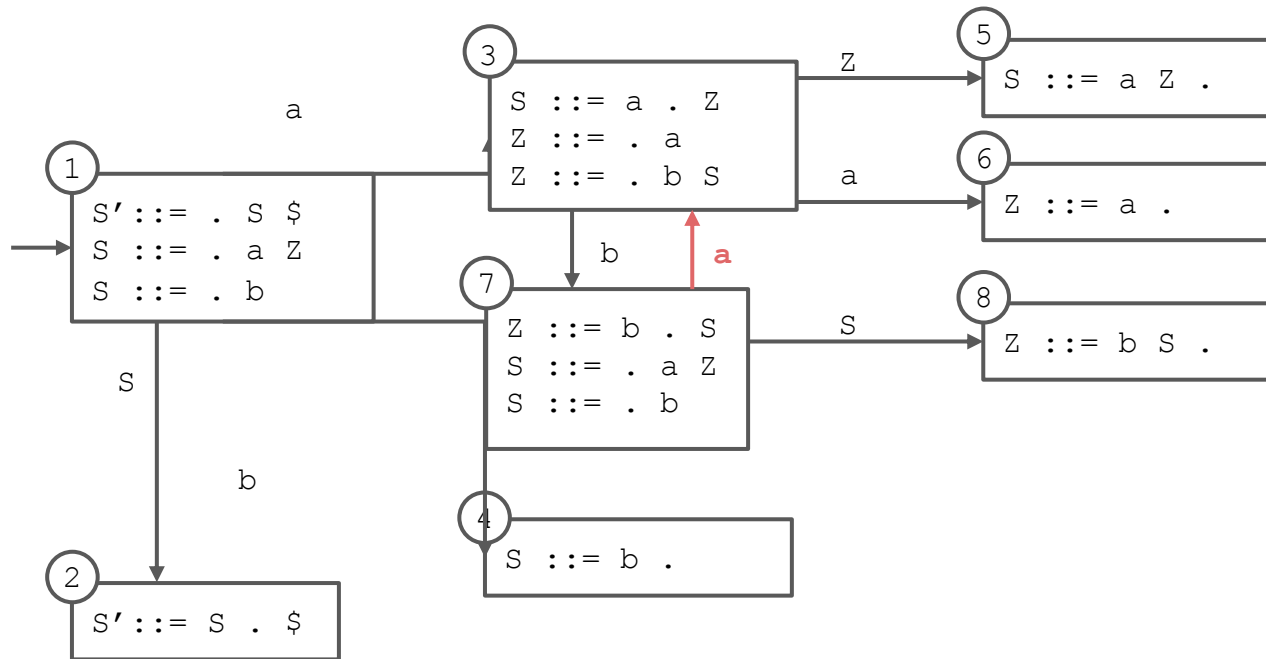
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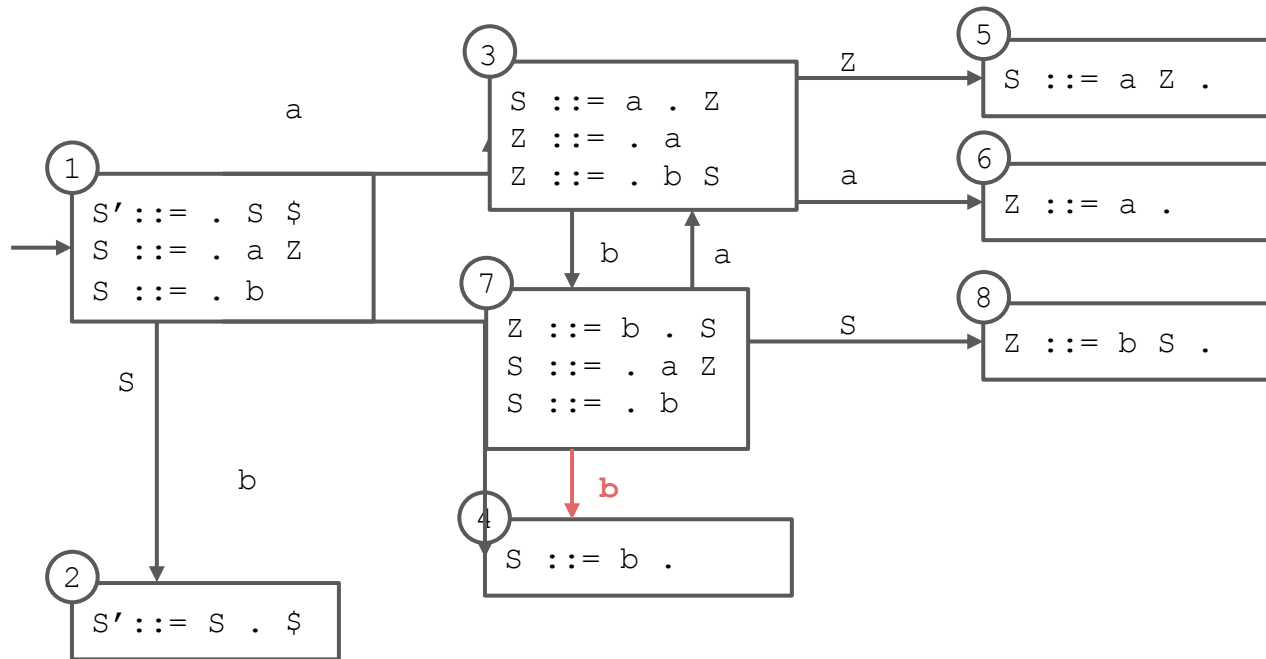
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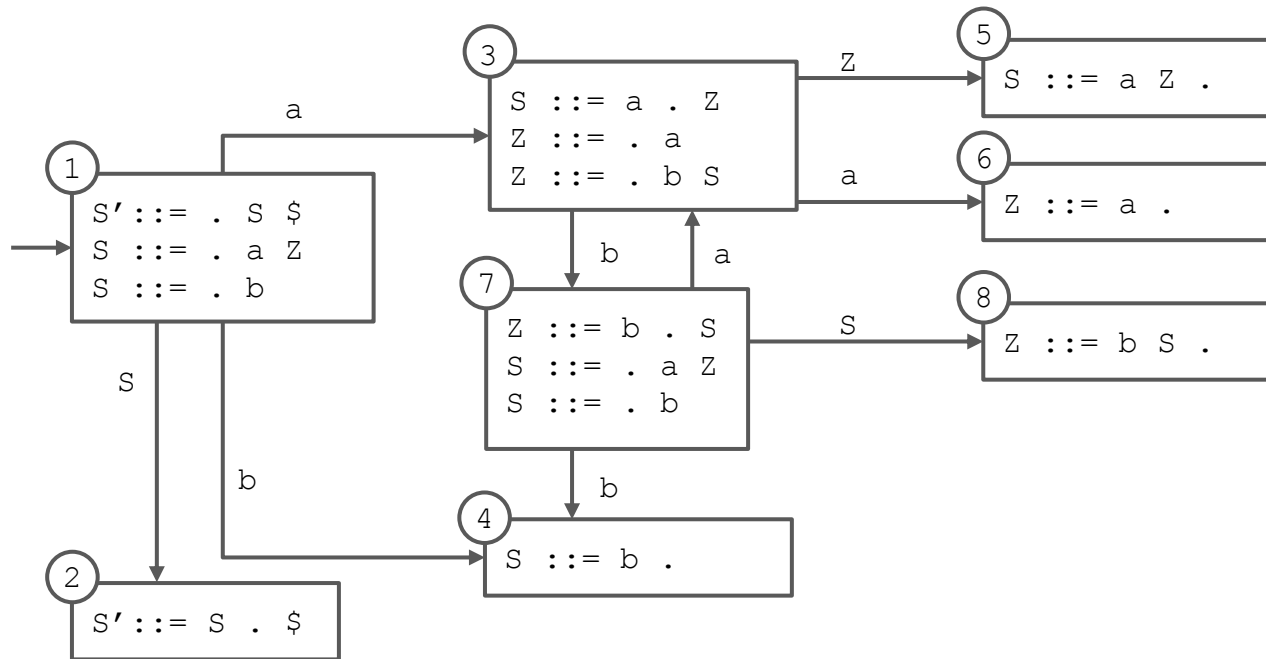
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# Completed State Diagram

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- 1.  $S ::= a Z$
- 2.  $S ::= b$
- 3.  $Z ::= a$
- 4.  $Z ::= b S$



# Converted to Table

## **s# means “shift and enter state #”**

- occurs when there is a transition on a terminal

## **r# means “reduce using production #”**

- occurs when a state contains an item with the location at the end of the right-hand side

## **g# means “go to state #”**

- occurs when there is a transition on a nonterminal

## **acc means “accept”**

- occurs when the start symbol (S here) has been completed and there is no more input

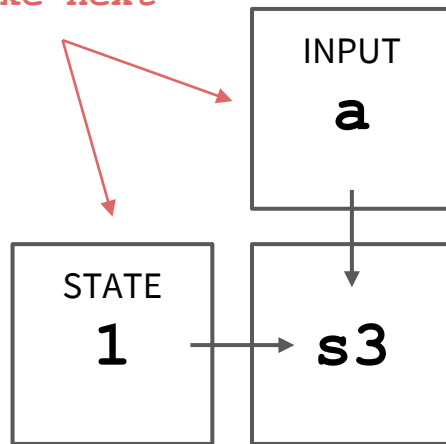
STATE	ACTION			GOTO	
	a	b	\$	S	Z
1	s3	s4		g2	
2			acc		
3	s6	s7			g5
4	r2	r2	r2		
5	r1	r1	r1		
6	r3	r3	r3		
7	s3	s4		g8	
8	r4	r4	r4		

# Parse Trace

STACK	INPUT	ACTION
\$ 1	a b a b b \$	

# Parse Trace

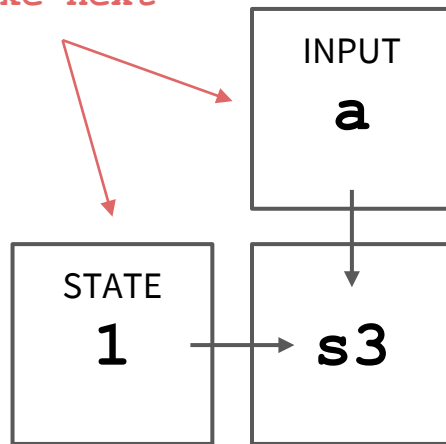
Row and column of table to look up: decides what action to take next



STACK	INPUT	ACTION
\$ 1 \$ 1 a	a b a b b \$ b a b b \$	SHIFT

# Parse Trace

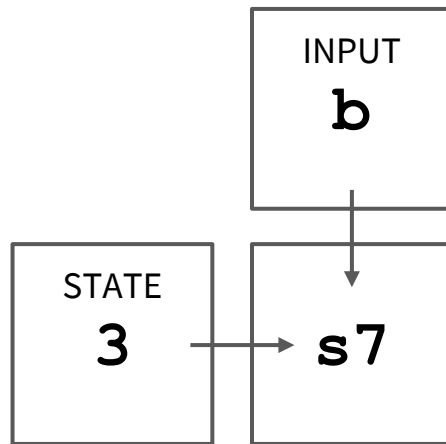
Row and column of table to look up: decides what action to take next



Shift, and enter state 3

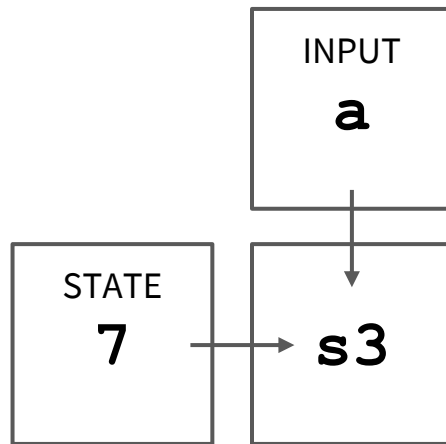
STACK	INPUT	ACTION
\$ 1 \$ 1 a 3	a b a b b \$ b a b b \$	SHIFT

# Parse Trace



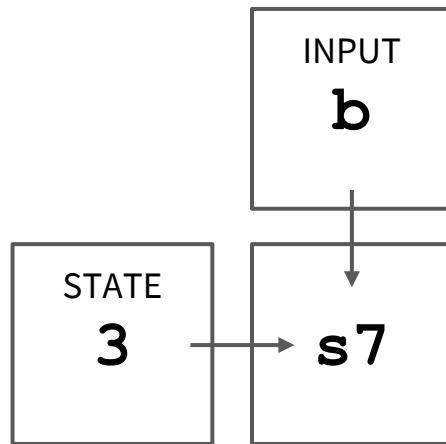
STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	<b>SHIFT</b>
\$ 1 a 3 b 7	a b b \$	

# Parse Trace



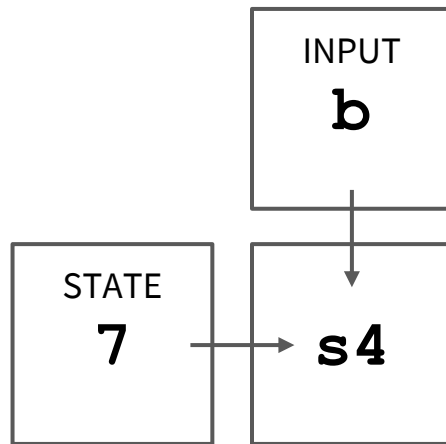
STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	<b>SHIFT</b>
<b>\$ 1 a 3 b 7 a 3</b>	<b>b b \$</b>	

# Parse Trace



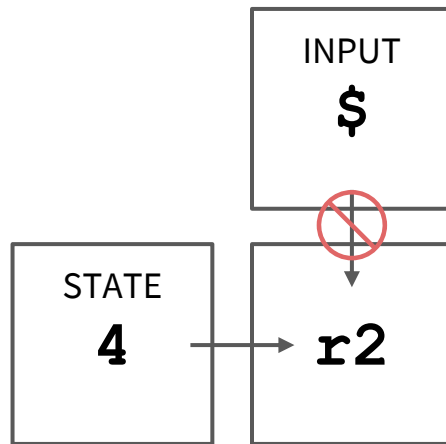
STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	<b>SHIFT</b>
<b>\$ 1 a 3 b 7 a 3 b 7</b>	<b>b \$</b>	

# Parse Trace



STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	<b>SHIFT</b>
<b>\$ 1 a 3 b 7 a 3 b 7 b 4</b>	<b>\$</b>	

# Parse Trace

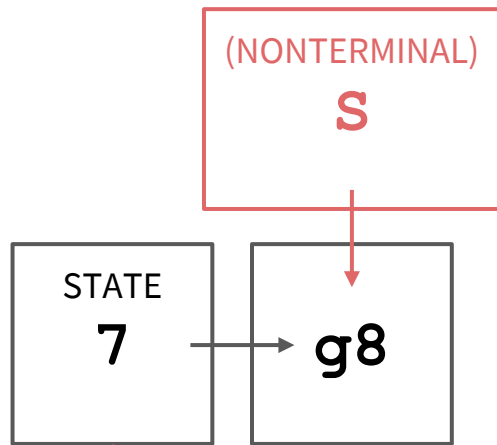


2.  $S ::= b$

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
<b>\$ 1 a 3 b 7 a 3 b 7 S</b>	<b>\$</b>	

For LR(0), the input doesn't technically matter here: state 8 always reduces by production 4 regardless of the next input

# Parse Trace

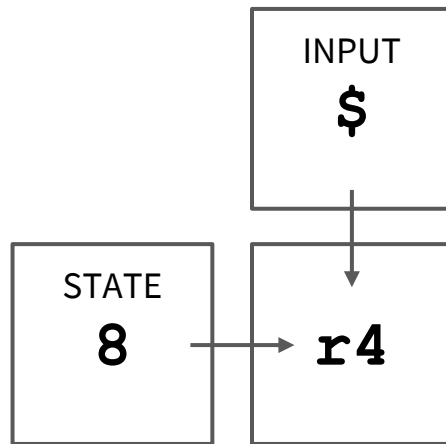


STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S <b>8</b>	\$	

After a reduction, we go back to a previous state on the stack and use the reduced non-terminal to determine what state to GOTO.

This allows the parser to run in  $O(n)$  time, since it doesn't have to re-evaluate the entire stack!

# Parse Trace



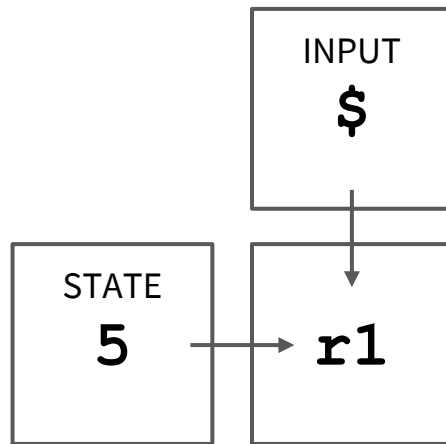
4. Z ::= b S

(and GOTO step:

s3 & Z □ g5)

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	<b>REDUCE</b>
<b>\$ 1 a 3 b 7 a 3 z 5</b>	<b>\$</b>	

# Parse Trace



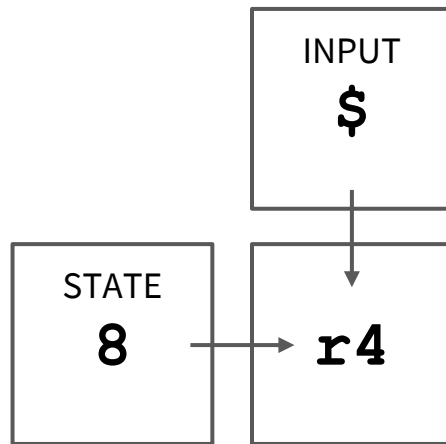
1.  $S ::= a Z$

(and GOTO step:

$s7 \ \& \ S \ \square \ g8$ )

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 Z 5	\$	REDUCE
<b>\$ 1 a 3 b 7 S 8</b>	<b>\$</b>	

# Parse Trace



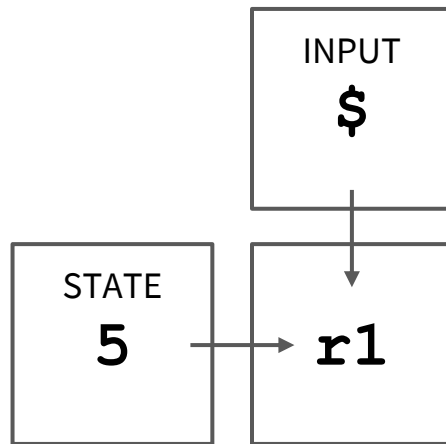
4.  $Z ::= b S$

(and GOTO step:

$s3 \ \& \ Z \ \square \ g5$ )

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 Z 5	\$	REDUCE
\$ 1 a 3 b 7 S 8	\$	<b>REDUCE</b>
<b>\$ 1 a 3 Z 5</b>	<b>\$</b>	

# Parse Trace



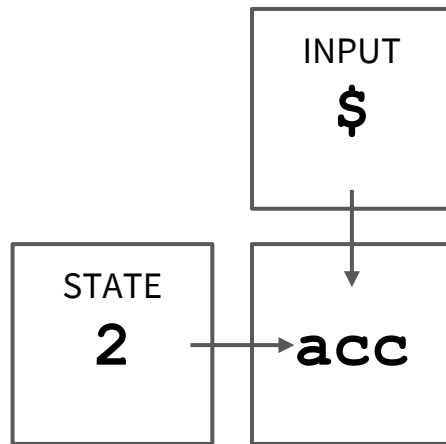
1.  $S ::= a Z$

(and GOTO step:

$s1 \ \& \ S \ \square \ g2$ )

STACK	INPUT	ACTION
\$ 1	a b a b b \$	SHIFT
\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 Z 5	\$	REDUCE
\$ 1 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 Z 5	\$	REDUCE
<b>\$ 1 S 2</b>	<b>\$</b>	

# Parse Trace



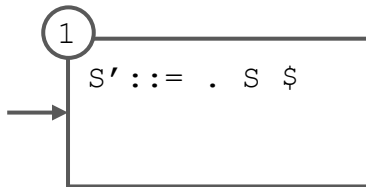
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\$ 1 a 3	b a b b \$	SHIFT
\$ 1 a 3 b 7	a b b \$	SHIFT
\$ 1 a 3 b 7 a 3	b b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7	b \$	SHIFT
\$ 1 a 3 b 7 a 3 b 7 b 4	\$	REDUCE
\$ 1 a 3 b 7 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 b 7 a 3 Z 5	\$	REDUCE
\$ 1 a 3 b 7 S 8	\$	REDUCE
\$ 1 a 3 Z 5	\$	REDUCE
\$ 1 S 2	\$	<b>ACCEPT</b>

## Problem 2 (On Worksheet)

0.  $S' ::= S \$$
1.  $S ::= a X a$
2.  $S ::= b X$
3.  $X ::= c$
4.  $X ::= S c$

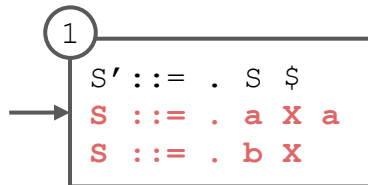
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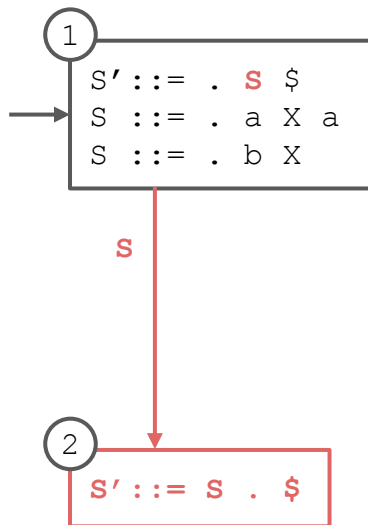
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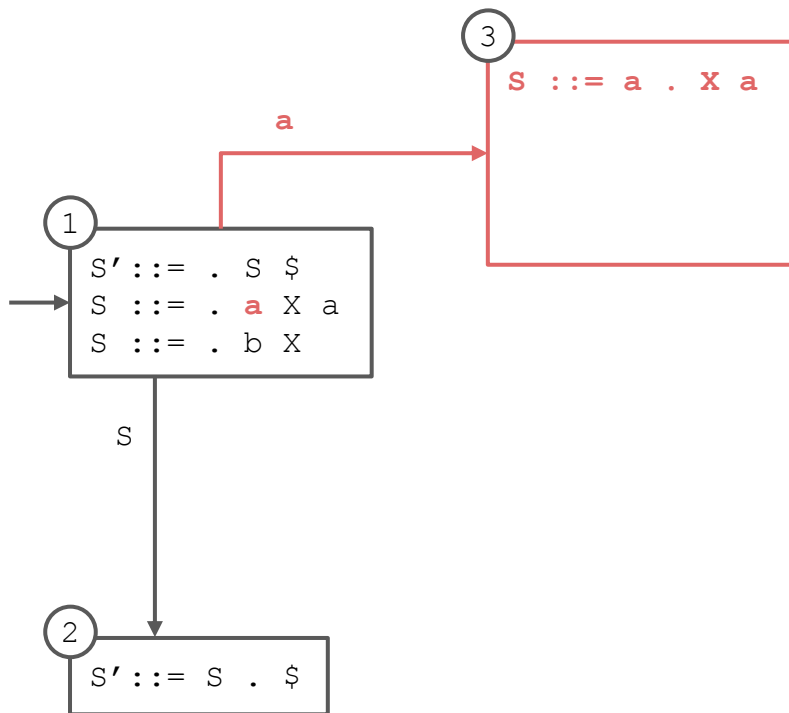
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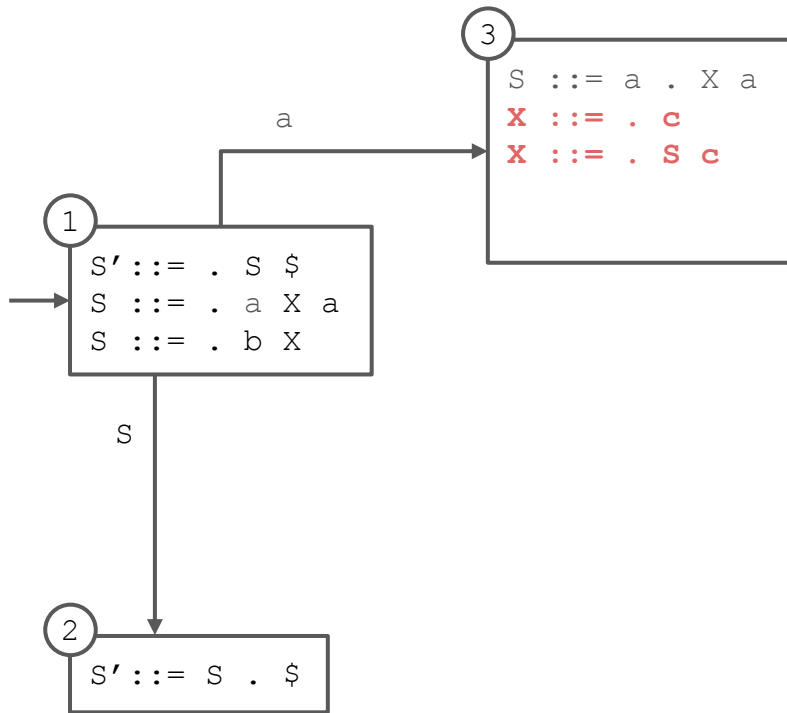
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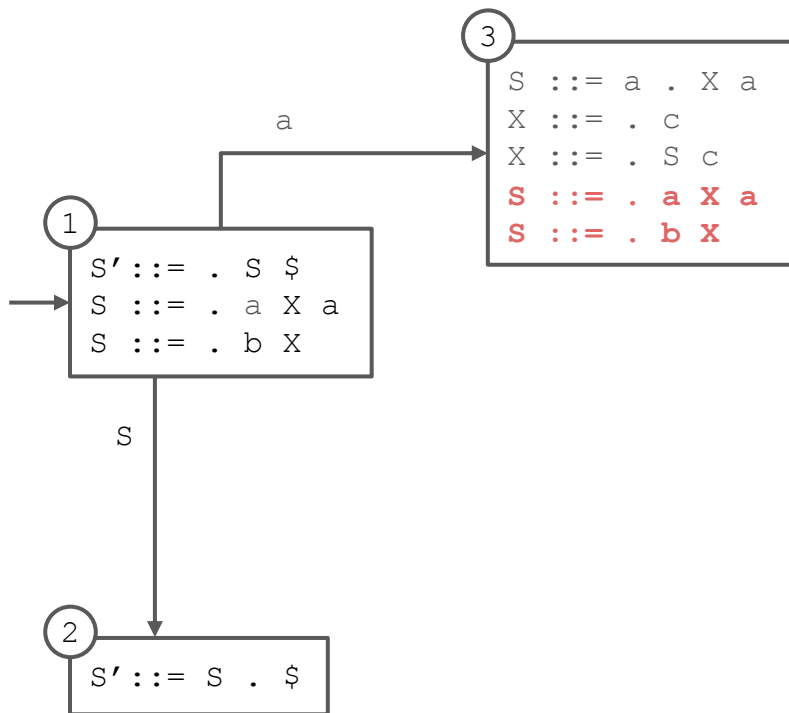
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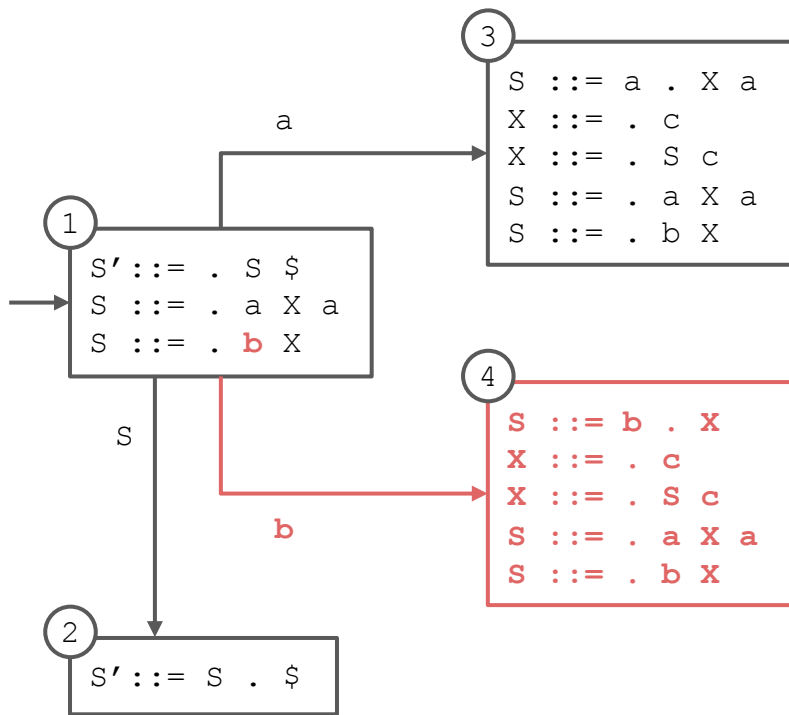
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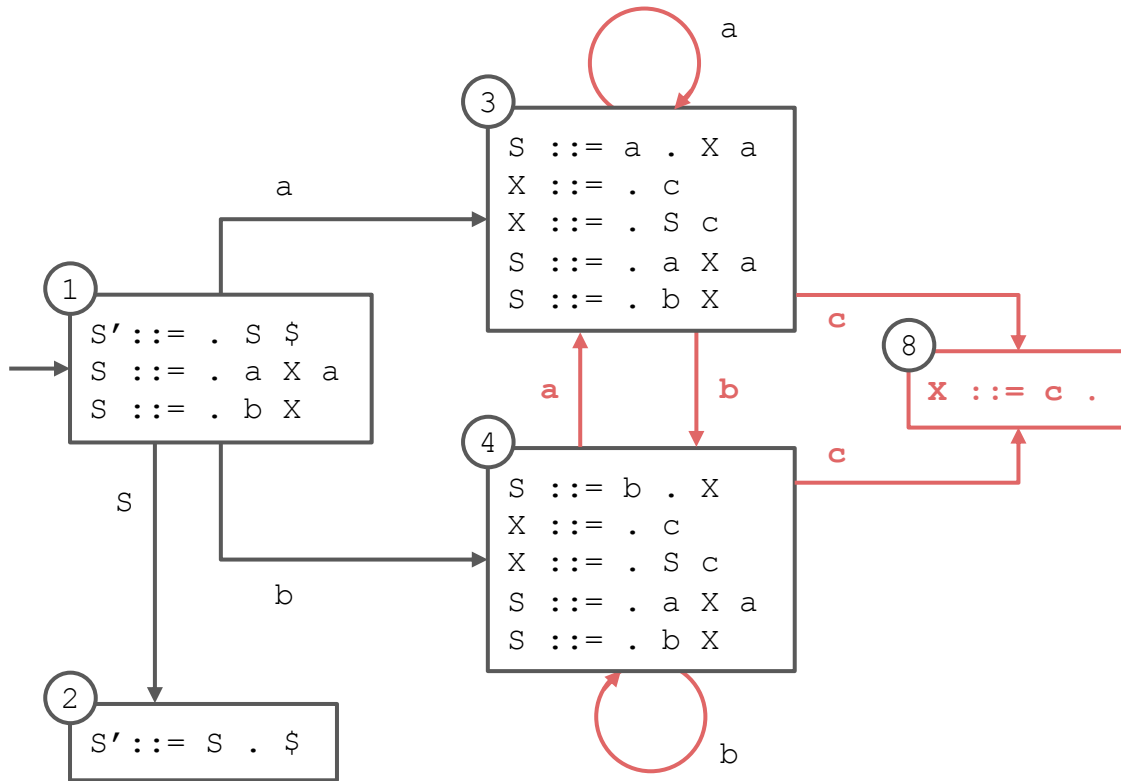
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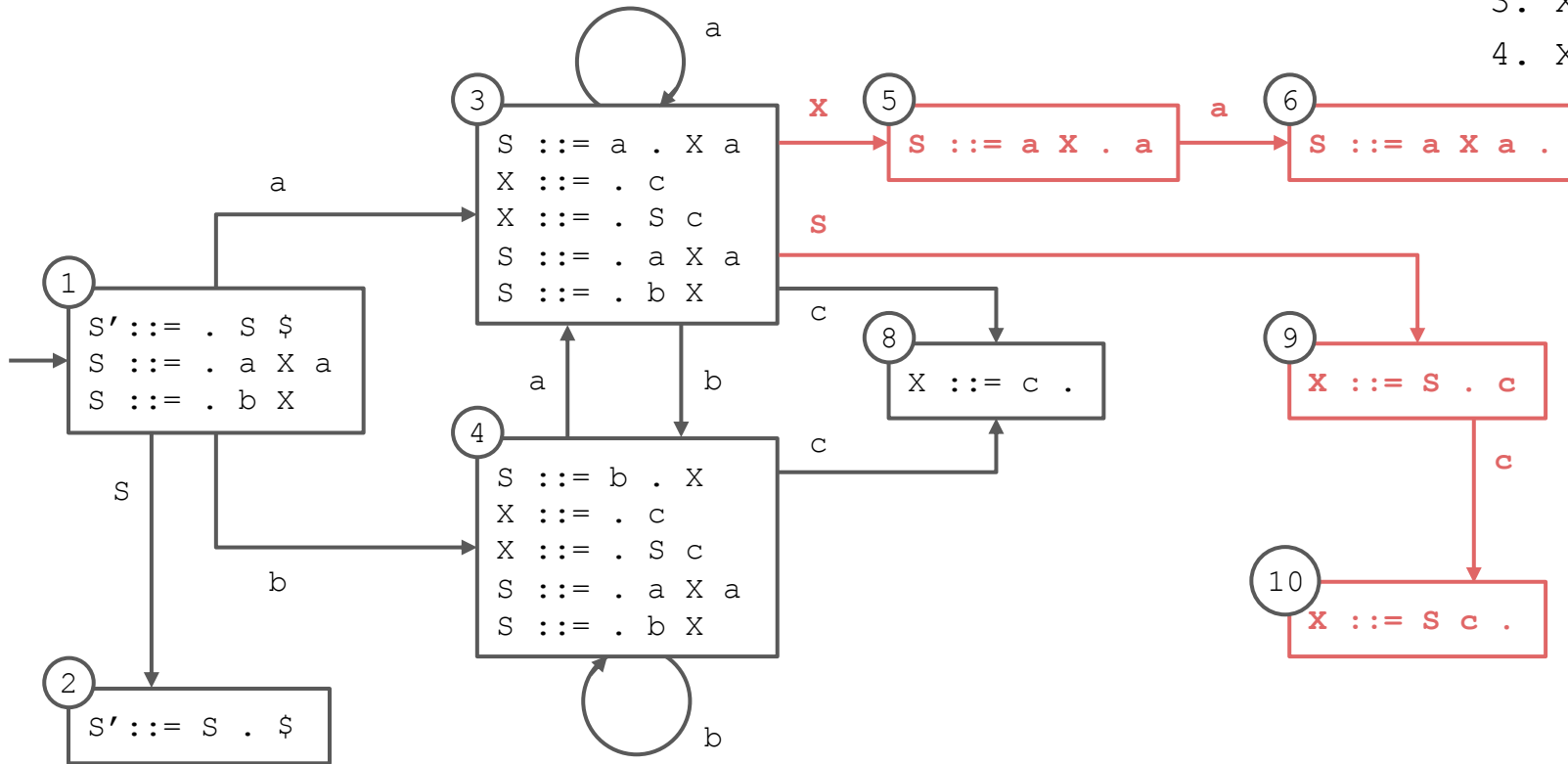
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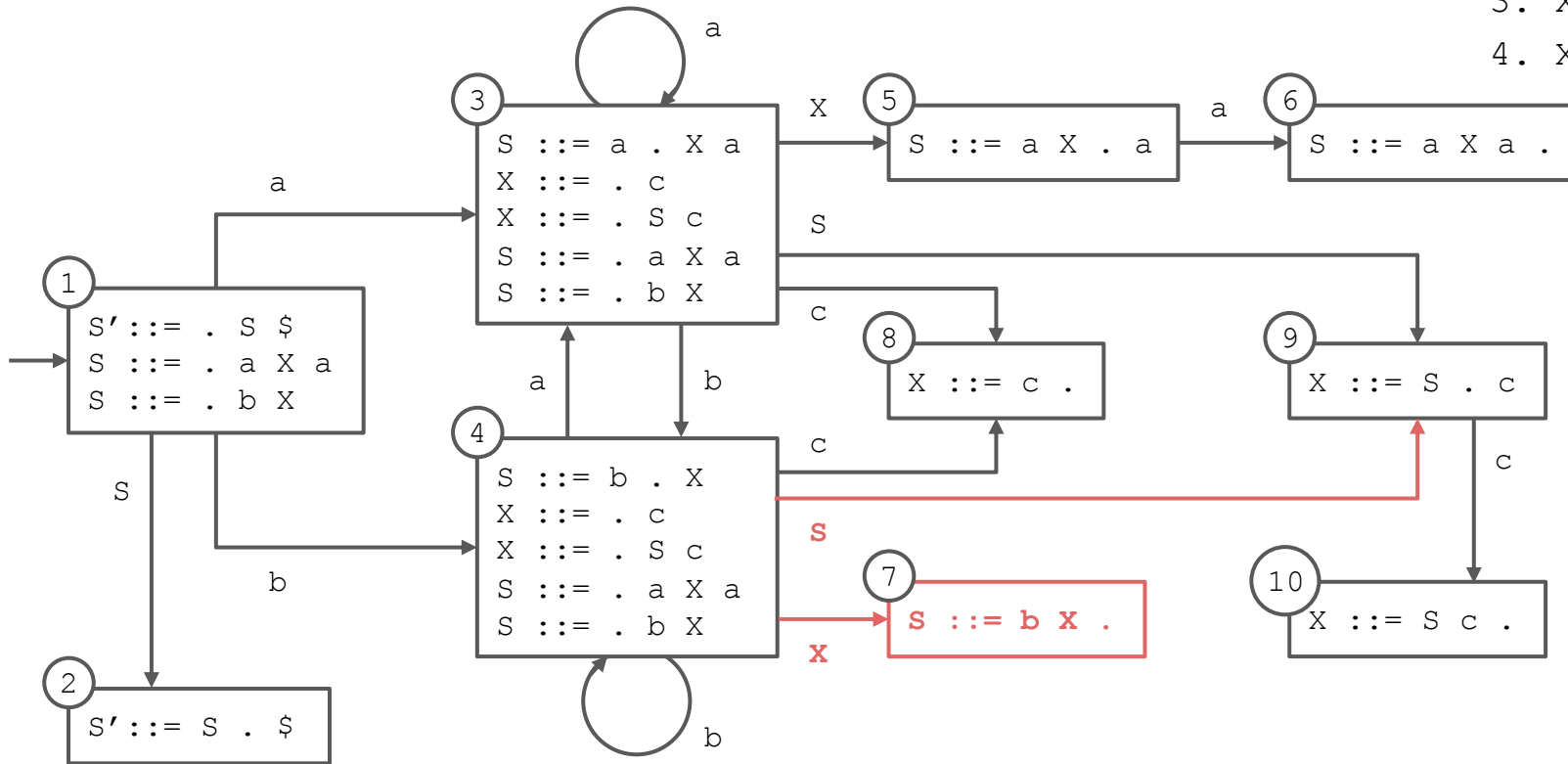
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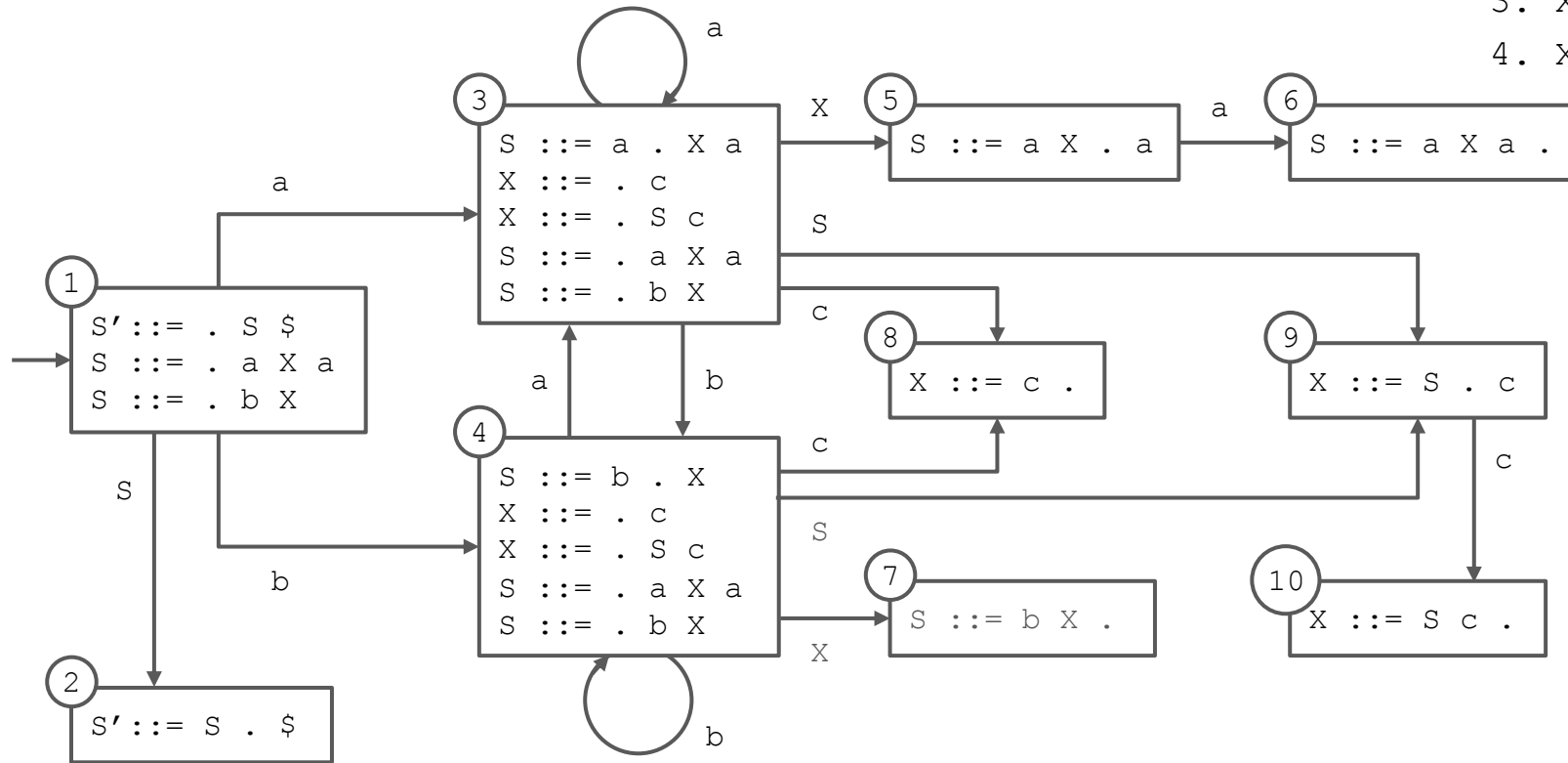
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# Completed State Diagram

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- 1.  $S ::= a X a$
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- 3.  $X ::= c$
- 4.  $X ::= S c$



# Converted to Table

**s# means “shift and enter state #”**

- occurs when there is a transition on a terminal

**r# means “reduce using production #”**

- occurs when a state contains an item with the location at the end of the right-hand side

**g# means “go to state #”**

- occurs when there is a transition on a nonterminal

**acc means “accept”**

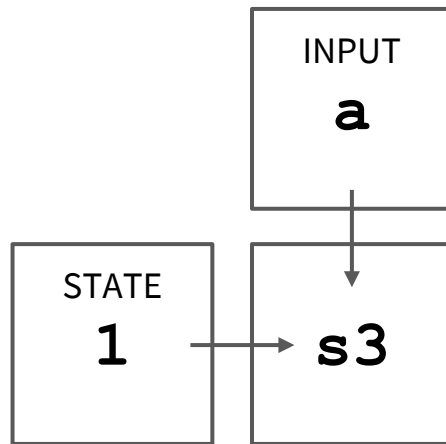
- occurs when the start symbol (S here) has been completed and there is no more input

STATE	ACTION				GOTO	
	a	b	c	\$	S	X
1	s3	s4			g2	
2				acc		
3	s3	s4	s8		g9	g5
4	s3	s4	s8		g9	g7
5	s6					
6	r1	r1	r1	r1		
7	r2	r2	r2	r2		
8	r3	r3	r3	r3		
9			s10			
10	r4	r4	r4	r4		

# Parse Trace

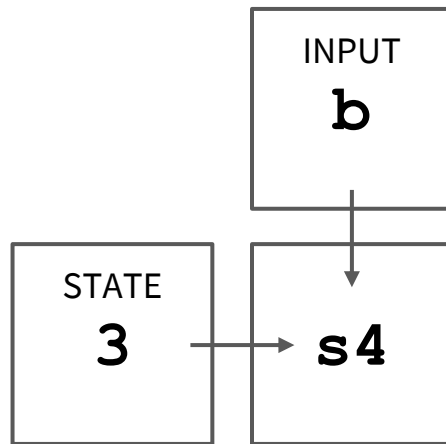
STACK	INPUT	ACTION
\$ 1	a b c c a \$	

# Parse Trace



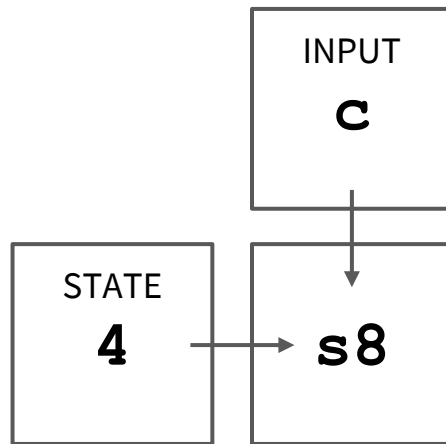
STACK	INPUT	ACTION
\$ 1 \$ 1 a 3	a b c c a \$ b c c a \$	SHIFT

# Parse Trace



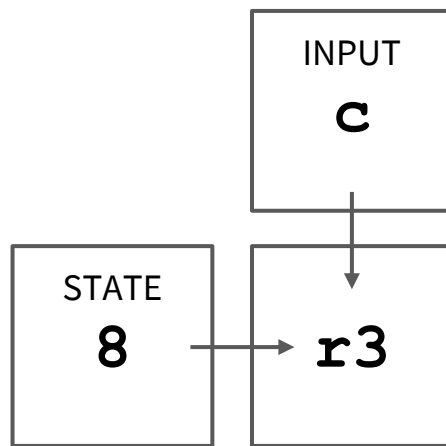
STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	<b>SHIFT</b>
\$ 1 a 3 b 4	c c a \$	

# Parse Trace



STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	<b>SHIFT</b>
<b>\$ 1 a 3 b 4 c 8</b>	<b>c a \$</b>	

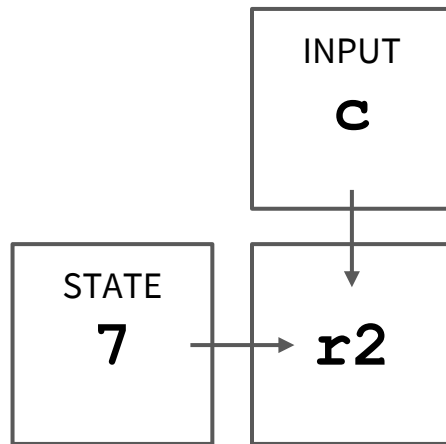
# Parse Trace



3.  $X ::= c$   
 (and GOTO step:  
 $s4 \ \& \ X \ \square \ g7$ )

STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	<b>REDUCE</b>
<b>\$ 1 a 3 b 4 X 7</b>	<b>c a \$</b>	

# Parse Trace



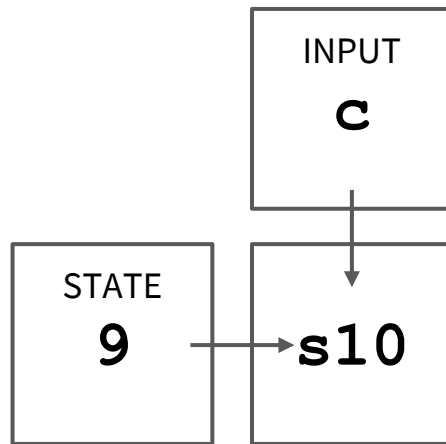
2.  $S ::= b X$

(and GOTO step:

$s3 \ \& \ S \ \square \ g9$ )

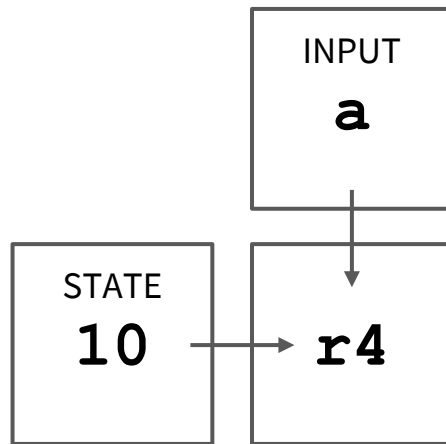
STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	<b>REDUCE</b>
<b>\$ 1 a 3 S 9</b>	<b>c a \$</b>	

# Parse Trace



STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	<b>SHIFT</b>
<b>\$ 1 a 3 S 9 c 10</b>	<b>a \$</b>	

# Parse Trace



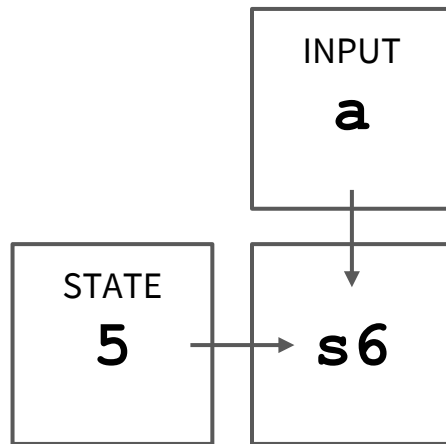
4.  $X ::= S c$

(and GOTO step:

s3 & X  $\square$  g5)

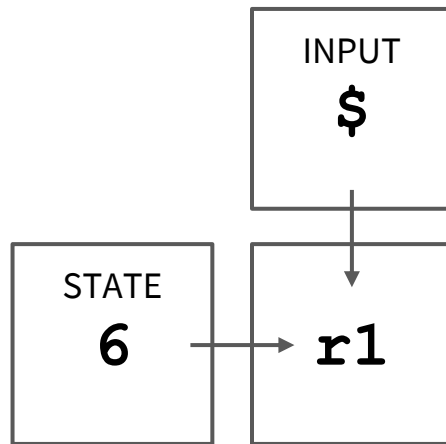
STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	REDUCE
\$ 1 a 3 X 5	a \$	

# Parse Trace



STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	REDUCE
\$ 1 a 3 X 5	a \$	<b>SHIFT</b>
<b>\$ 1 a 3 X 5 a 6</b>	<b>\$</b>	

# Parse Trace



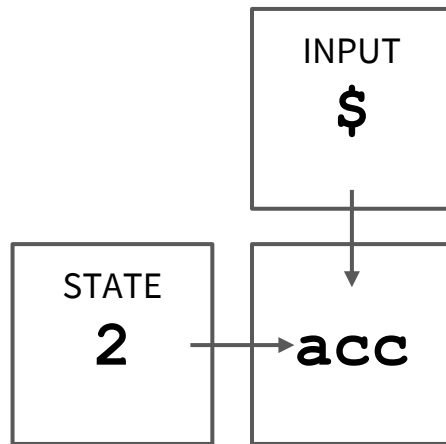
1.  $S ::= a X a$

(and GOTO step:

$s1 \ \& \ S \ \square \ g2$ )

STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	REDUCE
\$ 1 a 3 X 5	a \$	SHIFT
\$ 1 a 3 X 5 a 6	\$	REDUCE
\$ 1 S 2	\$	

# Parse Trace



STACK	INPUT	ACTION
\$ 1	a b c c a \$	SHIFT
\$ 1 a 3	b c c a \$	SHIFT
\$ 1 a 3 b 4	c c a \$	SHIFT
\$ 1 a 3 b 4 c 8	c a \$	REDUCE
\$ 1 a 3 b 4 X 7	c a \$	REDUCE
\$ 1 a 3 S 9	c a \$	SHIFT
\$ 1 a 3 S 9 c 10	a \$	REDUCE
\$ 1 a 3 X 5	a \$	SHIFT
\$ 1 a 3 X 5 a 6	\$	REDUCE
\$ 1 S 2	\$	<b>ACCEPT</b>